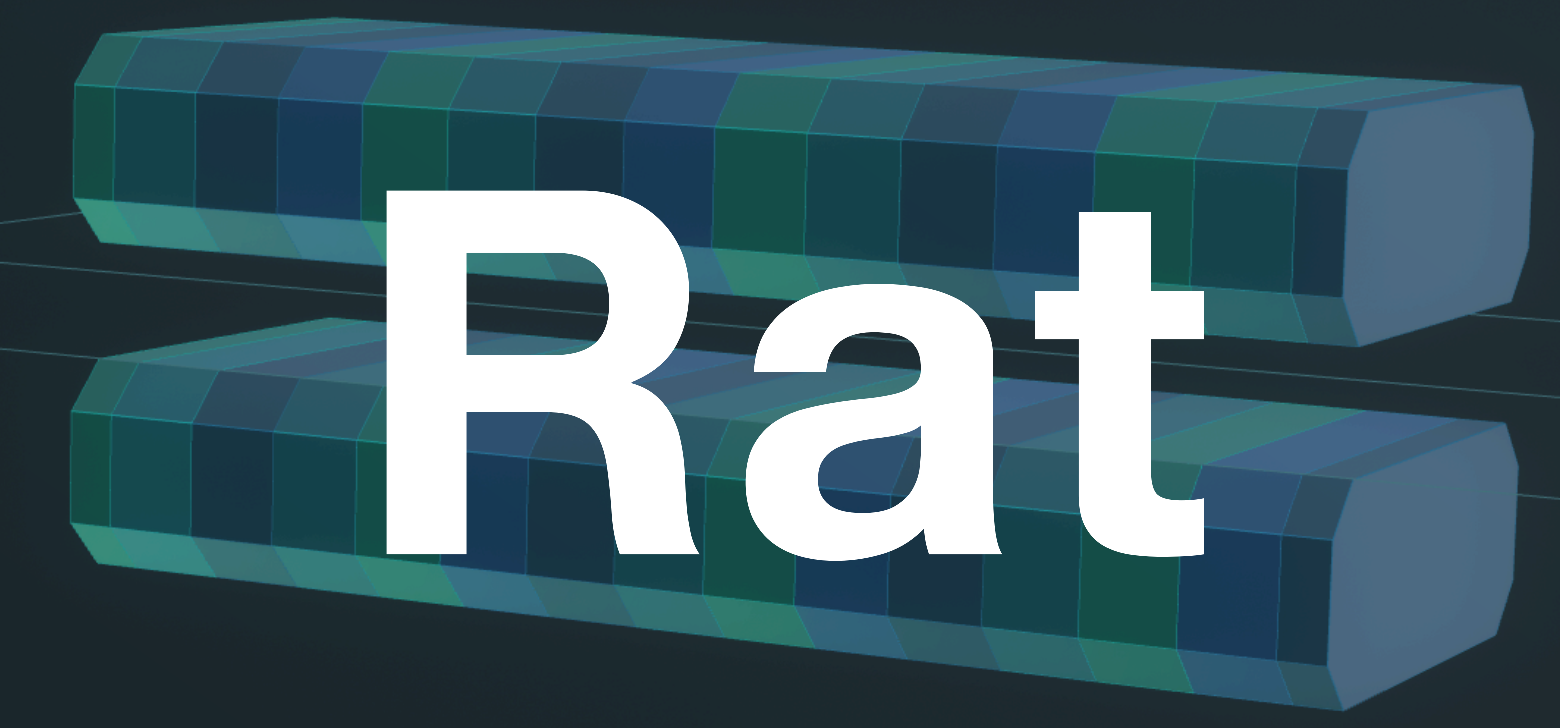
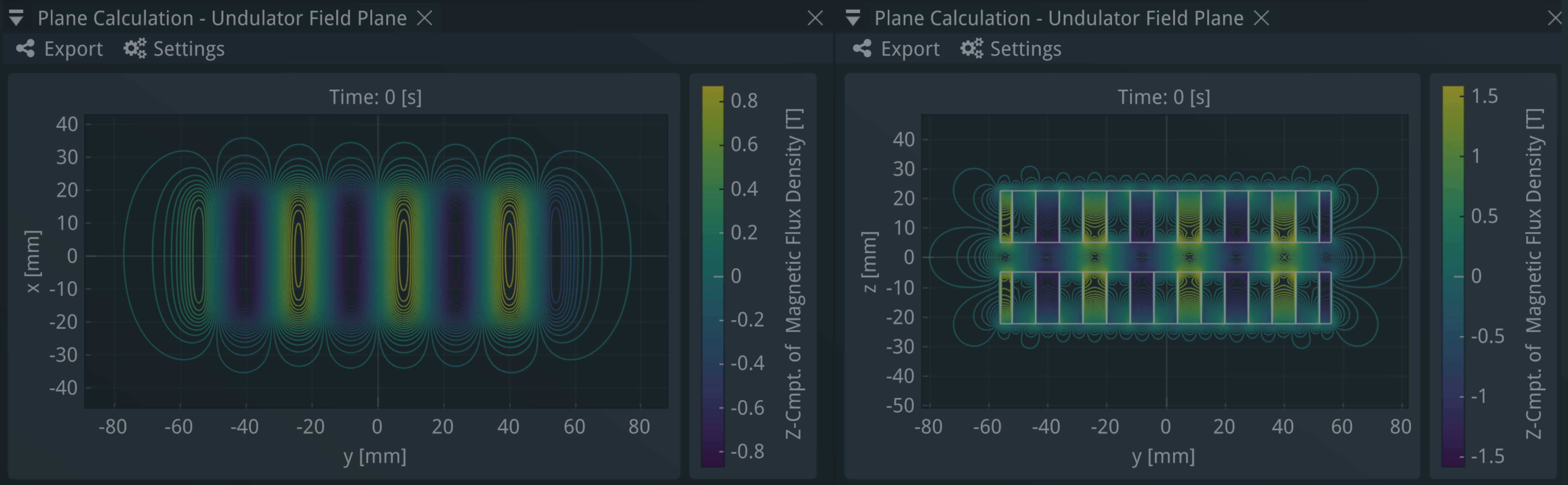


