Implementation of geometric transformation "patch" and associated passmethod in pyAT

Mengyu Su, Zhe Duan

IHEP, CHINA

With the help of Etienne Forest (KEK,IHEP)

Accelerator Toolbox Workshop

中国科学院高能物理研究所
Institute of High Energy Physics Chinese Academy of Sciences

PART ONE Background

PART TWO What's Patch

PART THREE Examples

PART FOUR Conclusion

- The 3D model of Accelerator
- An element is like a LEGO-block

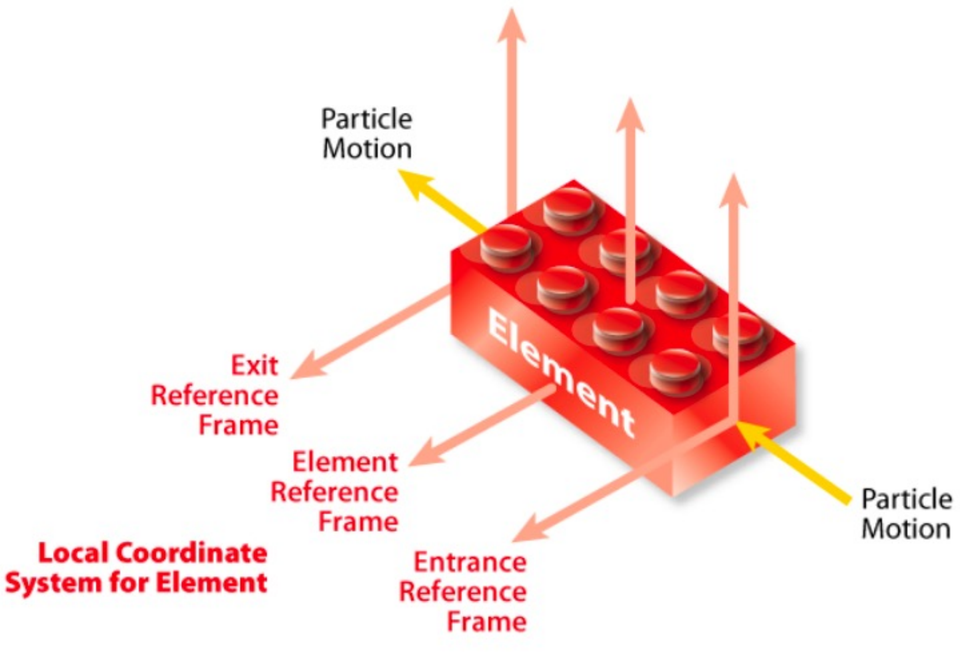

Fig1. LEGO-block element, with reference frames for the entrance, element body, and exit. [1]

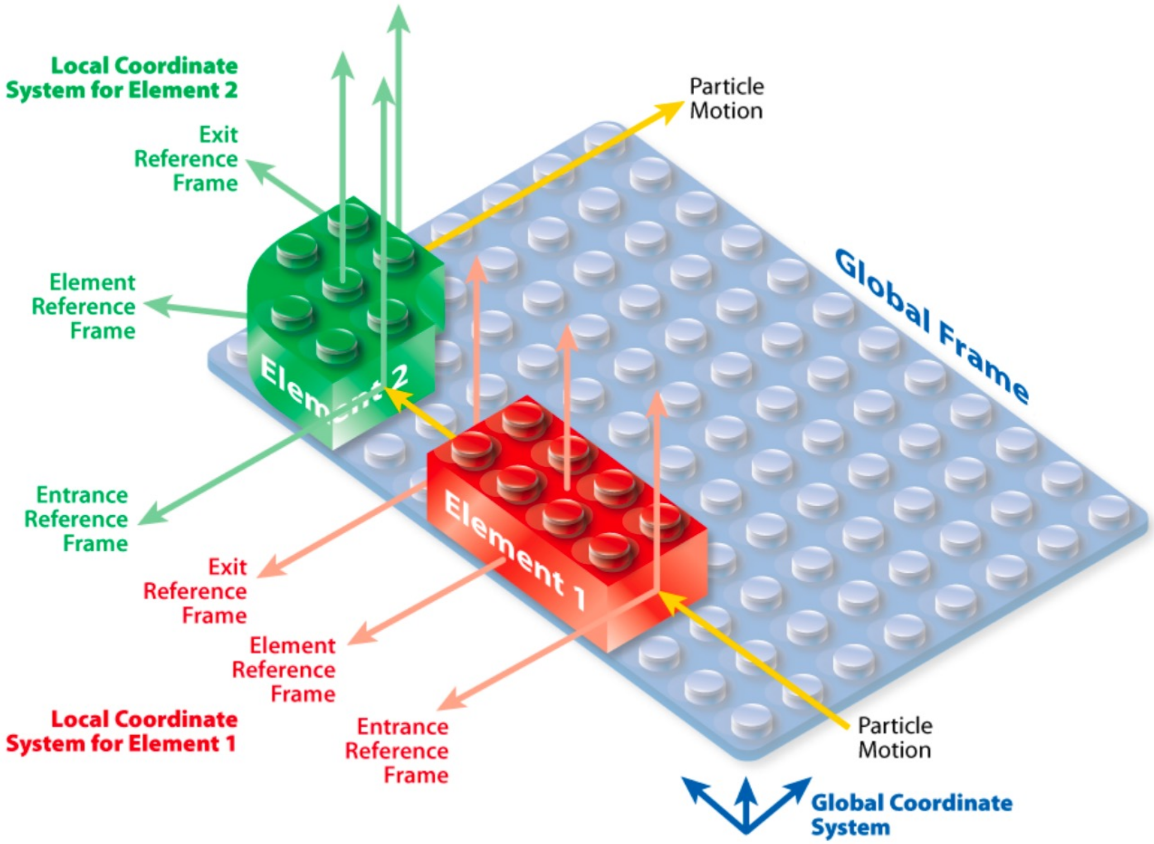

Fig2. LEGO-blocks elements on a base (global frame). [1]

- The 3D model of Accelerator
- What can we do when everything has been moved?
- What if a special design where the magnet is not place on the conventional position, such as Quadruple placed horizontally eccentrically to create a bending magnet which is **not** a kicker?

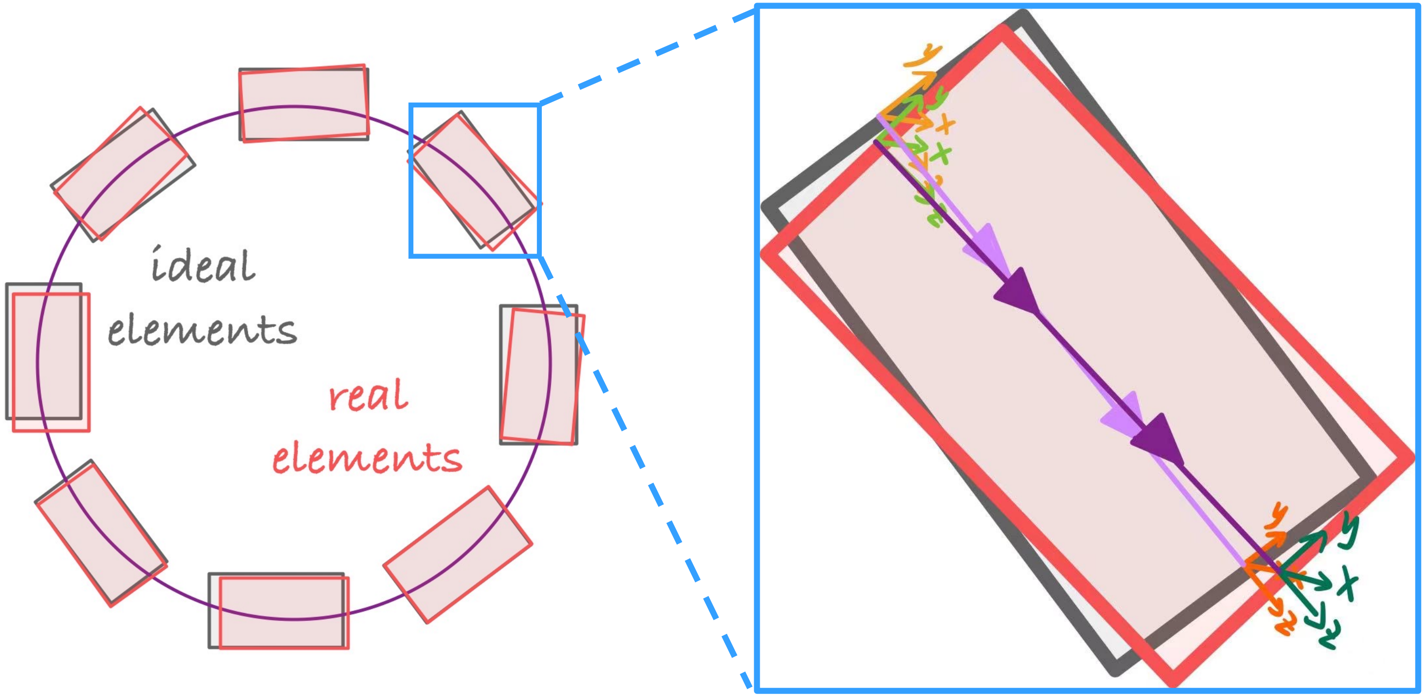

Fig3. The design/ideal accelerator and the real accelerator in tunnel

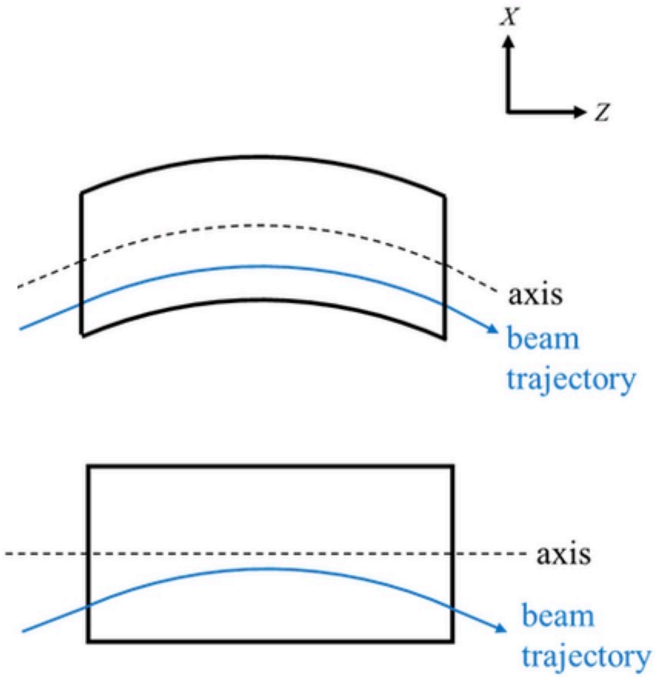

Fig4. A dipole with transverse field gradient by design is often realized by a quadrupole placed with a horizontal offset

- The solution of AT[2]
- Generally, AT provides T1,T2 and R1,R2 field in most PassMethods to describe translation or rotations of the 6D coordinates; Concerning magnetic fields errors, the structures PolynomB and PolynomA provide full access to all magnetic components.

