Accelerator Toolbox passmethods for more accurate modeling

10/2/2023

Accelerator Toolbox Workshop (at ESRF)

(Presented remotely)

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Outline

- A code-comparison effort in 2009
- Original AT passmethods for magnet modeling
- Quadrupole passmethods with fringe field
- Dipole passmethods
  - Dipole w/ straight geometry
- Comparison w/ Elegant

See Borland, Sun, and Huang, PRAB 22, 114601 (2019)

- Summary

See an earlier talk on the topic by X. Huang at Future Light Source 2012 (on March 5, 2012) at
https://www.jlab.org/conferences/FLS2012/talks/Mon/Huang_lattice_modelling.pptx
This talk focus on progress since then.

An earlier code comparison in 2008-2009

SLAC

- Work was initiated at the 1st NLBD Workshop (2008, also at ESRF!)
  - A joint effort involving several codes and spanning continents
- D. Einfeld presented findings at the 2nd NLBD workshop (Nov. 2009)

There were large discrepancies between the codes.

Linear Parameters for the Lattice ALBA

ALBA

Calculation for the Lattice ALBA

Parameter	Unit	MAD	Tracy II	BETA	ELEG.	DIMAD	AT	OPA	Accel.
Energy	GeV	3	3.000	3	3	3	3	3	3
Circumference	m	268.8003	268.8003	268.8000	268.8000	268.8000	268.8003	268.8000	268.8003
Horizontal Tune Q(x)		18.1790	18.1789	18.1791	18.1790	18.1790	18.1790	18.1790	18.1790
Vertical Tune (Qy)		8.3720	8.3715	8.3710	8.3379	8.3720	8.3720	8.3720	8.3720
Beta_x (β(x))		11.1986	11.1980	11.1950	11.1967	11.1960	11.1966	11.1970	11.1970
Beta_y (β(y))		5.9288	5.9270	5.9250	5.7711	5.9290	5.9287	5.9290	5.9288
Dispersion_x (η(x))		0.1461	0.1470	0.1462	0.1462	0.1460	0.1461	0.1462	0.1465
Horiz.-Natur.-Chromaticity ξ(x)		-39.4893	-39.4976	-39.4400	-39.4433	-39.4433	-39.4155	-39.6480	-39.6481
Vertic.-Natur.-Chromaticity ξ(y)		-28.0677	-28.1603	-28.7700	-29.4241	-28.7558	-28.7372	-26.8830	-26.8831
Momentum Compaction Factor (α)		8.8230E-04	8.7580E-04	8.8290E-04	8.8293E-04	8.8230E-04	8.8316E-04	8.8300E-04	8.8229E-04
Energy Spread (δE/E)		1.0489E-03	1.0600E-03	1.0500E-03	1.0515E-03	1.0500E-03	1.0512E-03	1.0490E-03	1.0515E-03
Natural emittance	nm*rad	4.4874	4.4880	4.48922	4.4571	4.4600	4.4545	4.4880	4.4570
Horiz.-Damping-Time (τ(x))	msec	4.0826	4.0830	4.0810	4.0550	4.0551	4.0531	4.0840	4.0550
Vert.-Damping-Time (τ(y))	msec	5.2908	5.2910	5.2880	5.2908	5.2910	5.2887	5.2910	5.2908
Long.-Damping-Time (τ(s))	msec	3.1048	3.1040	3.1030	3.1210	3.1211	3.1199	3.1050	3.1210
Energy Loss per Turn (U(0))	MeV	1.0168	1.0168	1.0170	1.0168	1.0168	1.0172	1.0167	1.0156
Horiz.-Partition Number (J(x))		1.2959	1.2960	1.29576	1.3048	1.3048	1.3048	1.2958	1.3048
Vert.-Partition Number (J(y))		1.0000	1.0000	1.00000	1.0000	1.0000	1.0000	1.0000	1.0000
Long.-Partition Number (J(s))		1.7041	1.7040	1.70424	1.6952	1.6952	1.6952	1.7042	1.6952
Synchr.-Integrat (I1)		0.2375	0.2354	0.2373	0.2373	0.2373	0.2374	0.2373	0.2373
Synchr.-Integrat (I2)		0.8916	0.8916	0.8916	0.8916	0.8916	0.8916	0.8916	0.8916
Synchr.-Integrat (I3)		0.1265	0.1265	0.1265	0.1265	0.1265	0.1265	0.1265	0.1265
Synchr.-Integrat (I4)		-0.2717		-0.2637	-0.2717	-0.2717	-0.2718	-0.2637	-0.2717
Synchr.-Integrat (I5)		3.9356E-04		3.9256E-04	3.9258E-04	3.9258E-04	3.9258E-04	3.9250E-04	3.9258E-04

Slide 10 of D. Einfeld's presentation at the 2nd NLBD workshop (Diamond)

Original AT passmethods for magnets

- Linear passmethods w/ exact momentum deviation modeling
  - BendLinearPass
  - QuadLinearPass
  - CorrectorPass

These are basically transfer matrix, but convert to x' and y' coordinates first and handle pass length properly.

- Fourth-order integrators
  - BndMPoleSymplectic4Pass –
    - for multipoles on circular reference orbit
  - StrMPoleSymplectic4Pass
    - for all straight multipoles, including quadrupole
- Thin-lens multipoles
  - ThinMPolePass
    - thin-lens multipoles

Quadrupole fringe field modeling

- Quadrupole fringe field was known to cause linear and nonlinear optics perturbation

A general Hamiltonian (including longitudinal field variation) can be derived using a proper magnetic field expansion(1).

$$H(s)=\frac{1}{2}(P_x^2+P_y^2)+\frac{1}{2}k(s)(x^2-y^2)-\frac{1}{4}k'(s)(x^2-y^2)(xP_x+yP_y)-\frac{1}{12}k''(s)(x^4-y^4)+O(X^6)$$

J. Irwin, C.X. Wang

The leading correction for a hard-edge model is from the last two terms, which are nonlinear(2).

The leading correction term from a soft fringe model is linear(3).

$$H(s)=H_0(s)+\tilde{H}(s)$$

A perturbation approach

The soft fringe results in contraction and expansion at the edges.

$$f_2=\frac{I_1}{2}(xP_x-yP_y) \qquad I_1=\int_{-\infty}^{\infty}\tilde{k}(s)(s-s_0)ds$$

leading contribution

→ matrix $\mathrm{diag}(e^{I_1},e^{-I_1},e^{-I_1},e^{I_1})$

For a symmetric quadrupole, the entrance edge has a reversed sign for I_1

(1) El-Kareh; Forest; Bassetti & Biscari
(2) Lee-Whiting, Forest & Milutinovic, Irwin & Wang, Zimmermann
(3) Irwin & Wang (PAC'95), D. Zhou (IPAC10).