# Rat

## Integral Methods for Magnetostatic Simulation




Nikkie Deelen and Jeroen van Nugteren, Little Beast Engineering — ID26 Workshop, Deauville, France — 2026

# Motivation

## Magnetostatic simulations for accelerator magnets and insertion devices involve:

- Open magnetic domains
- Long-range magnetic interactions
- Nonlinear ferromagnetic materials
- Permanent magnet assemblies
- Repeated optimization and field evaluation

Classical FEM approaches:

- Require meshing large surrounding air regions
- Introduce artificial outer boundaries
- Increase memory usage and nonlinear solve cost for large open-domain problems

Integral formulations:

- Only discretize magnetic regions
- Open-boundary conditions are inherently satisfied
- Naturally suited for magnetostatic field interactions

*Rat combines:*

Biot–Savart field evaluation with the Multi-Level Fast Multipole Method, a Volume Integral Method for non-linear materials, and parametric magnet geometry modelling

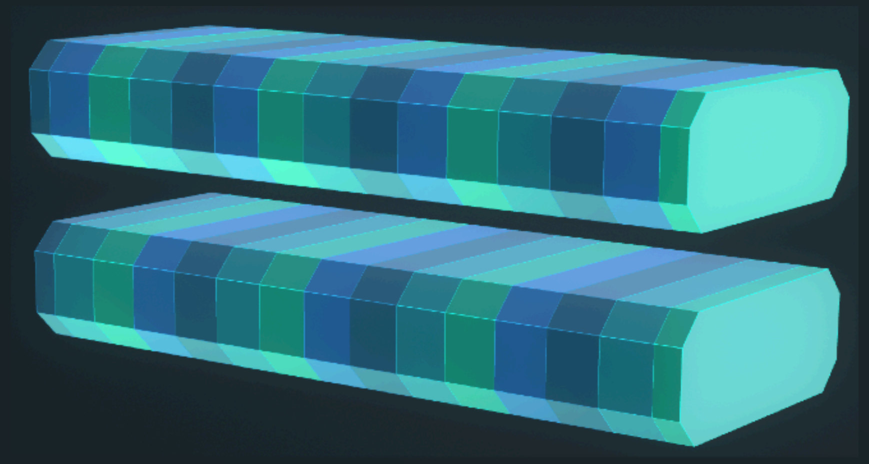
# What is Rat?

*RAT is an open-source framework for 3D magnet modelling and magnetostatic simulation based on integral formulations, fast multipole acceleration, and nonlinear magnetic material models.*

## 3D Geometry modelling

- Coil, cable, and solid geometries
- Hierarchical model construction
- Integrated post-processing tools