- Strong association between error (or non-conventional position) and the element
- Addresses the effect of error on phase space without changing the model of real space

- A more universal solution

The concept of 'Patch' was introduced in PTC by Etienne Forest [1,4]
It's easy to add a 'Patch' class in pyAT for python's object-oriented program

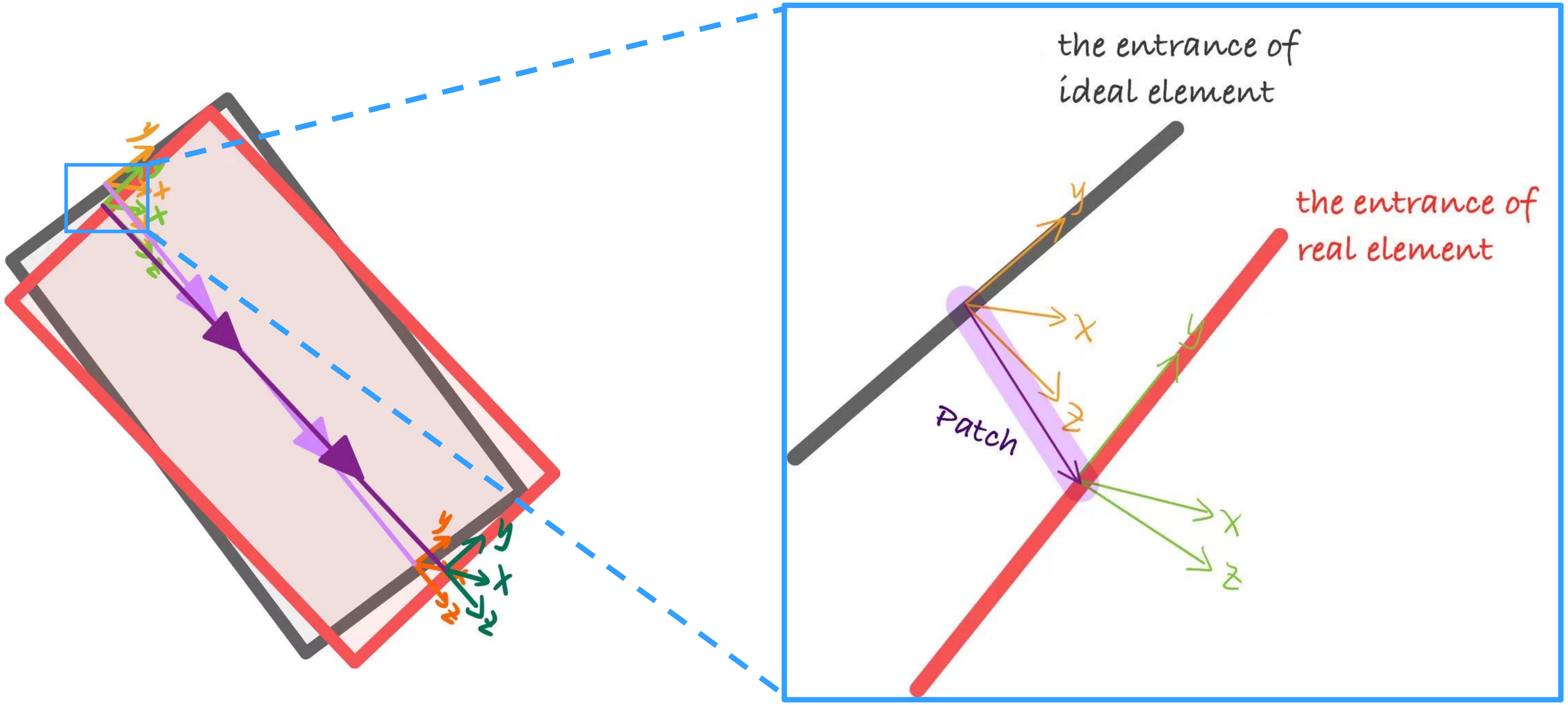

Fig5. The solution using Patch

- A new element —— Patch
- Translation & Rotation
- Misalignment can be also described by Patches. (Not true in PTC as we pointed out)

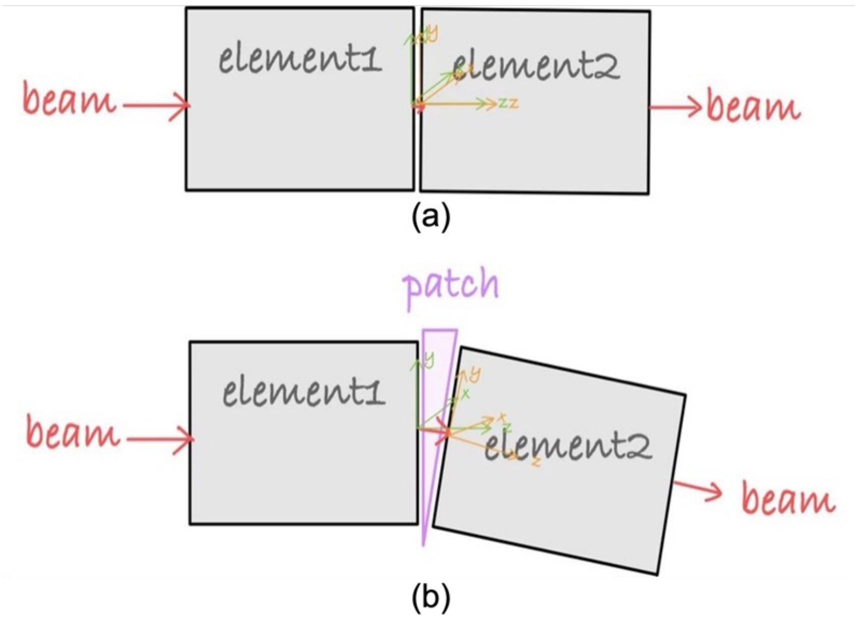

Fig6. Patch is a new element

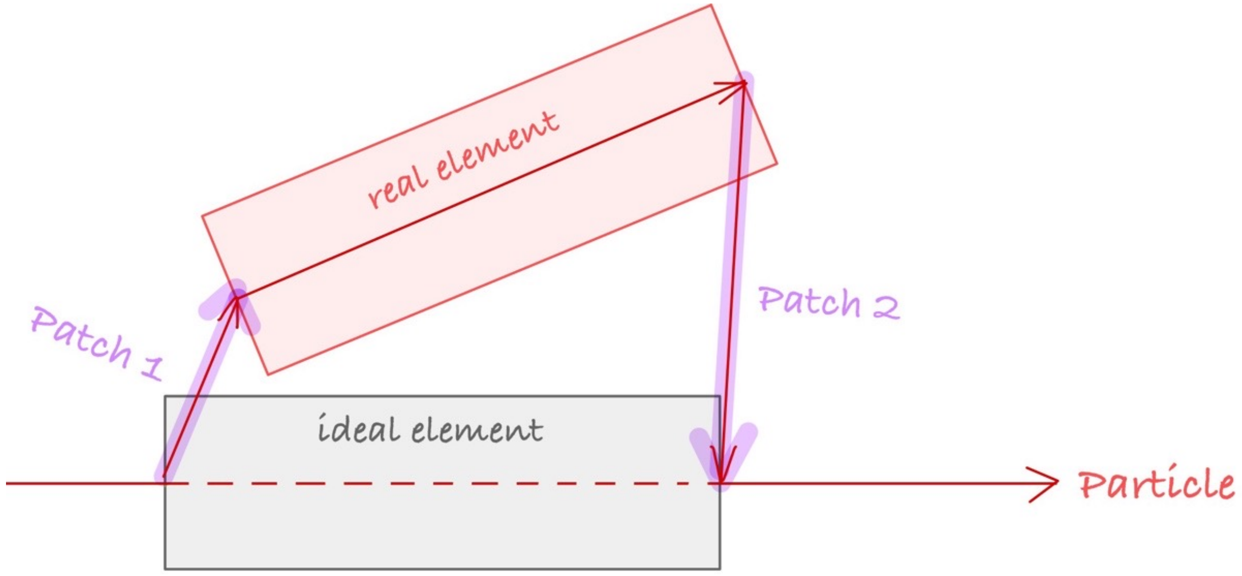

Fig7. Misalignment described by Patches

- Definition of geometric part[1,3,4,5,6]
  - In local coordinate system
  - Translation first
  - Then rotation （Z-Y-X intrinsic rotations）
    - PS. intrinsic rotations = rotated axis
      extrinsic rotations = static/fixed axis

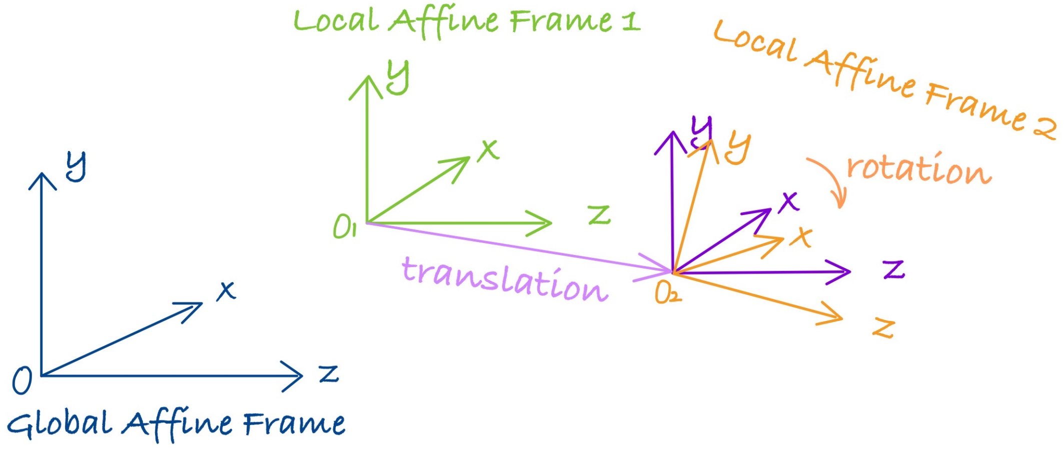

Fig8. Global Affine Frame, Local Affine Frame 1 and Local Affine Frame 2

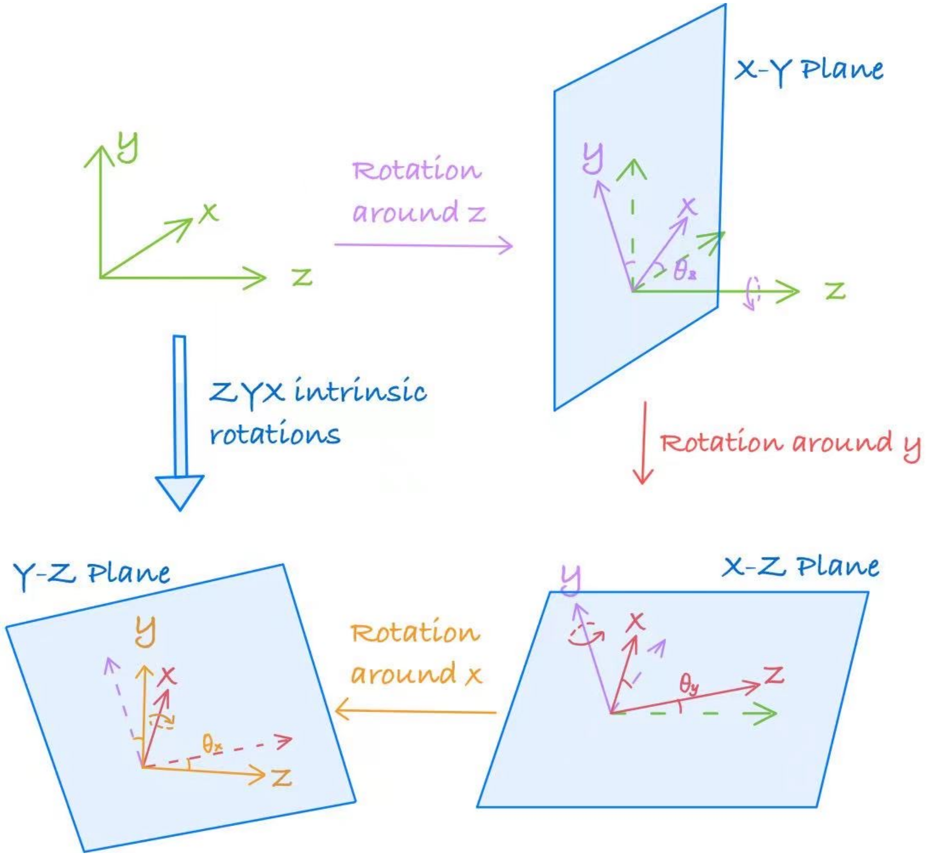

Fig9. Z-Y-X intrinsic rotations

- Euler Angle and Rotation Matrix

$$R = Z(\theta_z) * Y(\theta_y) * X(\theta_x) (1)$$

Where

$$X(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix} (2.a)$$

$$Y(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_x) \end{bmatrix} (2.b)$$