QuadLinearFPass – quadrupole passmethod w/ linear and nonlinear fringe field effect

- Was introduced in 2011
- For linear effect, apply the scaling transformation at edges
  - Use Irwin-Wang result on PAC'95
- Modeling of nonlinear effect

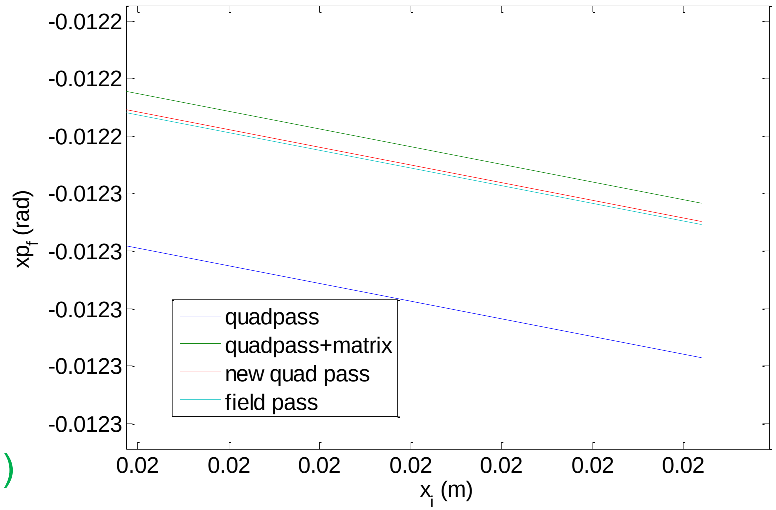
The generating function for the correction map (exit edge)

$$f_4 = \frac{1}{12(1+\delta)} k_0 (x^3 P_x + 3xy^2 P_x - y^3 P_y - 3x^2 y P_y) - \frac{1}{6(1+\delta)} k_{skew} (x^3 P_y + y^3 P_x)$$

Forest & Milutinovic

The function for the entrance edge has an opposite sign.

The skew quadrupole part corresponds to a 'kick map'! A normal quadrupole can thus be modeled by a pair of pi/4 rotation and a kick map.
This is the basis for the nonlinear part of the new AT quadrupole pass method.



See X. Huang, FLS'2012 (3/5/2012)

New quadrupole fringe field passmethods (2018)

- Needed for comparison with Elegant
- New passmethod - StrMPoleSymplectic4NPass
  - Also include exact drift in this passmethod
  - Nonlinear quadrupole fringe effect is modeled the same as in QuadLinearFP
- Differences from 2011 work – adopting D. Zhou's more detailed linear transformation

$$
\begin{aligned}
f_2^- \cong & -\frac{1}{2} I_0^- \left(x^2 - y^2\right) - I_1^- \left(x p_x - y p_y\right) \\
& -\frac{1}{2} I_2^- \left(p_x^2 - p_y^2\right) + \frac{1}{2} K_0 I_2^- \left(x^2 + y^2\right) \\
& +\frac{2}{3} K_0 I_3^- \left(x p_x - y p_y\right) + \frac{1}{2} \Lambda_2^- \left(x^2 + y^2\right) \\
f_2^+ \cong & -\frac{1}{2} I_0^+ \left(x^2 - y^2\right) - I_1^+ \left(x p_x - y p_y\right) \\
& -\frac{1}{2} I_2^+ \left(p_x^2 - p_y^2\right) + \frac{1}{2} \Lambda_2^+ \left(x^2 + y^2\right)
\end{aligned}
$$

$$
I_0^- = \int_{s_1}^{s_0} \tilde{K}(s) ds, \qquad I_1^- = \int_{s_1}^{s_0} \tilde{K}(s)(s - s_0) ds
$$

$$
I_2^- = \int_{s_1}^{s_0} \tilde{K}(s)(s - s_0)^2 ds, \qquad I_3^- = \int_{s_1}^{s_0} \tilde{K}(s)(s - s_0)^3 ds
$$

$$
I_0^+ = \int_{s_0}^{s_2} \tilde{K}(s) ds, \qquad I_1^+ = \int_{s_0}^{s_2} \tilde{K}(s)(s - s_0) ds
$$

$$
I_2^+ = \int_{s_0}^{s_2} \tilde{K}(s)(s - s_0)^2 ds, \qquad I_3^+ = \int_{s_0}^{s_2} \tilde{K}(s)(s - s_0)^3 ds
$$

$$
\Lambda_2^- = \int_{s_1}^{s_0} ds \int_{s}^{s_0} ds' \tilde{K}(s) \tilde{K}(s') (s' - s)
$$

$$
\Lambda_2^+ = \int_{s_0}^{s_2} ds \int_{s}^{s_2} ds' \tilde{K}(s) \tilde{K}(s') (s' - s)
$$

See D. Zhou, et al, IPAC'10 for more details

Test of quadrupole fringe passmethod

- A model with a simple profile is used as a test
  - For which the fringe integrals can be calculated analytically

$$I_{0p} = -I_{0m} = \frac{1}{3} K_0 \Delta, \quad (9a)$$

$$I_{1p} = 2 I_{1m} = \frac{2}{9} K_0 \Delta^2, \quad (9b)$$

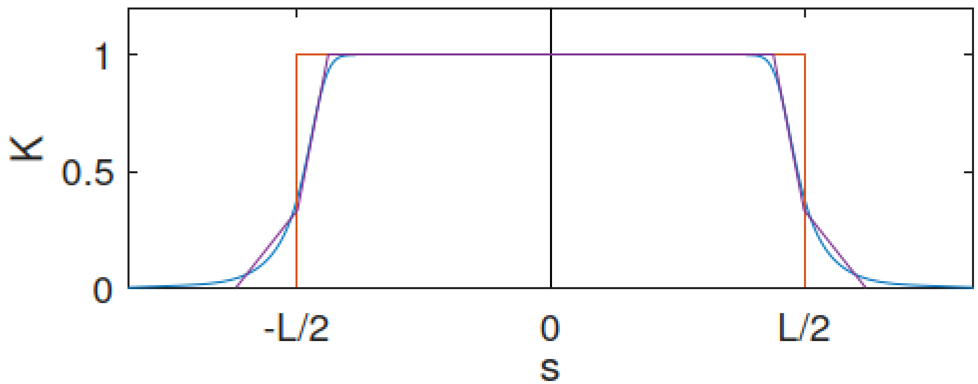
$$I_{2p} = -4 I_{1m} = \frac{2}{9} K_0 \Delta^3, \quad (9c)$$