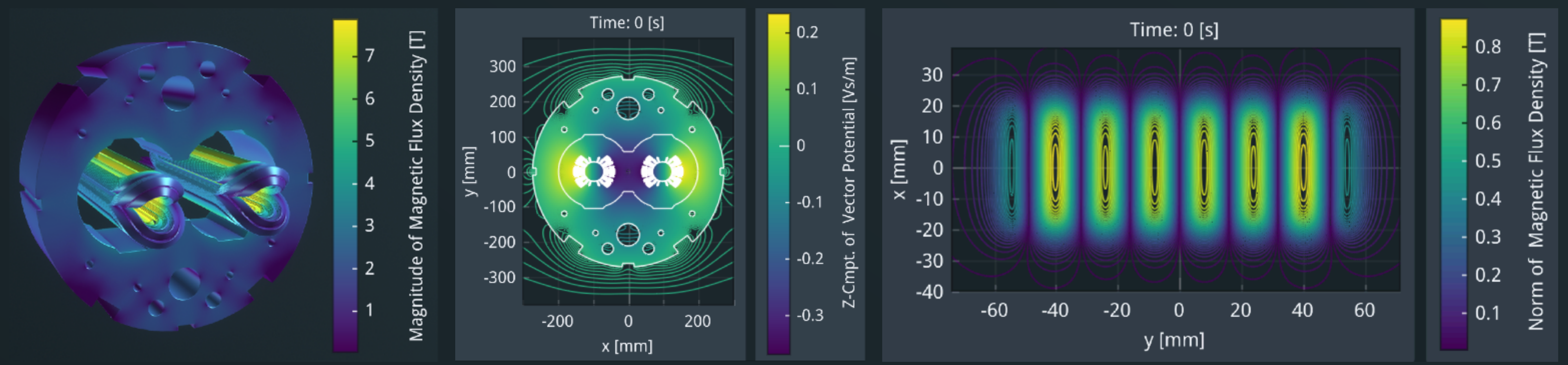
LHC dipole 3D



Simple undulator

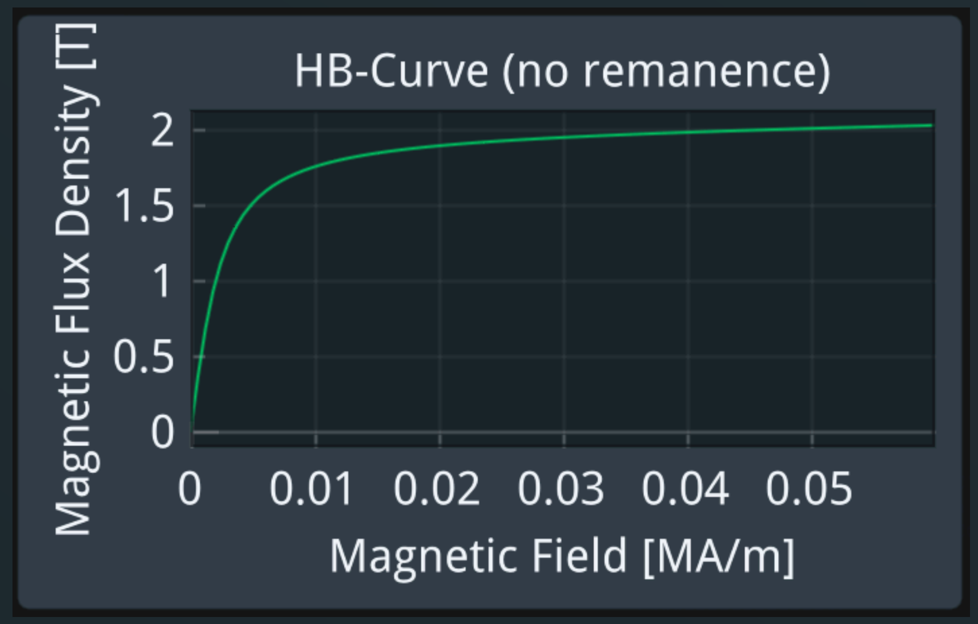
## Field computation

- Biot–Savart formulation
- Multi-Level Fast Multipole Method (MLFMM)
  - Efficient long-range interaction evaluation
- GPU acceleration




## Magnetic materials

- Volume Integral Method (VIM)
- Only magnetic regions discretized
- Nonlinear ferromagnetic materials
- Including permanent magnets




*Applications: accelerator magnets, insertion devices, electromagnets, and permanent magnet systems*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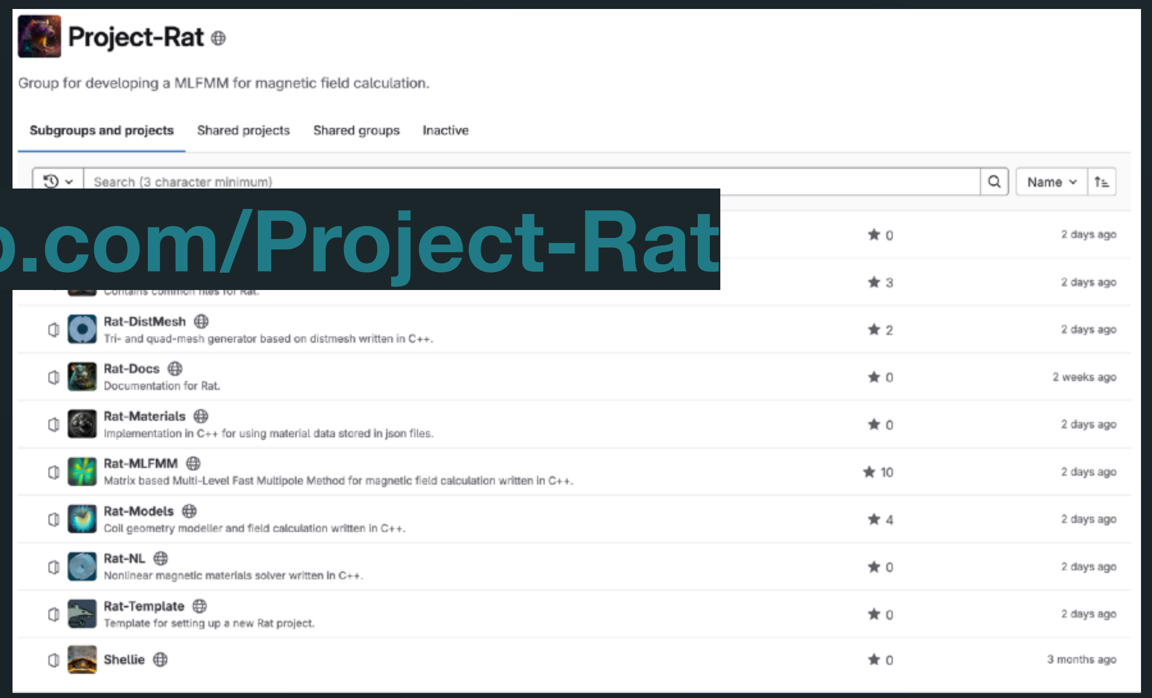
# What is Rat?