$$Z(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} (2.c)$$

properties: (use column vectors always)

$$T_G = R_1 T_1 (3.a)$$

$$R = A_{21} = A_{1G}^{-1} A_{2G} = R_1^{-1} A_{2G} (3.b)$$

$$R' = R_1 R R_1^{-1} (3.c)$$

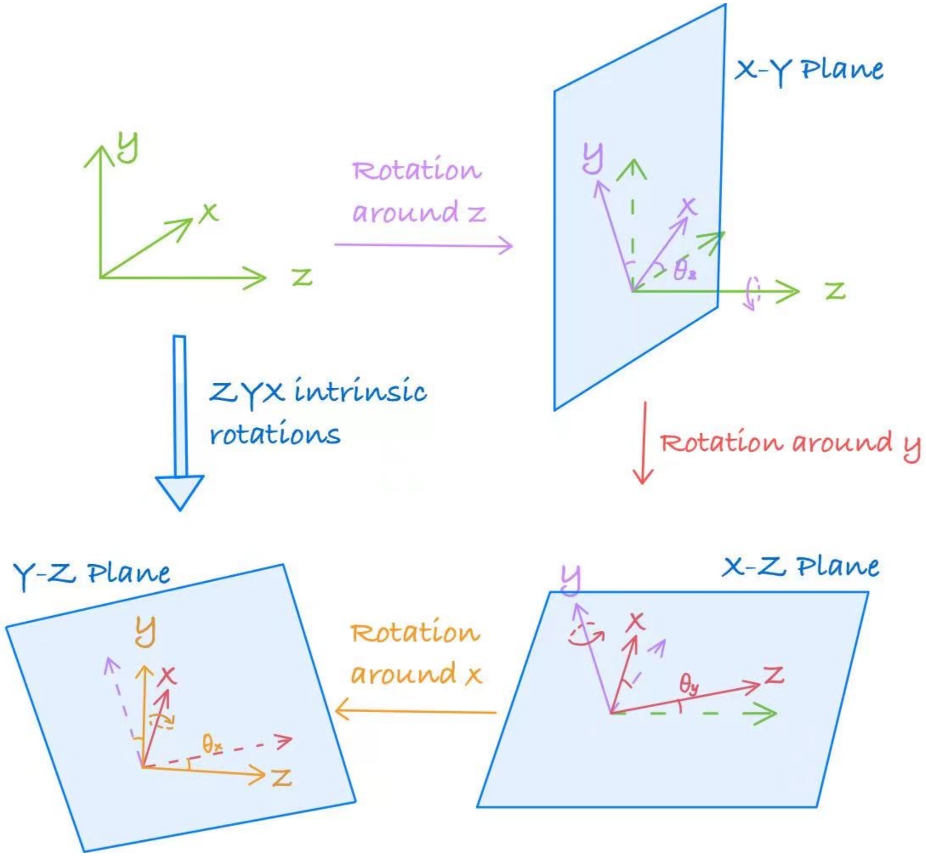

Fig9. Z-Y-X intrinsic rotations

- Implementation in code

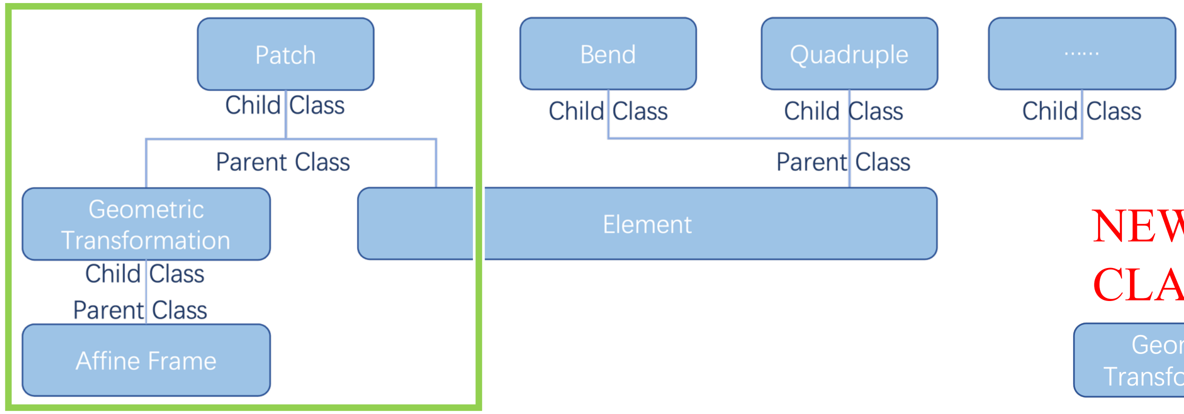

Fig10. The inheritance of each element class

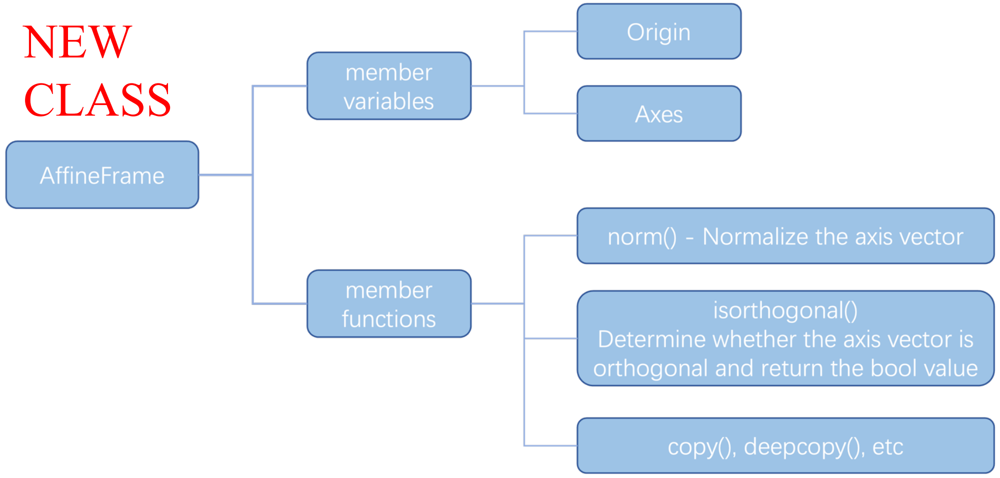

Fig11. AffineFrame class

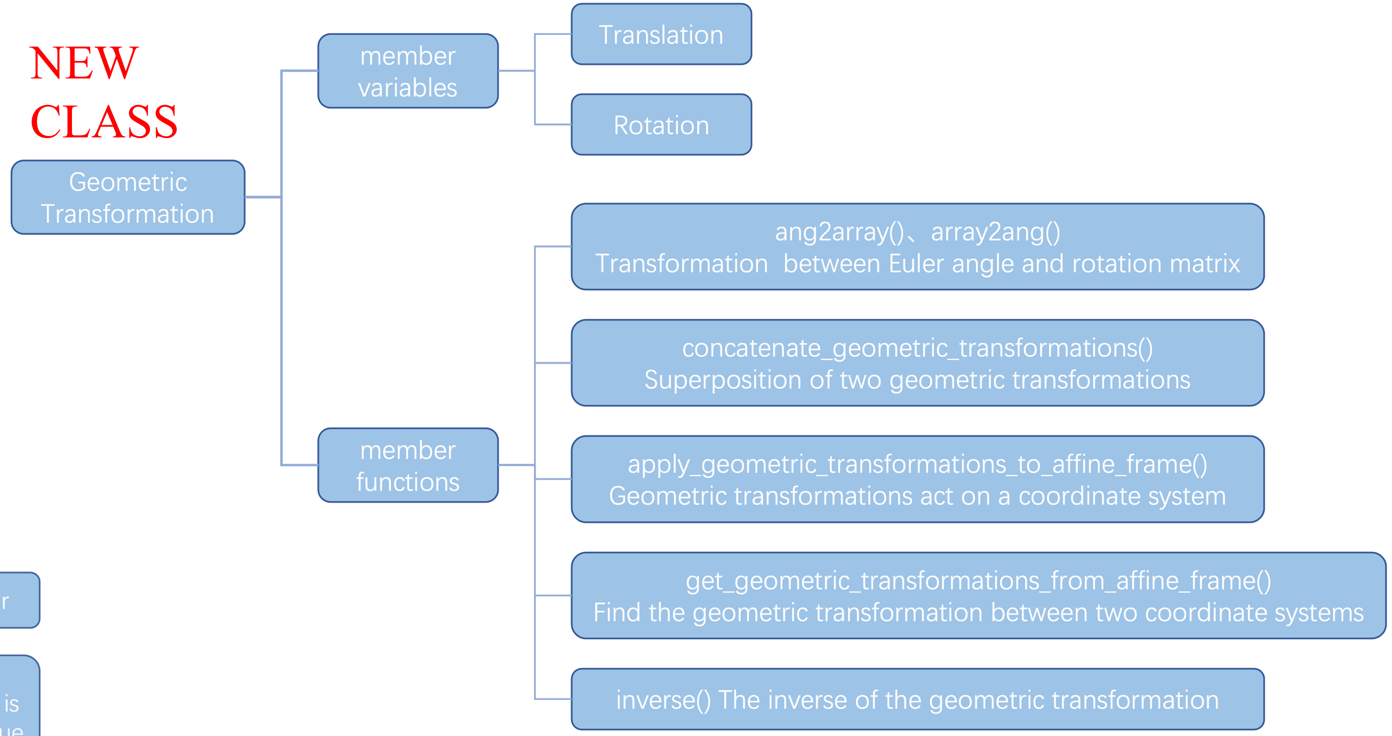

Fig12. GeometricTransformation class

- Implementation in code

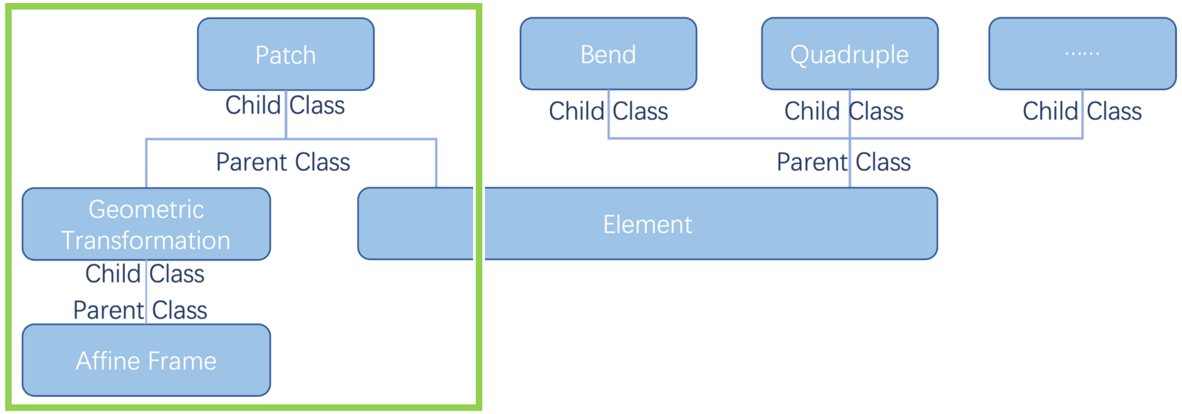

Fig10. The inheritance of each element class

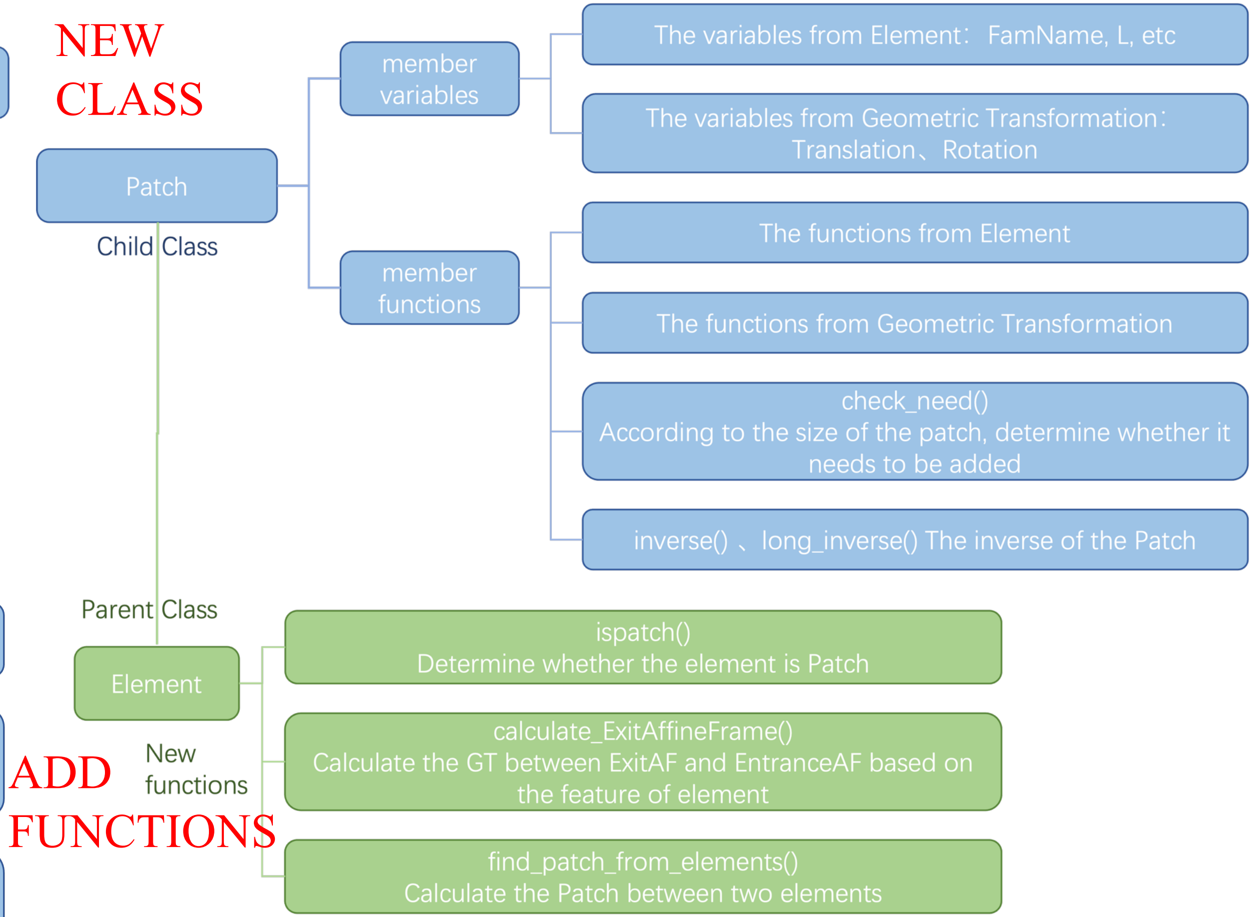

Fig13. Patch class and new functions in Element class

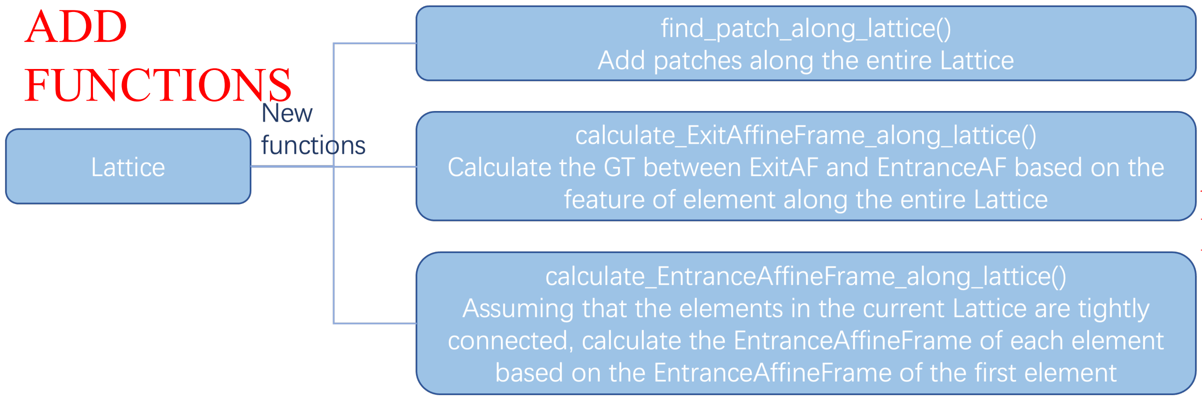

Fig14. New functions in Lattice class

- Passmethod[8,9]
- Coordinates and the Hamiltonian

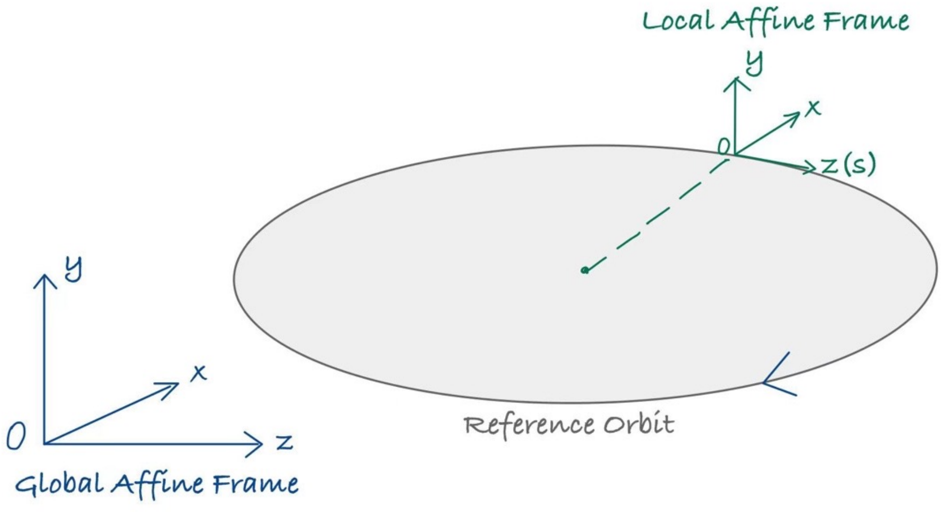

Fig15. Frenet-Serret Curvilinear Coordinate System[7]

*PS. The **Global Frenet-Serret Coordinate System and the reference orbit** are **utterly and completely** rejected by Forest. At best, in Forest's framework and PTC, it is a local coordinate system sometimes assigned to some magnets but of **no global significance** as far as patches and even Courant-Snyder theory are concerned. According to Forest, it is an ideological poison in the writing of tracking code.*

Phase space coordinates:

$$\vec{r} = \begin{cases} x \\ p_x = \dfrac{P_x}{P_0} \\ y \\ p_y = \dfrac{P_y}{P_0} \\ \delta = \dfrac{P - P_0}{P_0} \\ l = ct - s \end{cases} \quad (4)$$

In Accelerator, the Hamiltonian is

$$H = \delta - \left(1 + \frac{x}{\rho}\right)\left[(1+\delta)^2 - \left(p_x - \frac{eA_x}{p_0}\right)^2 - \left(p_y - \frac{eA_y}{p_0}\right)^2\right]^{\frac{1}{2}} - \frac{eA_s}{p_0} \quad (5)$$

where ρ is the curvature radius of the reference orbit, A is the vector potentials and $A_x = A \cdot \hat{x}, A_y = A \cdot \hat{y}, A_s = A \cdot \hat{s}$.