$= -4I_{2m}$

$$I_{2p} = 8 I_{1m} = \frac{4}{15} K_0 \Delta^4, \quad (9d)$$

$= 8I_{3m}$

$$\Lambda_2^+ = 2\Lambda_2^- = \frac{4}{135} K_0^2 \Delta^3. \quad (9e)$$



Test case w/ L=1 m, K=1 /m^2, $\Delta = 0.1$ m

In comparison to the slicing model w/ 16000 slices (slice length = 0.1 mm)
(using StrMPoleSymplectic4NPass)

```
AT - Line approx
-4.5991e-07,    2.7325e-06, 0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00
2.1099e-06, -4.5991e-07,    0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00
0.0000e+00, 0.0000e+00, 6.3483e-07, 3.4835e-06, 0.0000e+00, 0.0000e+00
0.0000e+00, 0.0000e+00, -1.8520e-06,    6.3483e-07, 0.0000e+00, 0.0000e+00
0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00
0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00, 0.0000e+00
```

Compared to QuadLinearFPass

- The old version seems to be a good approximation, too

To use QuadLinearFPass, set $I_{1a} = I_{1b} = I_{1p} + I_{1m}$.

-2.364058e-04,	1.782653e-04,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00
-1.332283e-04,	-2.484126e-04,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	-3.575765e-04,	-5.164855e-04,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	-3.766251e-04,	-3.232857e-04,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00

This is equivalent to use the new method but only with I1p and I1m

difference w/ StrMPoleSymplectic4NPass but only use I1p and I1m

-2.424098e-04,	1.782863e-04,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00
-1.332083e-04,	-2.424098e-04,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	-3.404302e-04,	-5.165137e-04,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	-3.765944e-04,	-3.404302e-04,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00
0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00,	0.000000e+00

- The linear effect in the Irwin-Wang model is dominant.
- D. Zhou’s refined linear fringe model does improve the accuracy.

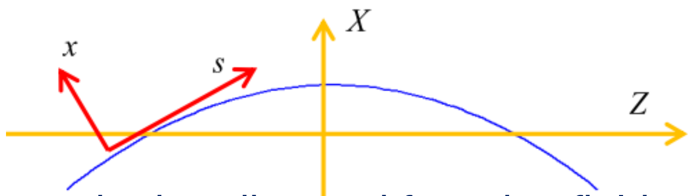
Usage of the passmethod

For the test case above

```
K = 1;
s0 = 0.5;
Delta = 0.1;

QF.FamName = 'QF';
QF.Length = 2*s0;
QF.K =K;
QF.PolynomB = zeros(1,4);
QF.PolynomA = zeros(1,4);
QF.PolynomB(2) = QF.K;

QF.MaxOrder = 3;
QF.NumIntSteps = 100;

QF.PassMethod = 'StrMPoleSymplectic4NPass';

QF.Iminus = [-1/30, 1/900, -1./18000 1./300000, 1./67500.];
QF.Iplus = [1./30., 1./450., 1./4500., 1./37500., 1./33750.];
QF.QuadEdgeFlag = [3 3];
```

Modeling a straight-geometry dipole

- The SPEAR3 dipole has a straight body (not a sector dipole)
  - Many other light source rings are the same

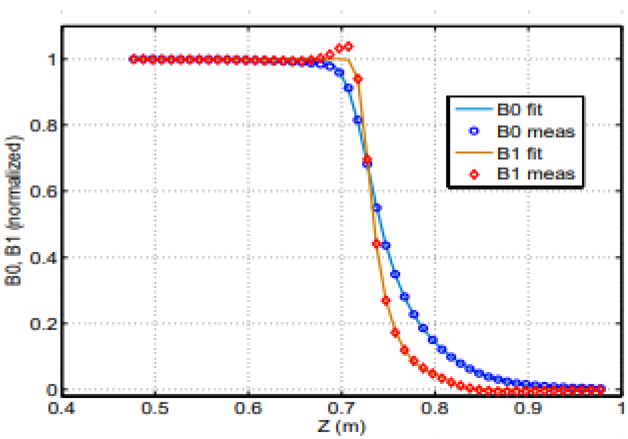


$$B_x = B_1 y \cos\theta(s), \quad B_s = -B_1 y \sin\theta(s),$$
$$B_y = (B_0 + B_1 X_{00}) - B_1\rho(1-\cos\theta(s)) + B_1 x \cos\theta(s).$$

The bending and focusing field components vary with distance, plus there is a longitudinal field component.

- A 3D magnetic field model and a field passmethod were used to model the SPEAR3 dipole

X. Huang, et al, IPAC'10



$$A_X = \frac{X^2 - Y^2}{4} B_0 \Theta_0'(Z), \quad A_Y = \frac{XY}{2} B_0 \Theta_0'(Z),$$
$$A_Z = -B_0 X \left(\Theta_0(Z) - \frac{1}{8}\Theta_0''(Z)(X^2+Y^2) \right), \quad (1)$$

Dipole field w/ fringe
(similarly for quadrupole, sextupole components)

The model predicts a 0.1% energy error which was later confirmed experimentally (K.P. Wootton, et al, Phys. Rev. ST Accel. Beams, 16, 074001 (2013))
But it can't be used for long-term tracking!

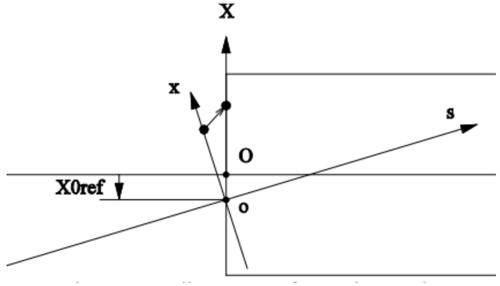
The CCBEND approach in Elegant

- Other approaches were tried to model such dipoles
  - E.g., to use Lie map thin-lens element to account for the differences with the sector dipole model (obtaining such map from Taylor map which is obtained by fitting tracking results, using a field model) Y. Li, X. Huang, IPAC'12
- But the CCBEND in Elegant gives a good solution
  - Coordinate rotation at the entrance and exit
  - Symplectic integration through dipole body, including dipole component
    - This is similar to the StrMPoleSymlectic4Pass

This solution was described in E. Forest's book, 'Beam dynamics – a new attitude and framework' (1998), p. 355.