## Rat Library

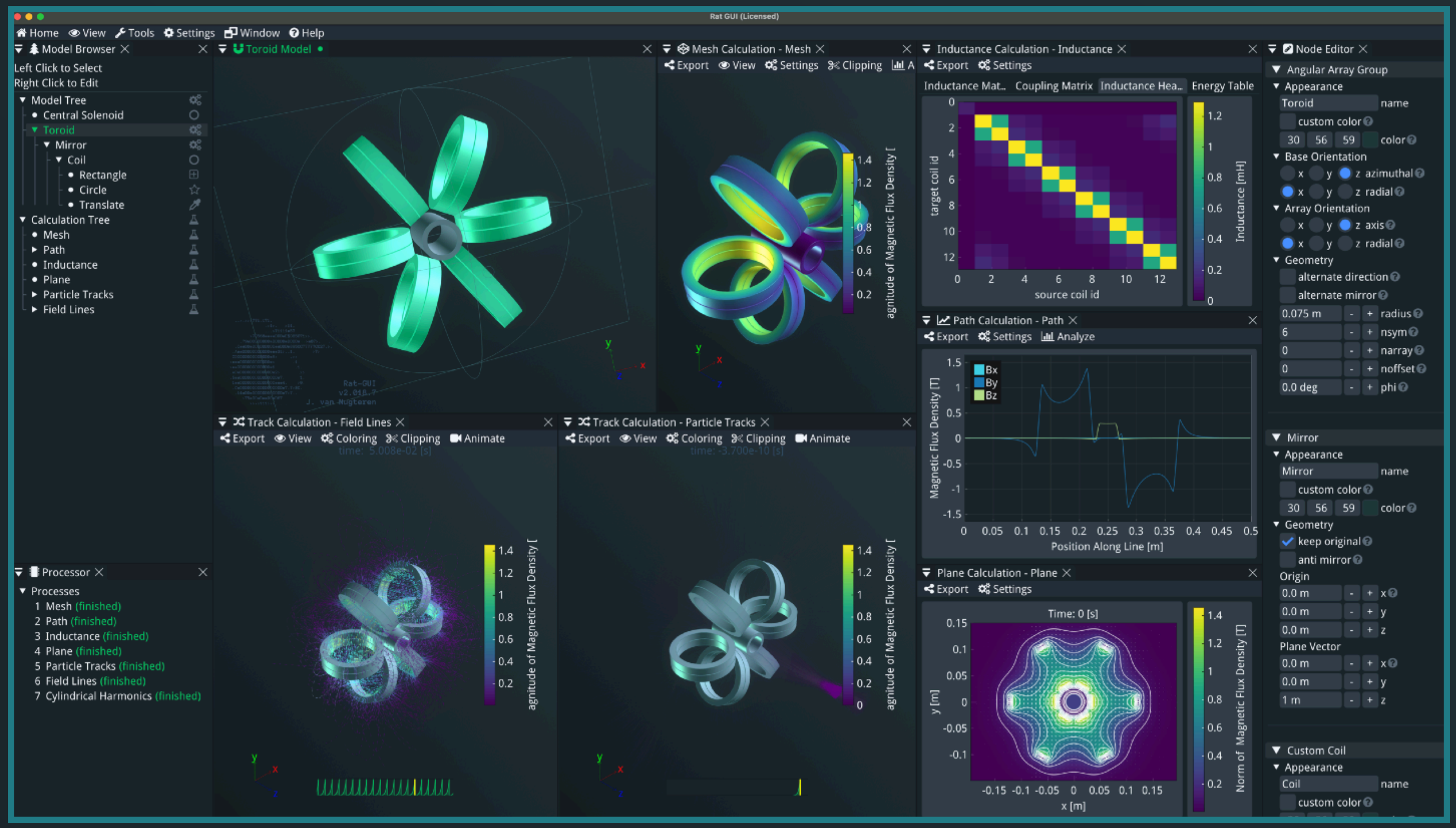
- **Open-source** modelling and simulation software for magnet engineers
- Written in modern, **object-oriented C++**
- Designed for **accurate simulations** of electromagnetic fields, inductance, quench protection, particle tracking and more
- **Modular and extensible:** build your own workflows
- Available on GitLab:

GitLab® https://gitlab.com/Project-Rat




## Rat Graphical User Interface

- A user-friendly graphical interface for Rat
- No coding required: full access to Rat's capabilities
- Visual modelling of coils, magnets, and iron structures
- Access to all modelling and calculation tools of the Rat library
- Same speed and accuracy, but ultimate convenience



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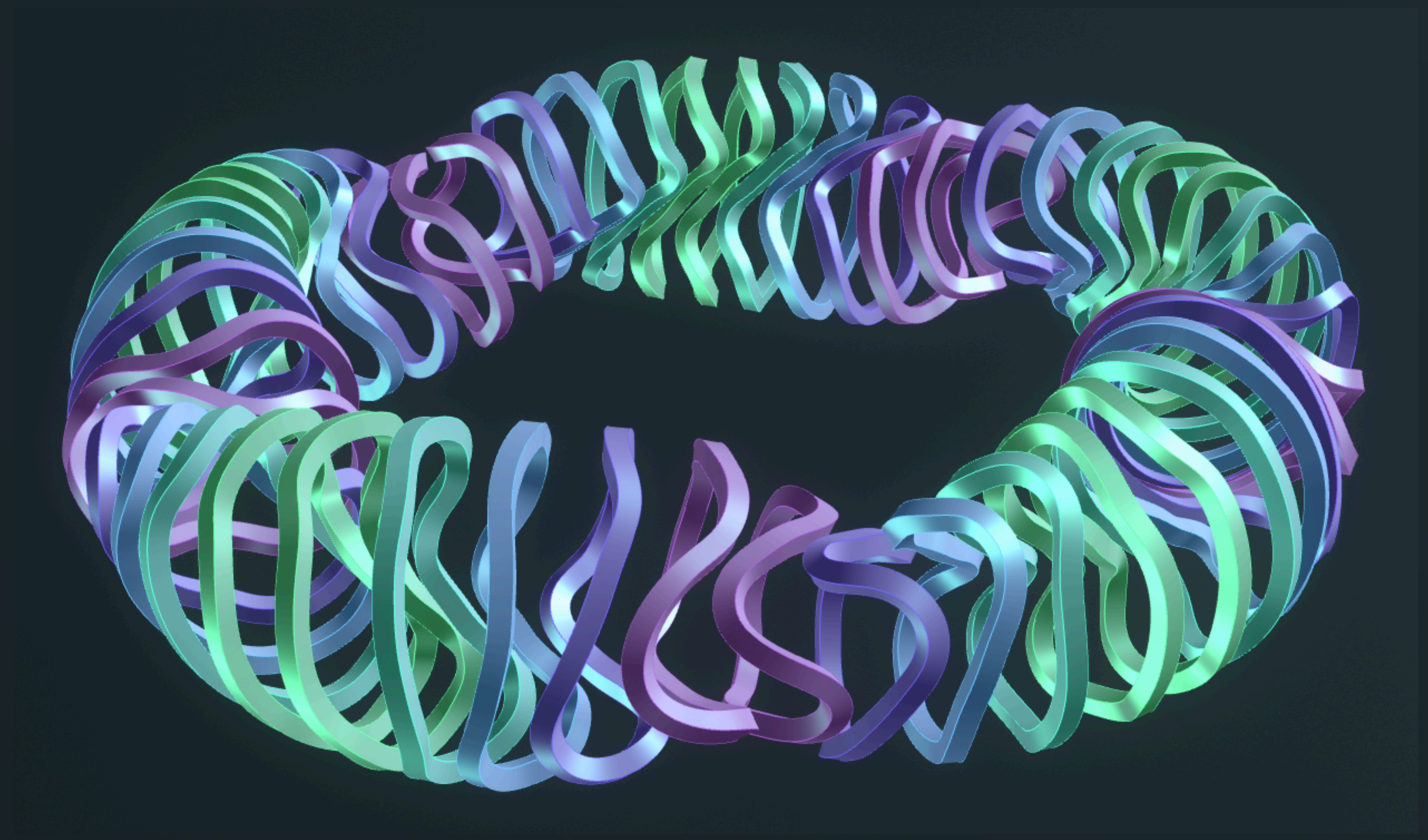
# Calculating Magnetic Fields With Multi-Level Fast Multipole Method