- Passmethod [8,9]
- Translation

We define the translation $T(\vec{d})$ by the Lie method:

$$T(\vec{d}) = \exp\left(:d_x p_x + d_y p_y + d_z\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}:\right) (6.a)$$

$$\overrightarrow{r^f} = T(\vec{d})\vec{r} (6.b)$$

in component form,

$$x^f = x - d_x + d_z \frac{p_x}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}} (7.a)$$

$$y^f = y - d_y + d_z \frac{p_y}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}} (7.b)$$

$$l^f = l + d_z \frac{(1+\delta)}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}} (7.c)$$

$$p_x^f = p_x, p_y^f = p_y, \delta^f = \delta (7.d)$$

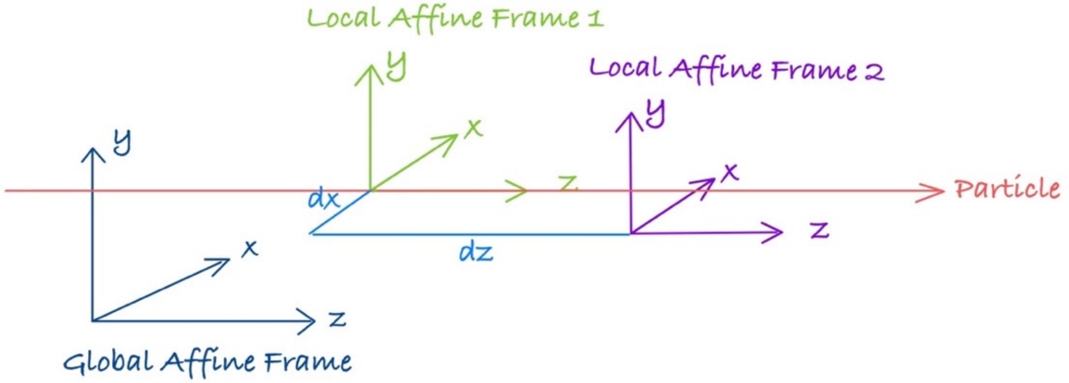

Fig16. A particle runs straight through a patch with dx and dz

- Passmethod [8,9]
- Rotation around Z axis
- By simple geometric relation,

$$x^f = x cos\theta_z + y sin\theta_z (8.a)$$
$$p_x^f = p_x cos\theta_z + p_y sin\theta_z (8.b)$$
$$y^f = -x sin\theta_z + y cos\theta_z (8.c)$$
$$p_y^f = -p_x sin\theta_z + p_y cos\theta_z (8.d)$$
$$\delta^f = \delta , l^f = l (8.e)$$

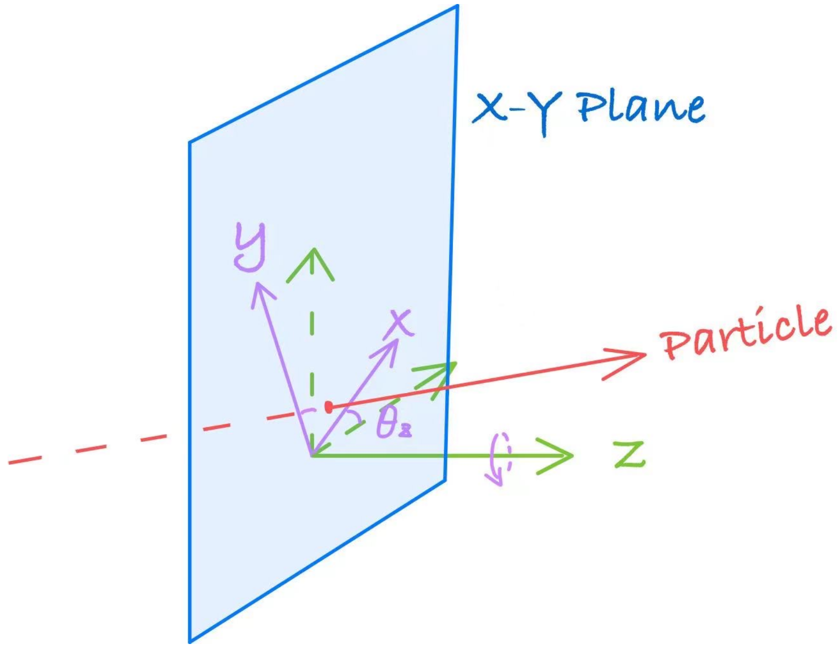

Fig17. A particle runs straight through a patch with only rotation around z

- Passmethod [8,9]
- Rotation around Y/X axis

- In particle's view, the rotations around X axis and around Y axis are symmetric
- We derive rotation in the ideal bend by taking appropriate limits: $\rho_c \to 0, s \to 0, \frac{s}{\rho_c} = \theta, b_0 \to 0$

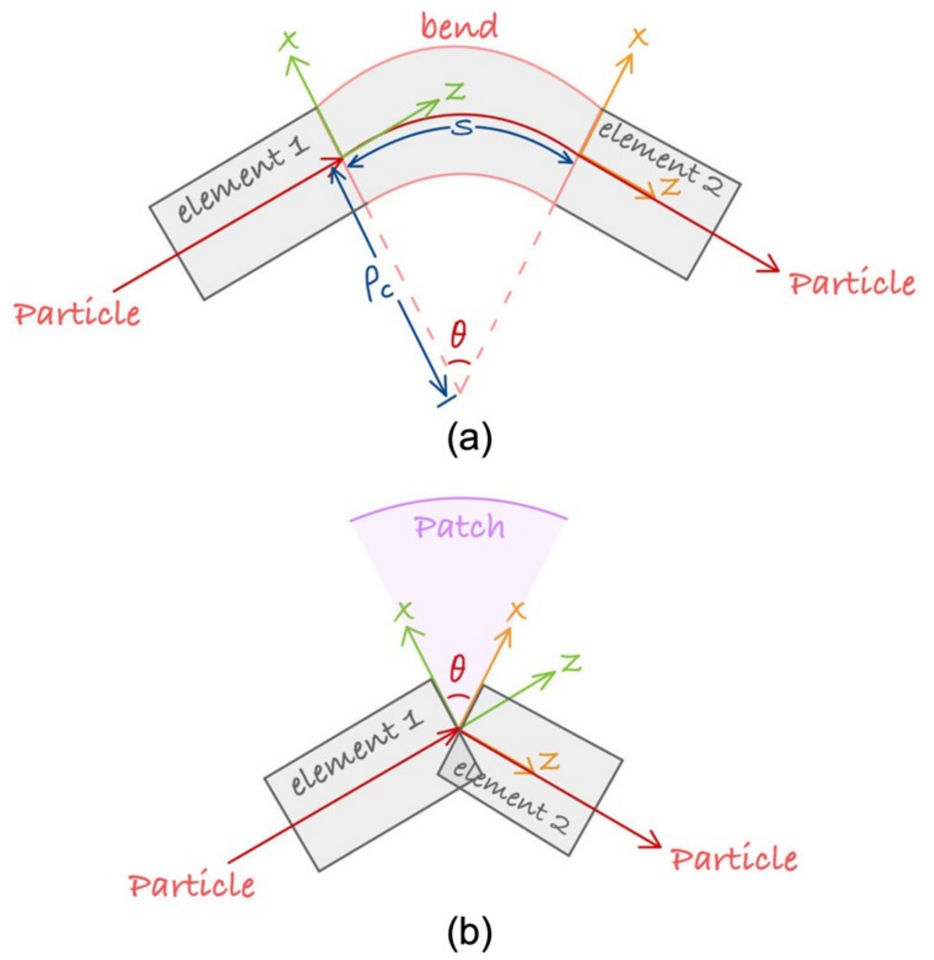

Fig18. A sector bend around Y axis (a) and a Patch (b)

The Hamiltonian in cylindrical coordinates for the body of the sector bend(rotate $-\theta$ around Y axis) is

$$H = -\left(1+\frac{x}{\rho_c}\right)\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} + b_0 x + b_0\frac{x^2}{2\rho_c} (9)$$

where ρ_c is the curvature of the frame of reference, $b_0 = \frac{qB_y}{p_0}$ is the normalized field strength

$$x^f = \frac{\rho_c}{b_0}\left(\frac{1}{\rho_c}\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} - \frac{dp_x^f}{ds} - b_0\right) (10.a)$$

$$p_x^f = p_x \cos\left(\frac{s}{\rho_c}\right) + \left[\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} - b_0(\rho_c + x)\right]\sin\left(\frac{s}{\rho_c}\right) (10.b)$$

$$y^f = y + \frac{p_y s}{b_0\rho_c} + \frac{p_y}{b_0}\left\{\arcsin\left(\frac{p_x}{\sqrt{(1+\delta)^2 - p_y^2}}\right) - \arcsin\left(\frac{p_x^f}{\sqrt{(1+\delta)^2 - p_y^2}}\right)\right\} (10.c)$$

$$p_y^f = p_y (10.d)$$

$$\delta^f = \delta (10.e)$$

$$l^f = l + \frac{(1+\delta)s}{b_0\rho_c} + \frac{(1+\delta)}{b_0}\left\{\arcsin\left(\frac{p_x}{\sqrt{(1+\delta)^2 - p_y^2}}\right) - \arcsin\left(\frac{p_x^f}{\sqrt{(1+\delta)^2 - p_y^2}}\right)\right\} (10.f)$$

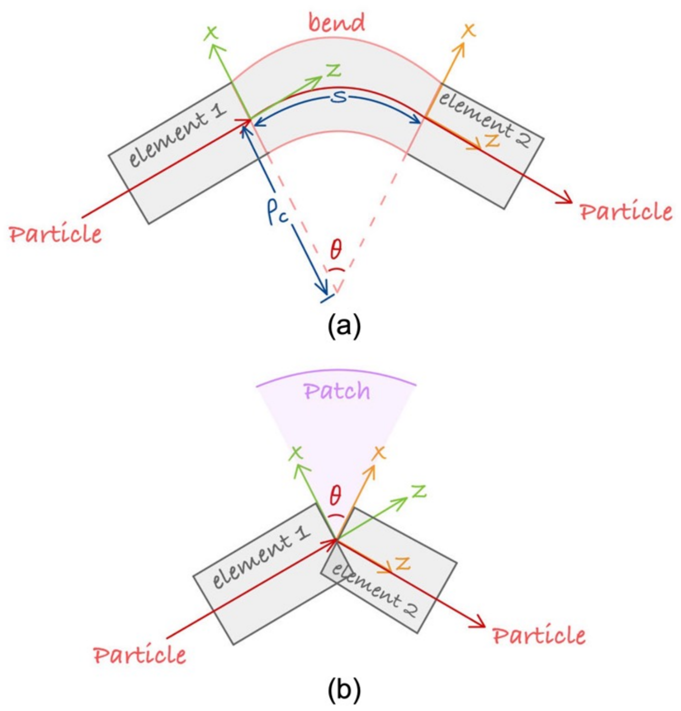

Fig18. A sector bend around Y axis (a) and a Patch (b)

Under the limits $\rho_c \to 0,\ s \to 0,\ \frac{s}{\rho_c} = \theta,\ b_0 \to 0$, we can derive the expressions for the rotation:

$$x^f = \frac{x}{cos\theta\left(1 - \frac{p_x tan\theta}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}}\right)} (11.a)$$

$$p_x^f = p_x cos\theta + sin\theta\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} (11.b)$$

$$y^f = y + \frac{p_y x\, tan\theta}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}\left(1 - \frac{p_x tan\theta}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}}\right)} (11.c)$$

$$p_y^f = p_y (3.9.d)$$

$$\delta^f = \delta (3.9.e)$$

$$l^f = l + \frac{(1+\delta) x\, tan\theta}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}\left(1 - \frac{p_x tan\theta}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}}\right)} (11.f)$$

Furthermore, for the rotation around X axis:

$$ROT_x(\theta, x, p_x, y, p_y, \delta, l) = ROT_y(-\theta, y, p_y, x, p_x, \delta, l) (12)$$