Implementation in AT

- Implemented in the new passmethod ‘BndStrMPoleSymplectic4Pass’
- Rotation at entrance and exit



Entrance:

$$X_2 = \frac{X\cos(\psi)}{\cos(\psi+\theta)},$$
$$\frac{dX}{dZ}|_2 = \tan(\psi+\theta),$$
$$Y_2 = Y + \frac{dY}{dZ}\frac{X\sin(\theta)}{\cos(\theta)-\frac{dX}{dZ}\sin(\theta)},$$
$$\frac{dY}{dZ}|_2 = \frac{dY}{dZ}\frac{1}{\cos(\theta)-\frac{dX}{dZ}\sin(\theta)},$$
$$s_2 = s + \frac{X\sin(\theta)}{\cos(\theta)-\frac{dX}{dZ}\sin(\theta)}\left[1+\left(\frac{dX}{dZ}\right)^2+\left(\frac{dY}{dZ}\right)^2\right]^{\frac{1}{2}}$$

With entrance angle θ and,

$$\psi = \frac{dx}{dz} = \frac{p_x}{\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}}$$

- Symplectic integration through the body w/ $(X, \frac{dX}{dZ}, Y, \frac{dY}{dZ}, \delta, \Delta z)$ coordinates
  - With PolynomA and PolynomB given on the straight reference axis
  - Fourth order symplectic integrator, but use exact drift with Hamiltonian

$$\mathrm{H} = (1+\delta) - \sqrt{(1+\delta)^2 - p_x^2 - p_y^2}$$

Need to specify entrance position by ‘X0ref’ and path length difference ‘RefDZ’

(The implementation details may differ from that of Elegant.)

Usage of BndStrMPoleSymplectic4Pass

- Determination of *X0ref* – by finding the reference orbit with the desired bending and entrance angles
  - This done numerically, e.g., by using an optimizer or solver

For symmetric, combined function with bend and quadrupole, an approximate first guess

$$X_{ref} = \rho(\cos\frac{\theta_0}{2} - 1) + \frac{L\theta_0}{24} - \frac{K_1 L^3 \theta_0}{720} - \frac{L\theta_0^3}{576} \quad \text{With} \quad \rho = \frac{L}{2\sin\theta_0/2}$$

Derived from M. Yoon, et al, Nucl. Instr. Meths. A 523 (2004) 9-18.

For SPEAR3 dipole, w/ $L = 1.5069434$ m, $K_1 = -0.31537858$ /m^2, and $\theta_0 = \pi/17$

Formula → $X_{ref} = -0.022919$ m, while

Numerical → $X_{ref} = -0.022997$ m

For APS-U Q4, w/ $L = 0.2110$m, $K_1 = 4.004396383$ /m^2, and $\theta_0 = -0.001706915788385$

Formula → $X_{ref} = 3.010244 \times 10^{-5}$ m, while

Numerical → $X_{ref} = 3.010281 \times 10^{-5}$ m

Comparison with BendLinearPass

- The equivalent sector dipole should have
  - The same bending angle
  - Arc length $L_{arc} = \rho\theta = \frac{L\theta}{2\sin\theta/2}$
  - Quadrupole K component, $K_{eff} = K_1 \frac{2}{\theta_0} \sin\frac{\theta_0}{2} \approx K_1(1 - \frac{\theta_0^2}{24})$

Using BndStrMPoleSymplectic4Pass

Rb =

1.3802	1.6840	0	0	0.1474	0
0.5373	1.3802	0	0	0.2084	0
0	0	0.6484	1.3337	0	0
0	0	-0.4346	0.6484	0	0
0	0	0	0	1.0000	0
0.2084	0.1474	0	0	0.0090	1.0000

Using BendLinearPass, w/ side negative drifts to correct the length

R0 =

1.3818	1.6858	0	0	0.1384	0
0.5368	1.3786	0	0	0.2068	0
0	0	0.6481	1.3343	0	0
0	0	-0.4345	0.6484	0	0
0	0	0	0	1.0000	0
0.2114	0.1578	0	0	0.0095	1.0000

Comparison of SPEAR3 standard dipole transfer matrix

Comparison to Elegant for APS-U lattice

- Work was done 2018-2019, as part of the effort to verify APS-U performance predictions.
  - The 41-pm lattice was used.
- Lattice model features: Longitudinal gradient bends, negative bends, CCBENDs, quadrupole fringe fields

# of elements	AT model	Elegant model
1120	BendLinearPass	CSBEND
160	BndStrMPoleSymplectic4Pass	CCBEND
800	StrMPoleSymplectic4NPass	KQUAD
480	StrMPoleSymplectic4NPass	KSEXT
6052*	LaDriftPass	EDRIFT

*With 1205 negative drifts.

Linear and nonlinear lattice parameters

- Good agreement between the two codes for a variety of parameters

Both linear and nonlinear parameters

TABLE III. APS-U lattice parameters calculated with ELEGANT and AT.

Parameter	ELEGANT	AT
Horizontal tune, ν_x	95.0999	95.0993
Vertical tune, ν_y	36.0999	36.1007
Momentum compaction (MCF), $\times 10^{-5}$	4.0406	4.0399
2nd-order MCF, $\times 10^{-4}$	1.2092	1.2091
Chromaticity, ξ_x	8.1183	8.1704
Chromaticity, ξ_y	4.7221	4.8739
Natural chrom., ξ_x^{nat}	−133.6488	−133.5874
Natural chrom., ξ_y^{nat}	−111.6335	−111.4689
Emittance (pm)	41.6612	41.6434
Energy loss per turn (MeV)	2.8688	2.8700
Momentum spread, σ_δ, $\times 10^{-3}$	1.3499	1.3494
Damping partition, J_x	2.2497	2.2495
Damping time τ_x (ms)	6.8446	6.8424
Horizontal tune, ν_x ($\delta_p = 0.04$)	95.4142	95.4410
Vertical tune, ν_y ($\delta_p = 0.04$)	36.3556	36.3927
Horizontal tune, ν_x ($x = y = 2$ mm)	95.2313	95.2324
Vertical tune, ν_y ($x = y = 2$ mm)	36.1186	36.1189

Borland, Sun, and Huang, PRAB 22, 114601 (2019)

Effects of CCBEND and quadrupole fringe

- Quadrupole fringe makes noticeable changes to tunes and chromaticities
- But CCBEND has relatively smaller effects

TABLE IV. Changes of lattice parameters when the quadrupole fringe field is turned off in the APS-U lattce, calculated with ELEGANT and AT.