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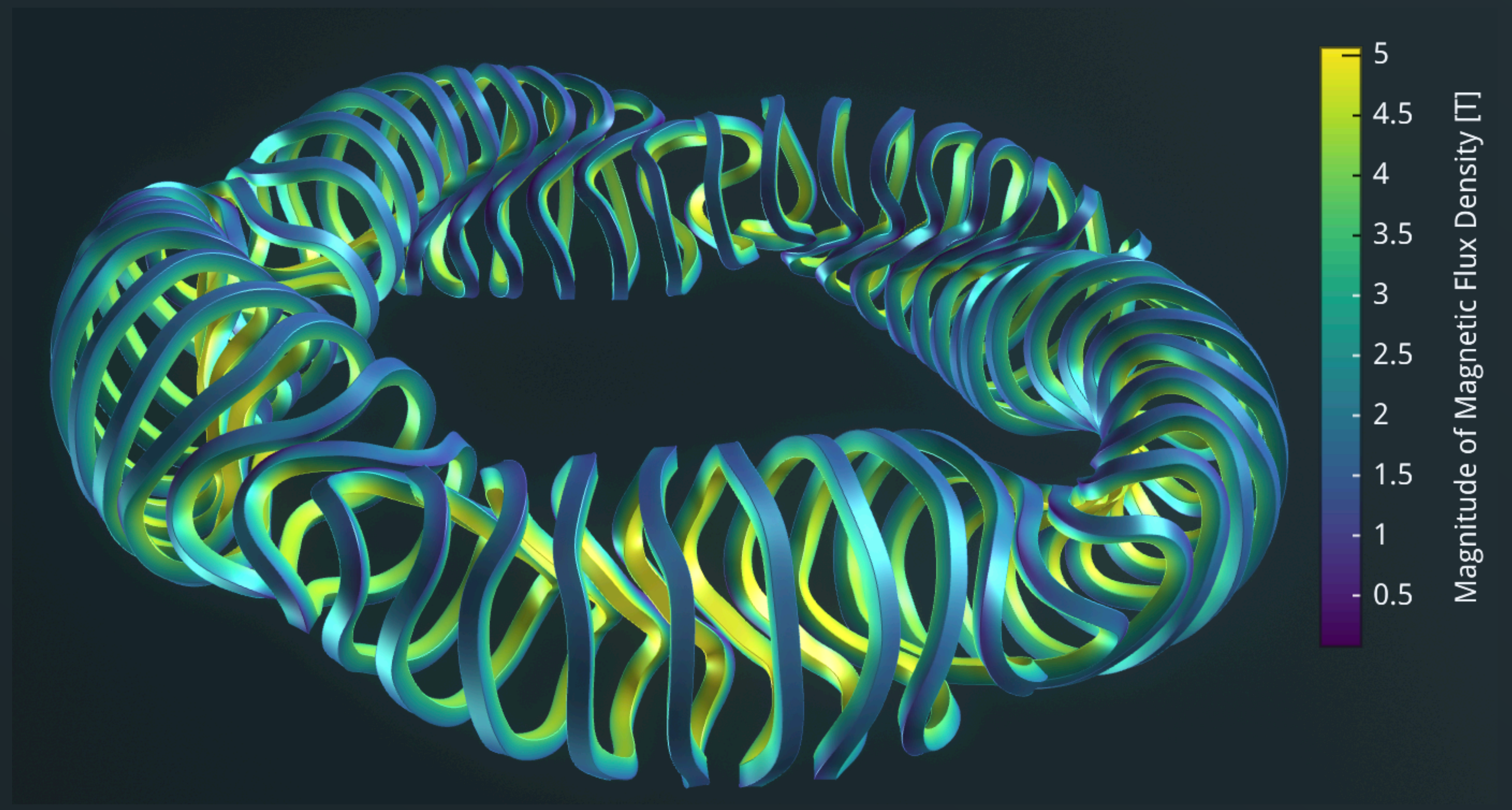
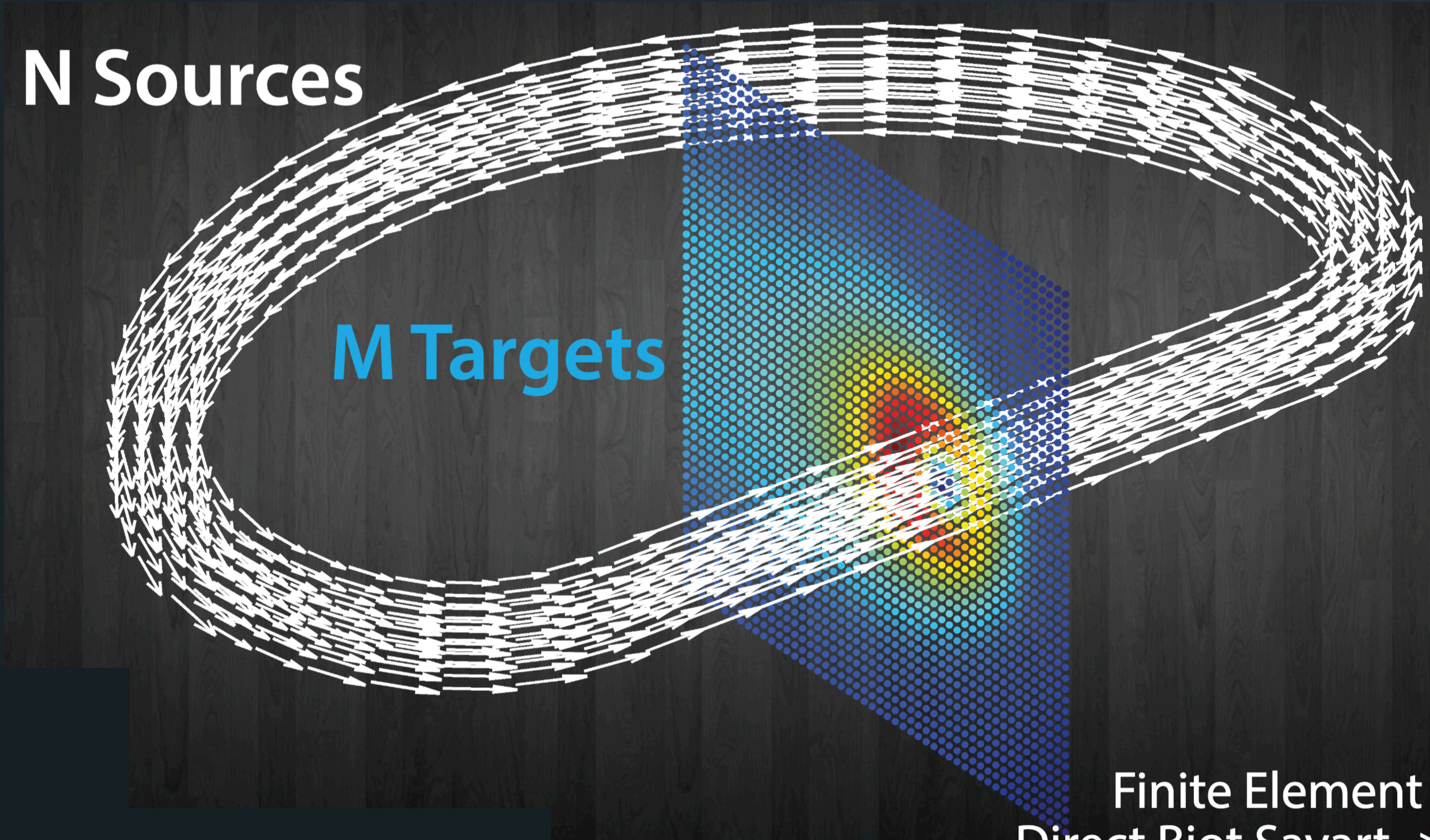