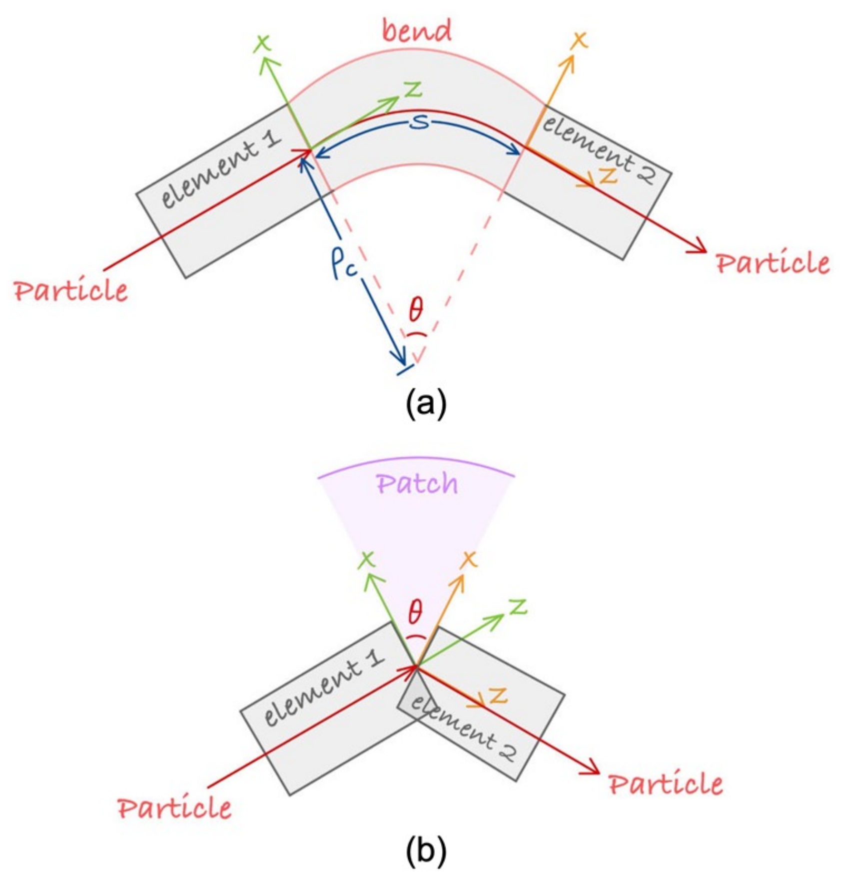

Fig18. A sector bend around Y axis (a) and a Patch (b)

- Implementation in code
  - Write it as 'PatchPass.py' ('PatchPass.c' is also available)
  - Toss it into our 'Passmethod Repository'

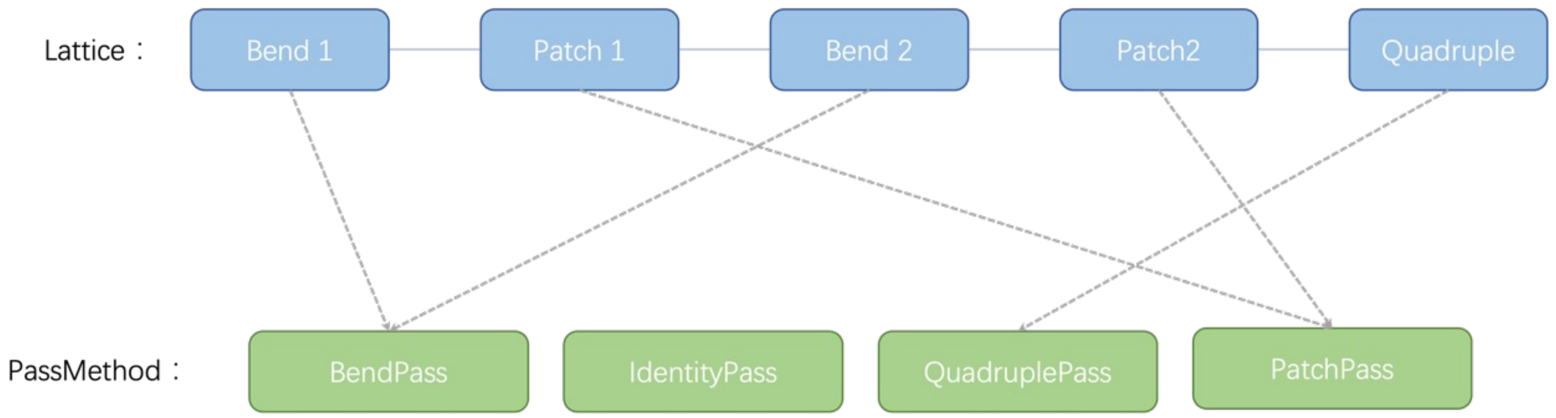

Fig19. Passmethod Repository

- High Energy Photon Source (HEPS) Booster

Fig20. Job site

The actual coordinate values under the global coordinate system

The theoretical coordinate values under a global coordinate system

The difference

全部设备装置坐标系下设备实际坐标值与理论值偏差（按束流排序）-梁静计算

	装置坐标系下实际坐标值			装置坐标系下理论值			装置坐标系下偏差		
点名	X1/mm	Y1/mm	高程Z1/mm	X2/mm	Y2/mm	高程Z2/mm	X1-X2/mm	Y1-Y2/mm	Z1-Z2/mm
						max	3.517	2.363	0.248
						min	-1.350	-1.703	-0.853
						标准值	1.223	1.287	0.299
BS1B01F1	-74346.278	-42266.484	59194.5665	-74349.212	-42268.164	59194.953	2.934	1.68	-0.386
BS1B01F2	-75337.099	-43306.797	59194.5496	-75339.96	-43308.448	59194.945	2.861	1.651	-0.395
BS1B01F3	-75517.697	-43134.886	59194.5607	-75520.561	-43136.569	59194.981	2.864	1.683	-0.42
BS1B01F4	-74527.031	-42094.474	59194.613	-74529.959	-42096.153	59194.964	2.928	1.679	-0.351
BS1B01EN	-75435	-43222.416	58999.592	-75437.882	-43224.11	59000	2.882	1.694	-0.408
BS1B01EX	-74435.236	-42172.392	58999.632	-74438.149	-42174.057	59000	2.913	1.665	-0.368
BS1QD02F1	-72884.732	-40676.371	59243.8172	-72887.828	-40678.021	59243.779	3.096	1.65	0.038
BS1QD02F2	-73028.262	-40834.213	59244.1327	-73031.346	-40835.878	59244.092	3.084	1.665	0.041
BS1QD02F3	-73242.475	-40642.678	59243.7157	-73245.588	-40644.363	59243.672	3.113	1.685	0.044
BS1QD02F4	-73098.799	-40483.818	59244.1838	-73101.915	-40485.5	59244.153	3.116	1.682	0.031
BS1QD02EN	-73164.138	-40770.134	59000.044	-73167.224	-40771.807	59000	3.086	1.673	0.044
BS1QD02EX	-72962.648	-40547.869	59000.033	-72965.756	-40549.522	59000	3.108	1.653	0.033
BS1BPM03F1	-71378.077	-38892.158	59025.7071	-71381.593	-38894.025	59026.047	3.516	1.867	-0.34

Fig21. Measured data of the devices in the HEPS booster

- Model for the designed booster (abbreviated as 'ideal model') use both pyAT & PTC
- Model for the measured booster (abbreviated as 'real model') use both pyAT & PTC

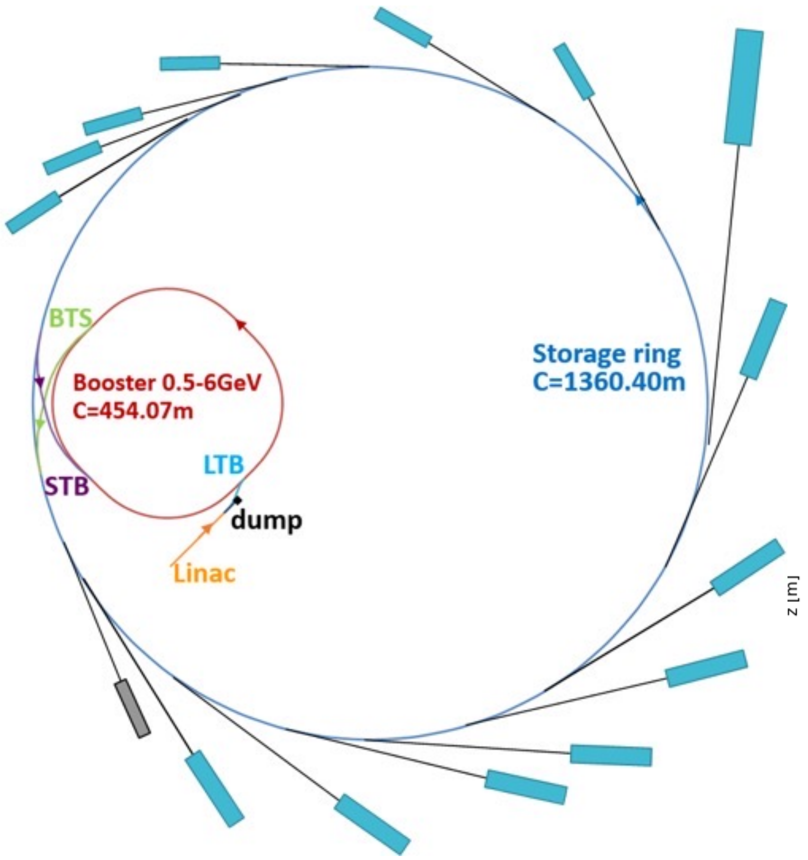

Fig22. The HEPS design

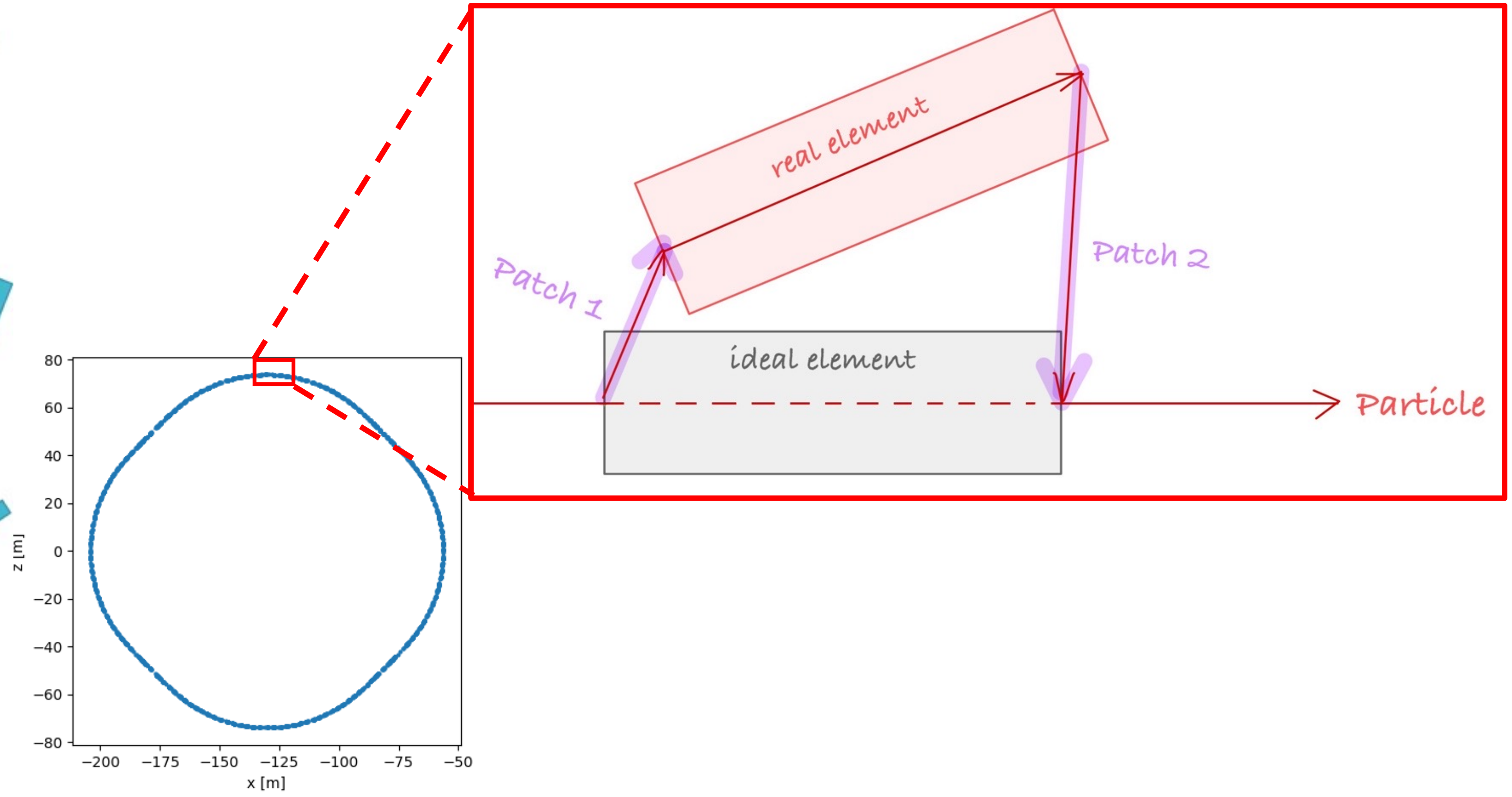

Fig23. Misalignment along the Lattice

- Close Orbit
- RMS:

$$\sigma_u = \sqrt{\frac{\sum_{i=1}^{N}\left(u_{i pyAT} - u_{i\,PTC}\right)^2}{N}} \quad (u = x, y)(13)$$