Parameter	ELEGANT	AT
Horizontal tune, $\Delta\nu_x$	0.0579	0.0582
Vertical tune, $\Delta\nu_y$	0.1360	0.1374
Chromaticity, $\Delta\xi_x$	−0.0071	−0.0688
Chromaticity, $\Delta\xi_y$	0.6184	0.4942
Horizontal tune, $\Delta\nu_x$ ($\delta_p = 0.04$)	0.1022	0.0690
Vertical tune, $\Delta\nu_y$ ($\delta_p = 0.04$)	0.1792	0.1754
Horizontal tune, $\Delta\nu_x$ ($x = y = 2$ mm)	0.0738	0.0735
Vertical tune, $\Delta\nu_y$ ($x = y = 2$ mm)	0.1407	0.1415

Effect of quadrupole fringe field

TABLE V. Changes of lattice parameters when the APS-U straight dipoles are replaced by sector dipoles, calculated with ELEGANT and AT.

Parameter	ELEGANT	AT
Horizontal tune, $\Delta\nu_x$, $\times10^{-5}$	−0.86	−0.86
Vertical tune, $\Delta\nu_y$, $\times10^{-5}$	0.27	0.55
Chromaticity, $\Delta\xi_x$	−0.1187	−0.0021
Chromaticity, $\Delta\xi_y$	0.0791	0.0030
$\Delta\nu_x$ ($\delta_p = 0.04$), $\times10^{-3}$	−6.46	0.68
$\Delta\nu_y$ ($\delta_p = 0.04$), $\times10^{-3}$	5.02	0.13
$\Delta\nu_x$ ($x = y = 2$ mm), $\times10^{-4}$	2.23	−0.25
$\Delta\nu_y$ ($x = y = 2$ mm), $\times10^{-4}$	−2.45	−0.17

Effect of CCBEND

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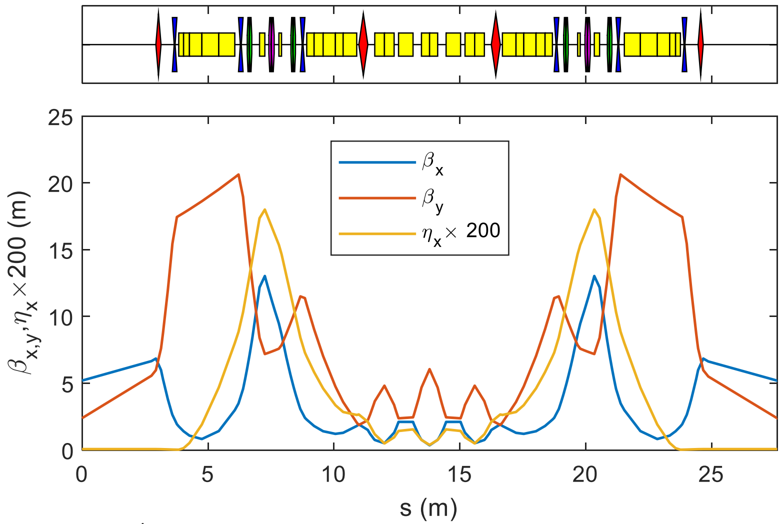
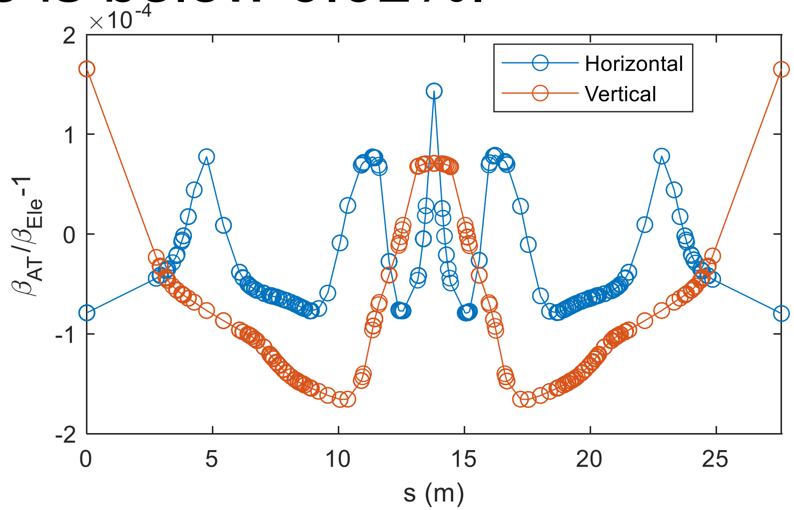
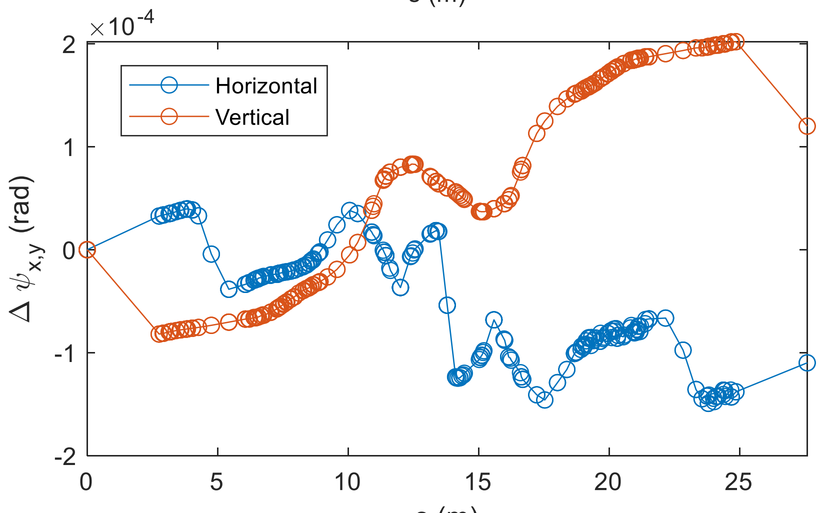
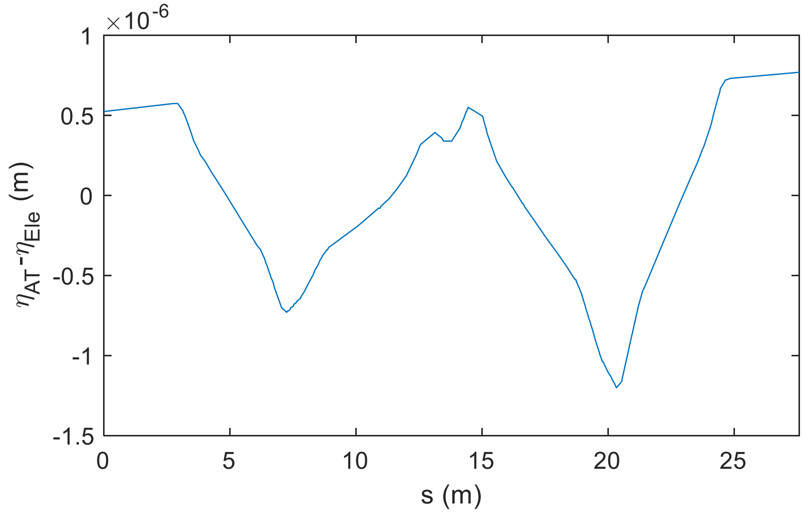
Linear lattice functions (one cell)

- Relative beta function difference is below 0.02%.