# Calculating the magnetic field from a coil

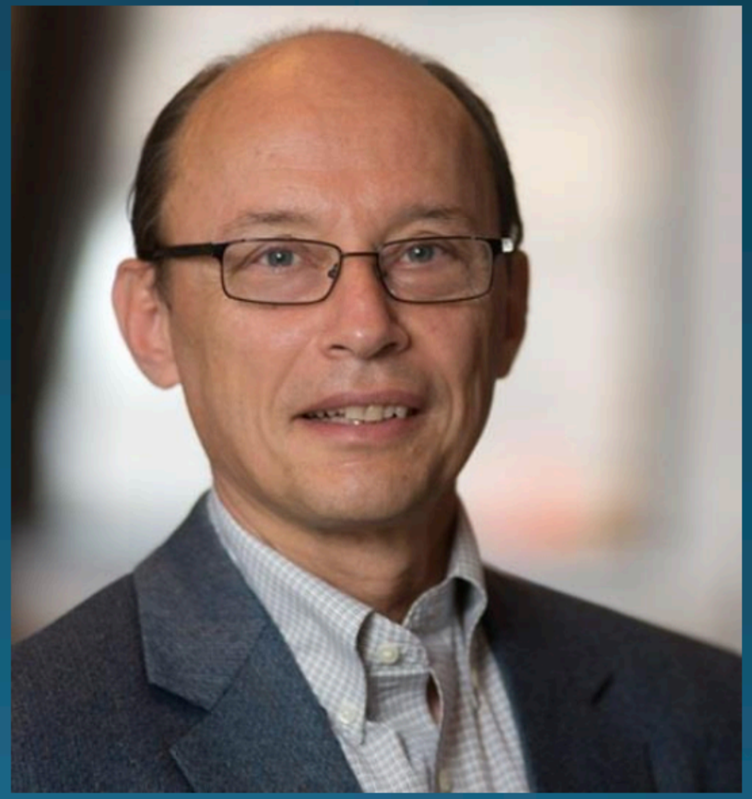



Finite Element Method?
Direct Biot Savart -> O(NxM)
MLFMM -> O(N+M)