Tab2. The RMS of X and Y close orbit between pyAT and PTC

σ_x	1.4054E-6
σ_y	2.2867E-6

Tab1. The close orbit at s=0

		x(m)	p_x	y(m)	p_y	δ	l(m)
Ideal Model		8.8667E-14	-6.1296E-14	0.0000	0.0000	0.0000	0.0000
Real Model	pyAT	-2.8487E-4	2.2824E-4	4.3742E-3	-6.7828E-4	0.0000	0.0000
	PTC	-2.8234E-4	2.2754E-4	4.3758E-3	-6.7850E-4	0.0000	0.0000

- Moreover, this approach is capable to model more complicated machine layout
- A ring contains only quadruple and drift, but no bend!

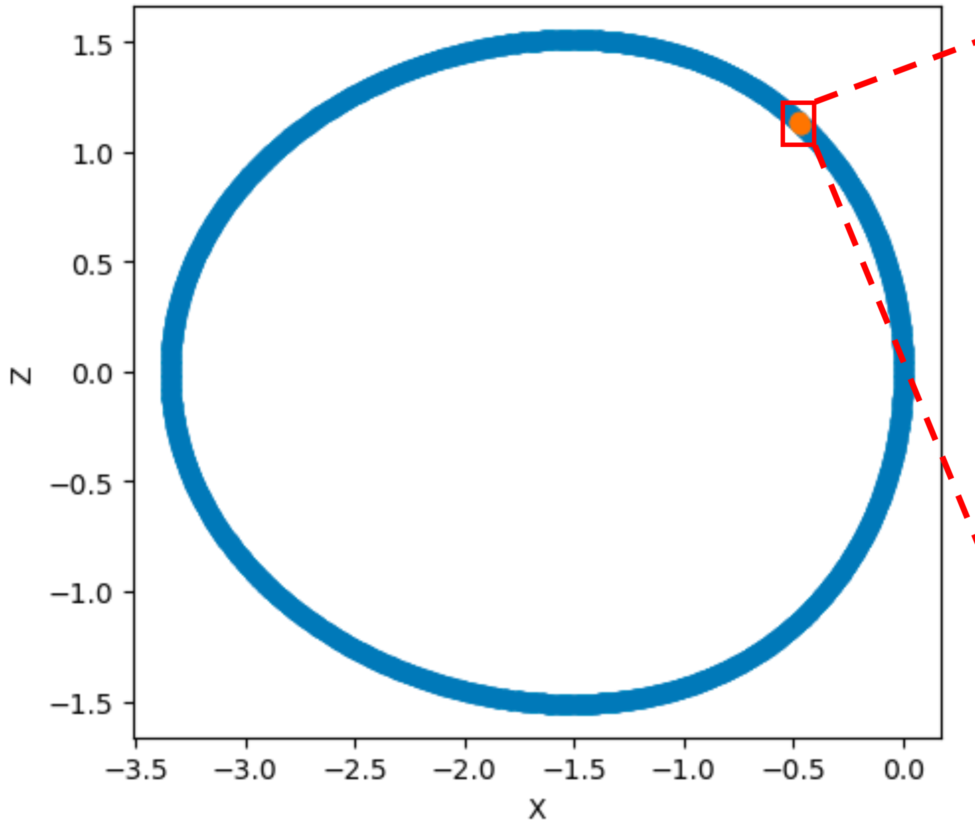

Fig28. A ring that contains only quadruple and drift

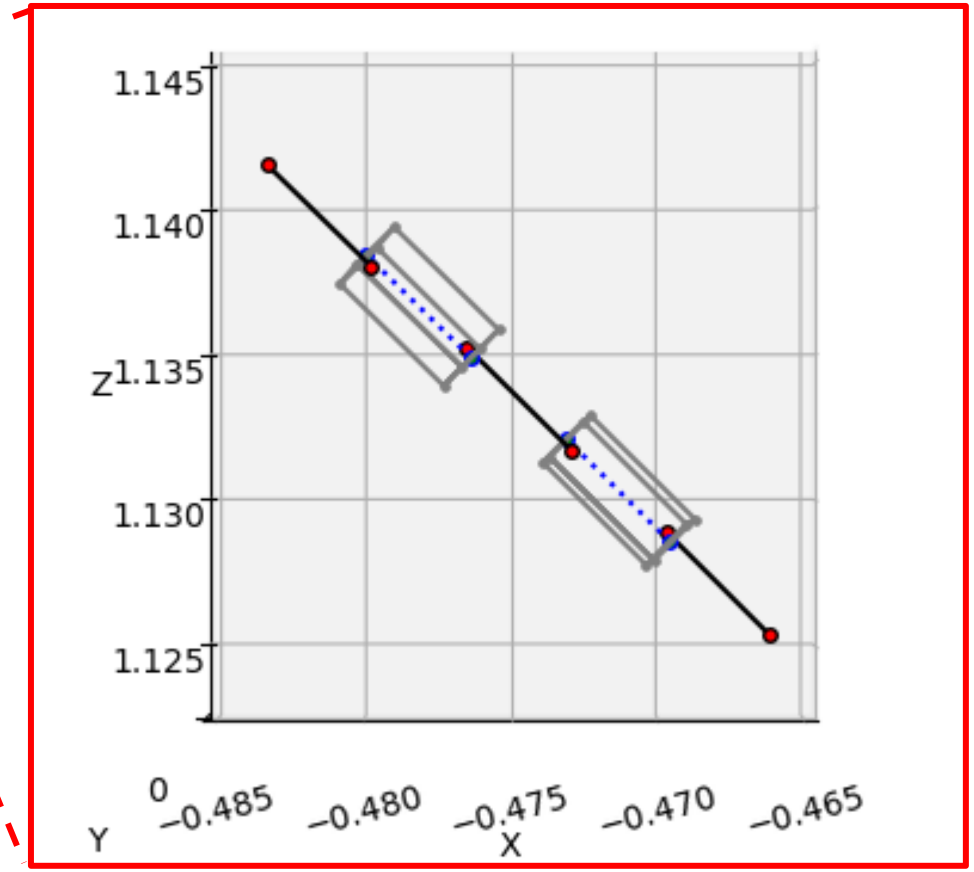

Fig29. An arbitrary part of the ring: Drift- Quadruple-Drift-Quadruple-Drift

- Moreover, this approach is capable to model more complicated machine layout
- A model like a roller coaster

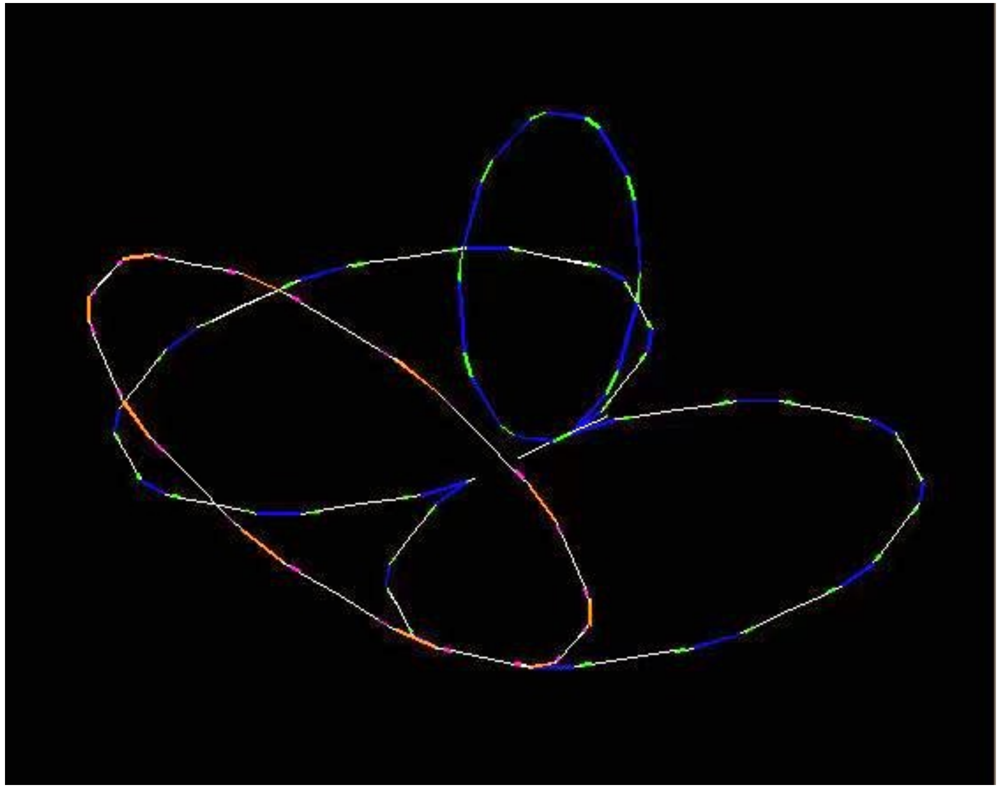

Fig30. A roller coaster model (From Professor Étienne Forest), in which the usual reference orbit is absent.