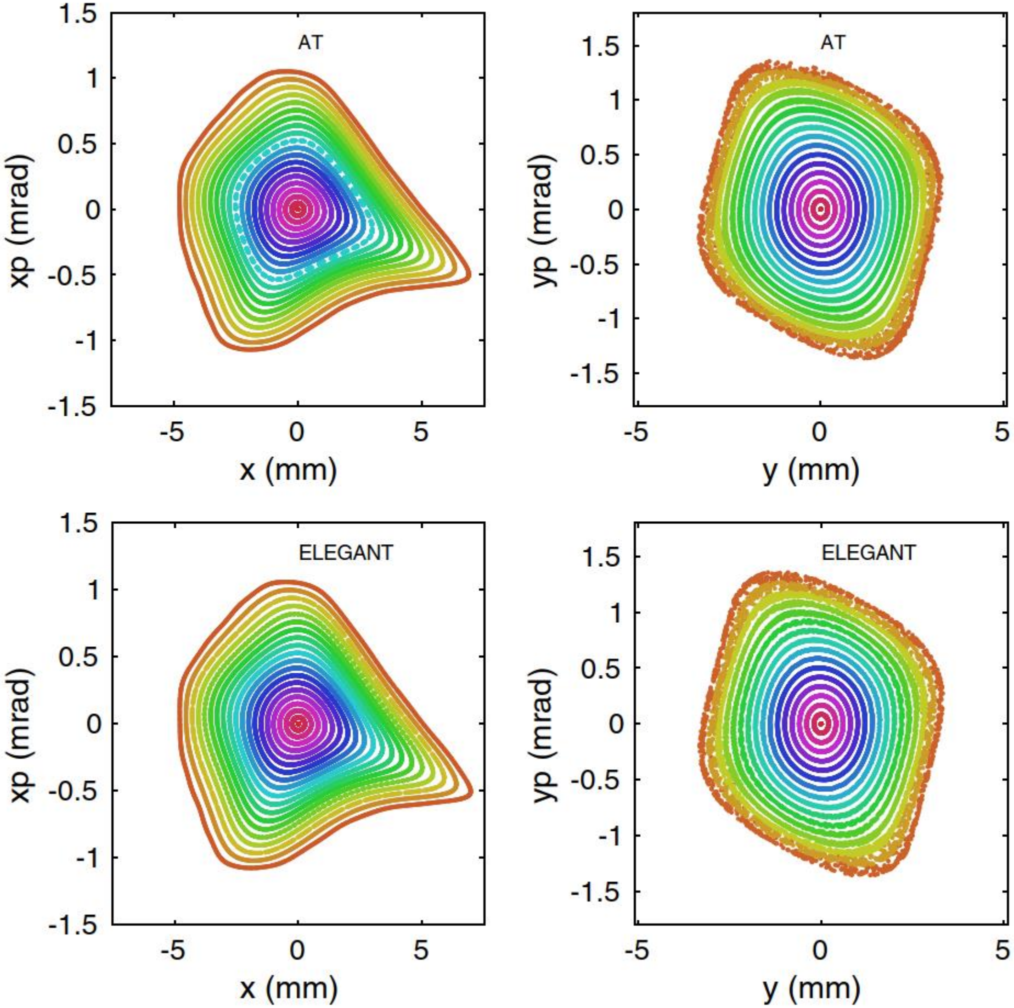






Nonlinear behavior – phase space

- Agreement extends to the edge of the phase space



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Nonlinear behavior – frequency map in (x, y)

- The tune footprints are in good agreement, but tune diffusion rates differ.