# Top 10 Algorithms of the 20th Century

- According to SIAM [1]
    1. 1946 - Monte Carlo method
    2. 1947 – Simplex method for linear programming
    3. 1950 – Krylov subspace iteration methods
    4. 1951 – Decompositional approach to matrix computations
    5. 1957 – Fortran optimizing compiler
    6. 1959 – QR algorithm
    7. 1962 – Quicksort
    8. 1965 – Faster Fourier Transform
    9. 1977 – Integer relation detection algorithm
    10. 1987 – Fast Multipole Algorithm (FMM)

Fathers of FMM



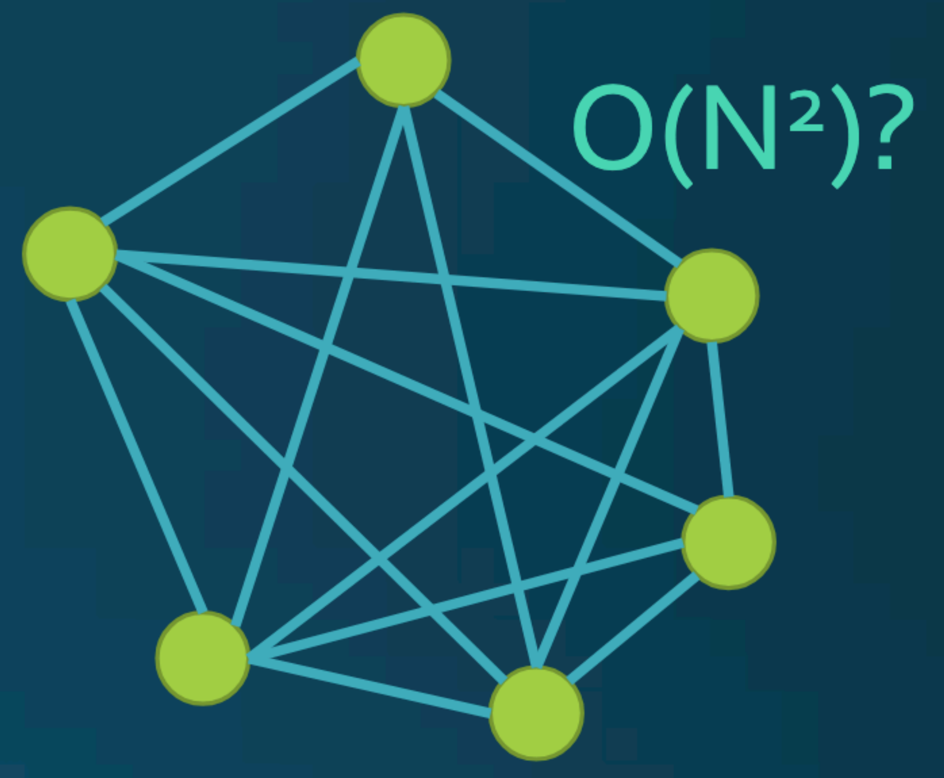
Leslie Greengard

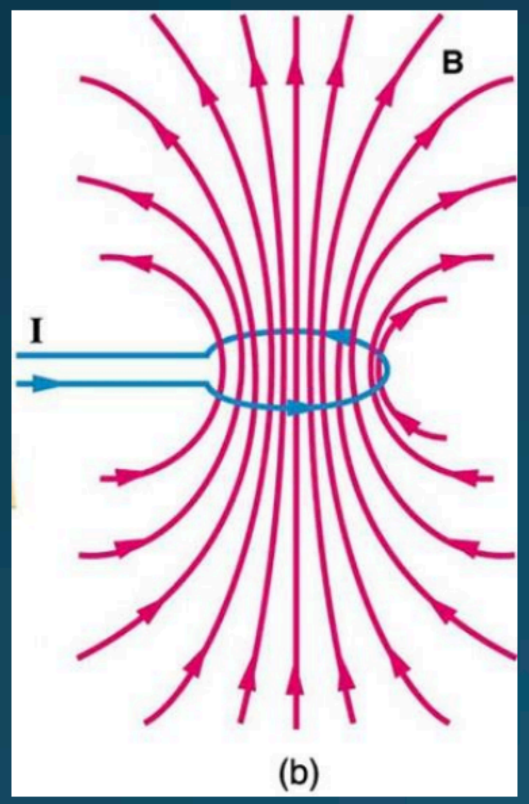
Vladimier Rokhlin

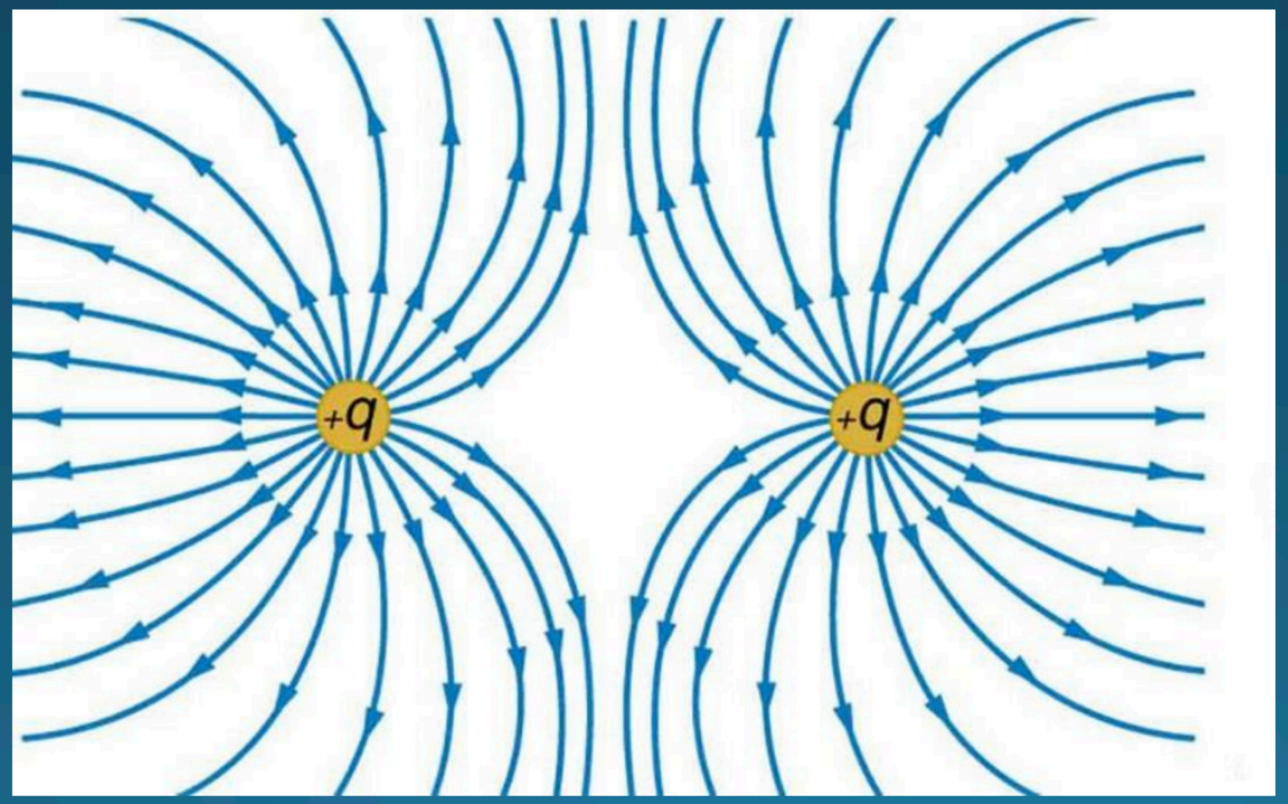
[1] Barry A. Cipra, "The Best of the 20th Century: Editors Name Top 10 Algorithms", SIAM News, vol 33, number 4