- Conclusion
  - Add 'Patch' into pyAT
  - For misalignment, the AT with patch can do as well as PTC
- Future TODO list：
  - Based on our 'Patch' work, refine the AT_mat part
  - Submit our code to the AT official code repository for review
  - Applications for CEPC

Tab3. The tolerance of alignment for a 100km ring[10]

Initial Mechanical alignment		
Length scale	tolerance	
6	20 to 50 um	mechanical installation tolerance of components on quad/sext girder - main issue is corrector and
50	200 um	mechnical installation and alignment of girder to girder - need to be able to transport first beam
200	500 um	mechnical installation
1000	2 mm	mechnical installation smoothed around the ring
10000	5 mm	Installation tolerance based on surface alignment network and GPS

▼ References

1. Dan T. Abell. PTC Library User Guide[M]. Colorado, USA: Tech-X Corporation, 2015:3-12.
2. Simone Liuzzo, Error setting and correction using Accelerator Toolbox 2.0, ESRF, Grenoble, 2016
3. W A H Rogers. pyAT: A Python Build of Accelerator Toolbox[A]. Gianluigi Arduini. Proceedings of IPAC2017[C], Copenhagen, Denmark: Bella Conference Center, 2017
4. Étienne Forest. Locally accurate dynamical Euclidean group[J]. Phys. Rev. E, 1997,55,4,(4665)
5. Laurent Deniau. The MAD-X Program (Methodical Accelerator Design) Version 5.08.01 User's Reference Manual[M]. Geneva, Switzerland: CERN,2022:1-22
6. David Sagan. The Bmad reference manual[M]. New York, USA: Cornell University,2022:2-28
7. S Y Lee. Accelerator Physics (Fourth Edition)[M]. Hackensack, USA: World Scientific Publishing Co. Pte. Ltd.,2021:34-40
8. Gennady Stupakov, Gregory Penn. Classical Mechanics and Electromagnetism in Accelerator Physics[M]. Cham, Switzerland: Springer International Publishing AG,2018:63-74
9. Étienne Forest. THE CORRECT LOCAL DESCRIPTION FOR TRACKING IN RINGS. Particle Accelerators,1994,45,65-94
10. Tor Raubenheimer, FCC Arc Alignment Approaches, FCC Week 2023 June 5-9, 2023

THANKS
For Your Attention

Mengyu Su, Zhe Duan

IHEP, CHINA

With the help of Etienne Forest (KEK,IHEP)

▼ Supplement

- High Energy Photon Source (HEPS) Booster
- β function & dispersion function

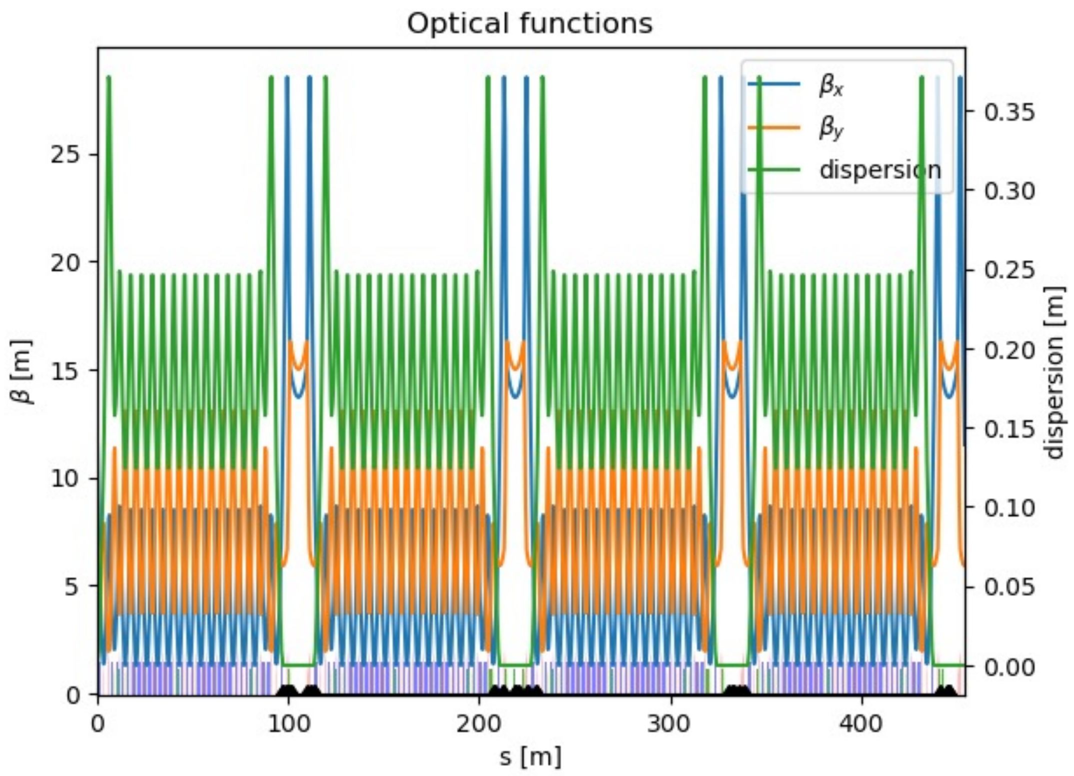

Fig24. Ideal Model

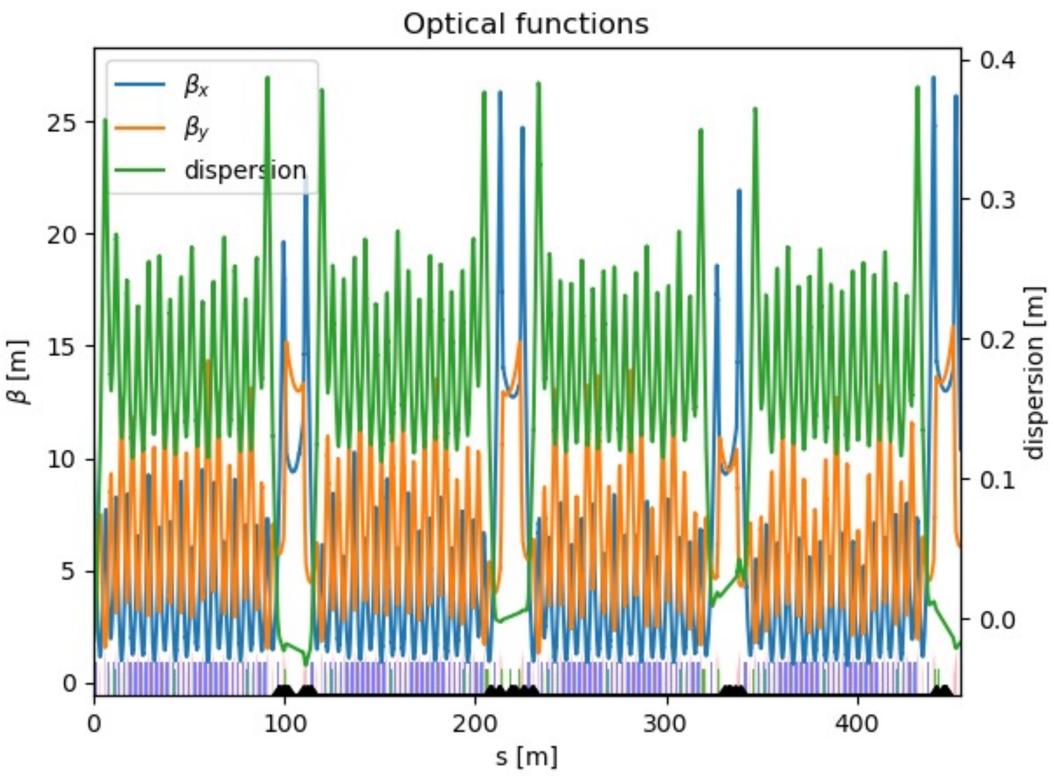

Fig25. Real Model

- High Energy Photon Source (HEPS) Booster
- β function & dispersion function

- Relative difference of β function $\frac{\Delta\beta}{\beta} = \frac{\beta_{pyAT} - \beta_{ideal}}{\beta_{ideal}}$ (4.6)
- Difference of dispersion function $\Delta dispersion = dispersion_{pyAT} - dispersion_{ideal}$ (4.7)

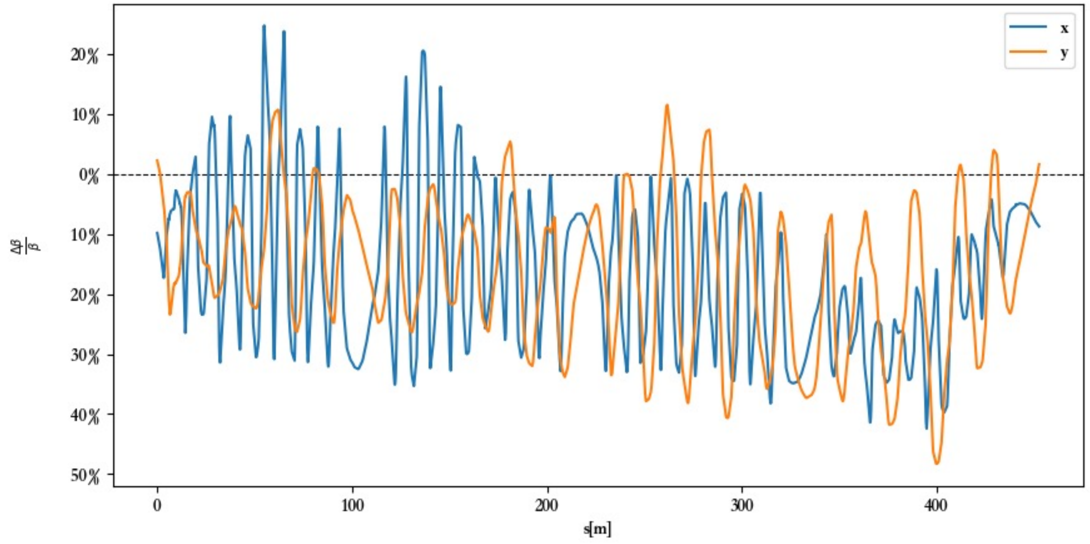

Fig26. Relative difference of β function

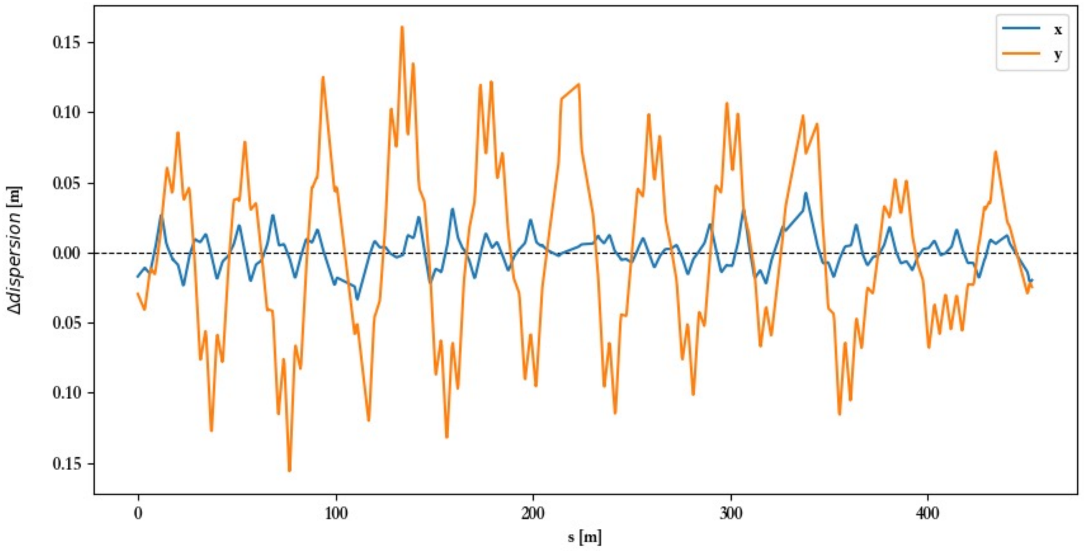

Fig27. Difference of dispersion function