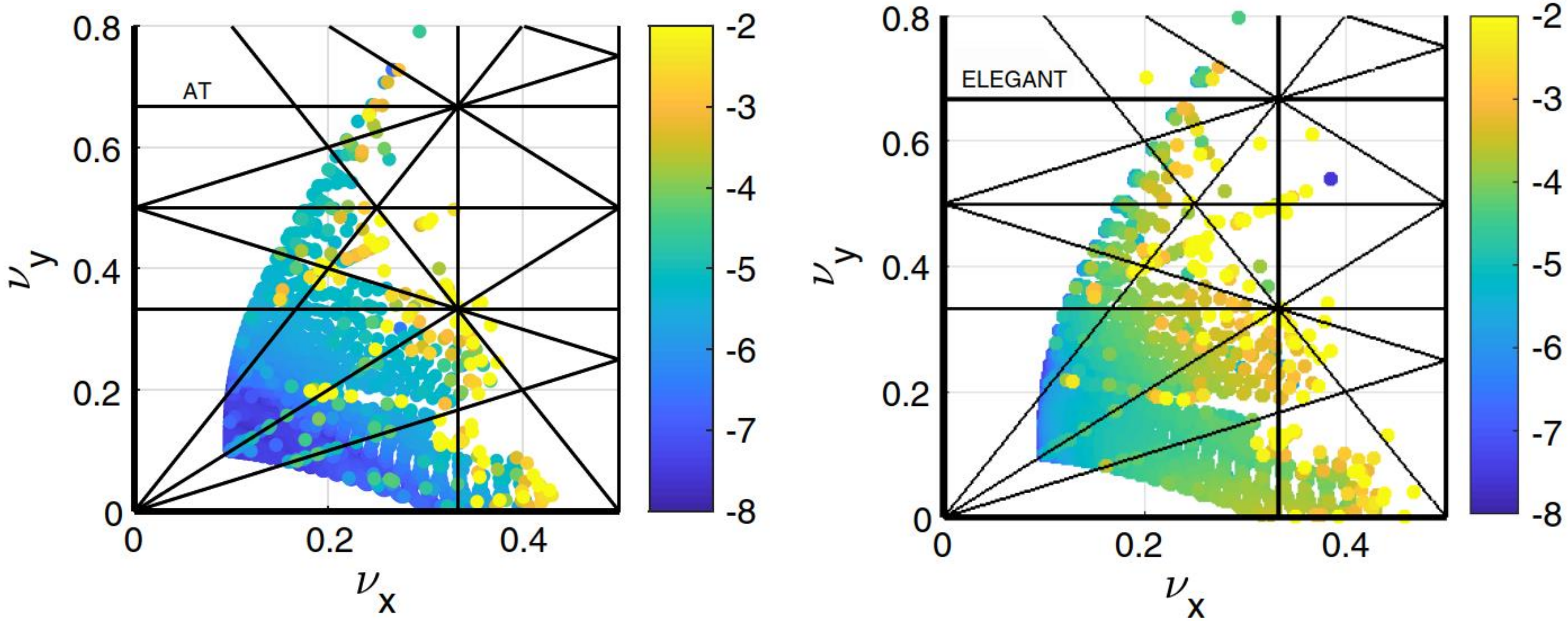

FIG. 8. Tune diagram for frequency map analysis in the x-y plane. The color code represents the detuning over 1024 turns, $\log_{10}\left(\sqrt{\Delta\nu_x^2 + \Delta\nu_y^2}\right)$, where $\Delta\nu_x$ and $\Delta\nu_y$ are tune changes from the first 512 turns to the second 512 turns.

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Nonlinear behavior – off-momentum stability region

- Good agreement in (x, $\Delta p/p$) space, too.

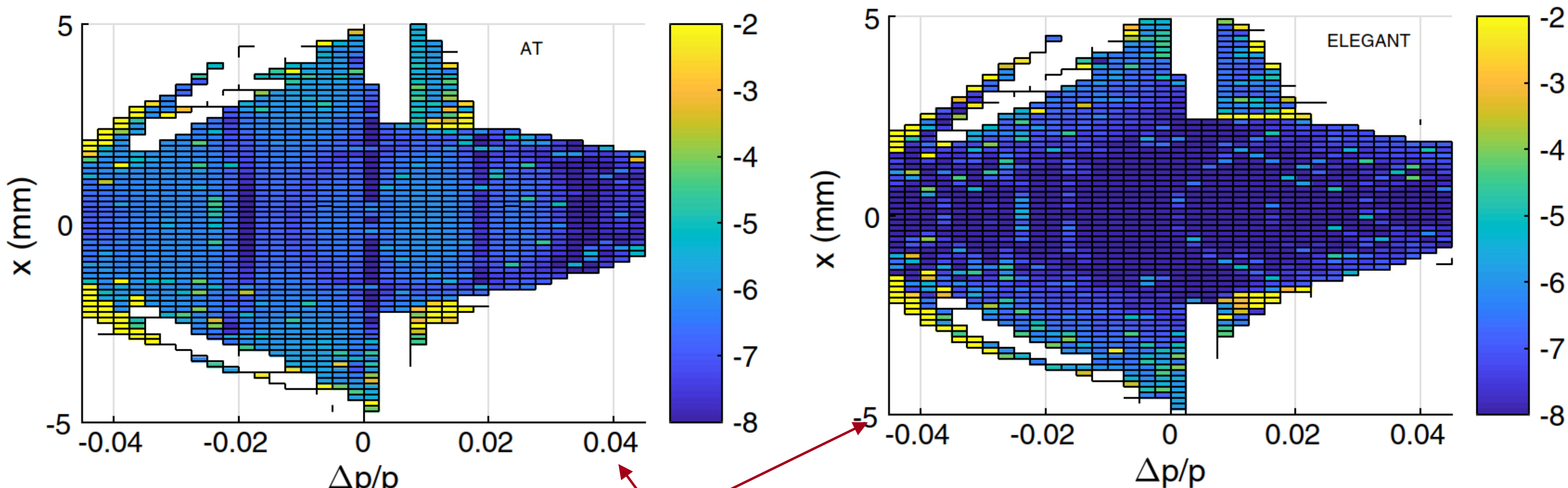

FIG. 9. Comparison of stability region in the x-$\frac{\Delta p}{p}$ plane as calculated by AT (top) or ELEGANT (bottom). Color code represents tune diffusion as defined in Eq. (13).

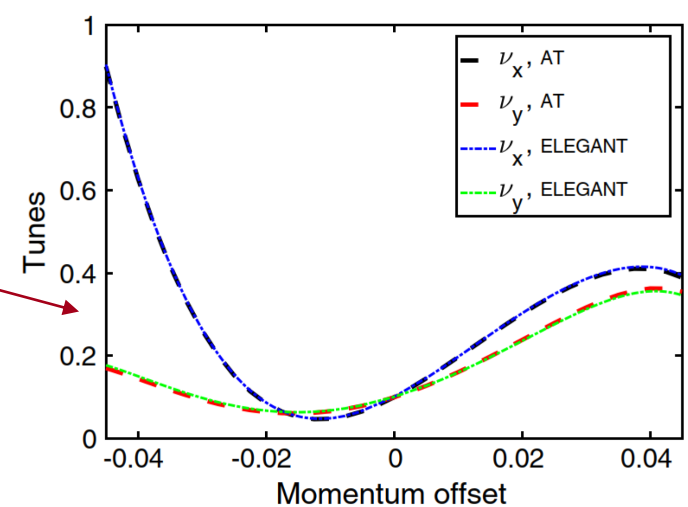

FIG. 10. Comparison of betatron tunes as a function of momentum deviation calculated by AT and ELEGANT.

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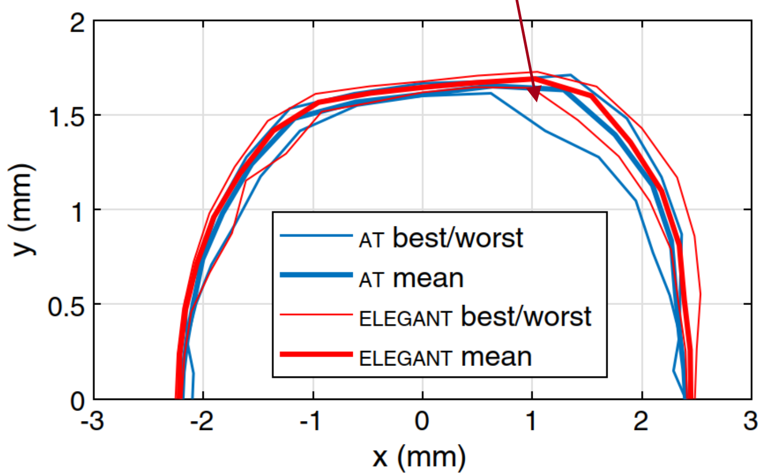
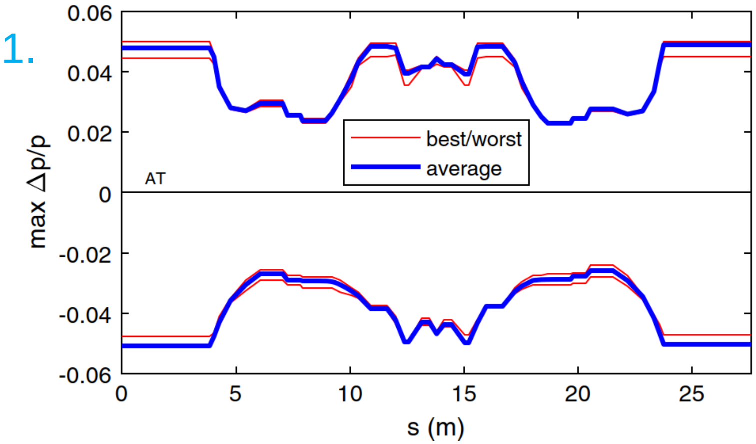
Comparison of DA and LMA (w/ the same error seeds)

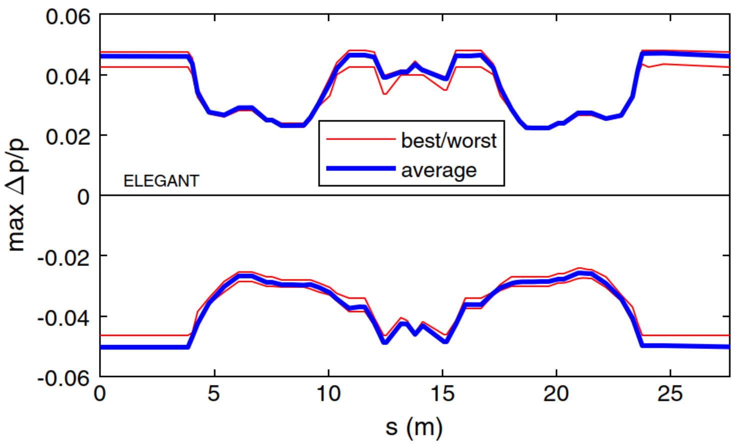
- Linear errors (normal and skew quadrupole components) are added at sextupole locations
  - Rms beta beating at 1%, and with 100% coupling

Unstable region with tunes (0.2, 0.2), see slide 21.





The predictions of DA and LMA by the two codes are very similar.



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More accurate symplectic integration for sector dipoles

- Hamiltonian for a sector dipole

$$H = 1 + \delta - (1 + hx)\frac{A_s}{B\rho} - (1 + hx)\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}$$

- In AT, BndMPoleSymplectic4Pass is used to model dipoles with higher field components
  - It uses drift-kick split of the Hamiltonian but integrates the drift without considering the curvature of the reference orbit,

In which AT uses drift $H_1 = \frac{p_x^2 + p_y^2}{1+\delta}$, while it should be $H_1 = 1 + \delta - (1 + hx)\sqrt{(1+\delta)^2 - p_x^2 - p_y^2},$

Exact solution to the latter exists, but requires evaluation of sin, cos, tan, functions.

  - And, it does not use the vector potential* for the curved reference orbit

$$\begin{aligned} A_s = & -B_0\left(x - \frac{hx^2}{2(1+hx)}\right) - B_1\left(\frac{1}{2}(x^2 - y^2) - \frac{h}{6}x^3 + \frac{h^2}{24}(4x^4 - y^4) + \cdots\right) \\ & -B_2\left(\frac{1}{6}(x^3 - 3xy^2) - \frac{h}{24}(x^4 - y^4) + \cdots\right) - B_3\left(\frac{1}{24}(x^4 - 6x^2y^2 + y^4) + \cdots\right) \\ & + \cdots. \end{aligned}$$

*F. C. Iselin, Physical Methods Manual for the MAD Program, September 1994

An attempt to fix this - BndMPoleSymplectic4E2Pass

- Expand the bend Hamiltonian

$$H = -(1+hx)\frac{A_s}{B\rho} - (1+\delta)hx + (1+hx)\frac{p_x^2+p_y^2}{2(1+\delta)}$$

- The drift Hamiltonian has approximate solution

$$\begin{aligned}
H_1 &= (1+hx)\frac{p_x^2+p_y^2}{2(1+\delta)}, \\
x_2 &= x + \frac{1+hx}{1+\delta}p_x L_d + \frac{hL_d^2}{4(1+\delta^2)}(p_x^2 - p_y^2), \\
p_{x2} &= p_x - \frac{hL_d}{2(1+\delta)}(p_x^2+p_y^2), \\
y_2 &= y + \frac{1+hx}{1+\delta}p_y L_d + \frac{hL_d^2}{2(1+\delta)^2}p_x p_y, \\
z_2 &= z + \frac{1+hx}{2(1+\delta)^2}(p_x^2+p_y^2)L_d,
\end{aligned}$$

This can be worked to higher orders

X. Huang, SSRL-AP-Note 021 (2009)

cont'd

- While solution to the vector potential term can also been approximated

$$H_2 = -(1+hx)\frac{A_s}{B\rho} - (1+\delta)hx.$$

Including correction terms up to ocutpoles

$$\begin{aligned}\Delta p_x/L_k &= -(K_1+h^2)x - K_2(x^2-y^2) - K_3(x^3-2xy^2) \\ &\quad -h\Big(K_1(x^2-\frac{1}{2}y^2) + K_2(x^3-\frac{4}{3}xy^2)\Big) \\ \Delta p_y/L_k &= K_1y + 2K_2xy + K_3(2x^2y-y^3) + h\Big(K_1xy + \frac{4}{3}K_2x^2y + \frac{1}{6}(hK_1-2K_2)y^3\Big) \\ \Delta z/L_k &= hx,\end{aligned}$$

Problem: these approximations are not symplectic

Can we show that the deviation from the symplectic condition is small and negligible?
Or once it is non-symplectic, it is not worth considering?

X. Huang, SSRL-AP-Note 021 (2009)

Summary

- AT had additions of new passmethods to more accurately model magnets
  - Exact drift (LaDriftPass)
  - Fourth-order integrator with quadrupole fringe fields (linear and nonlinear) and exact drift (StrMPoleSymplectic4NPass)
  - Fourth-order integrator on straight geometry with bending field (BndStrMPoleSymplectic4Pass)
- With these additions, excellent agreement with Elegant was seen when applied to the APS-U lattice
  - For both linear and nonlinear properties
- More work may be needed for symplectic integration in sector dipoles