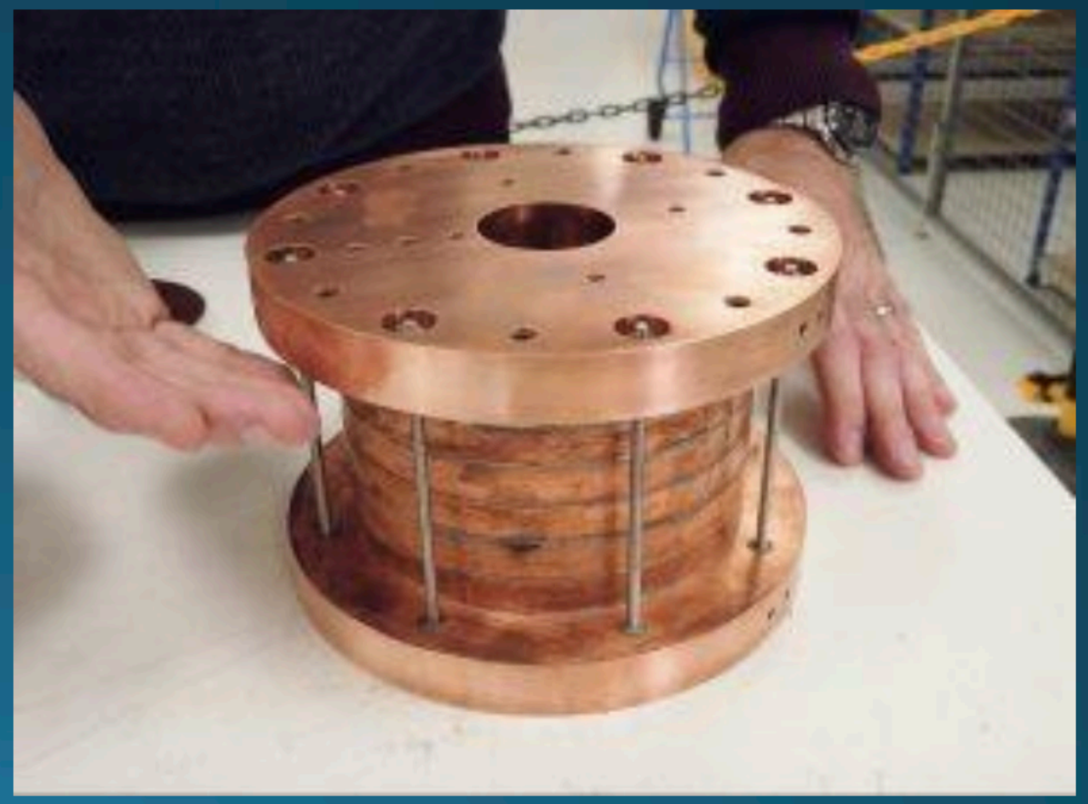
# What is so great about it?

- It can solve basically all N-body problems involving some form of field with O(N) complexity!
  - Electro Statics – Calculate electric potential and field from point charges
  - Magneto-Statics – Calculate magnetic vector potential and field from line currents/magnetic moments (Biot Savart)
  - Electrical Network Solver – Calculate induced voltage based on current change in all elements (must have for NI coils)
  - Astrophysics – Calculate gravitational potential and field
  - Plasma physics?
  - Many more. (=

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# Rat for Coils: Biot-Savart with MLFMM

## In integral magnetostatics:

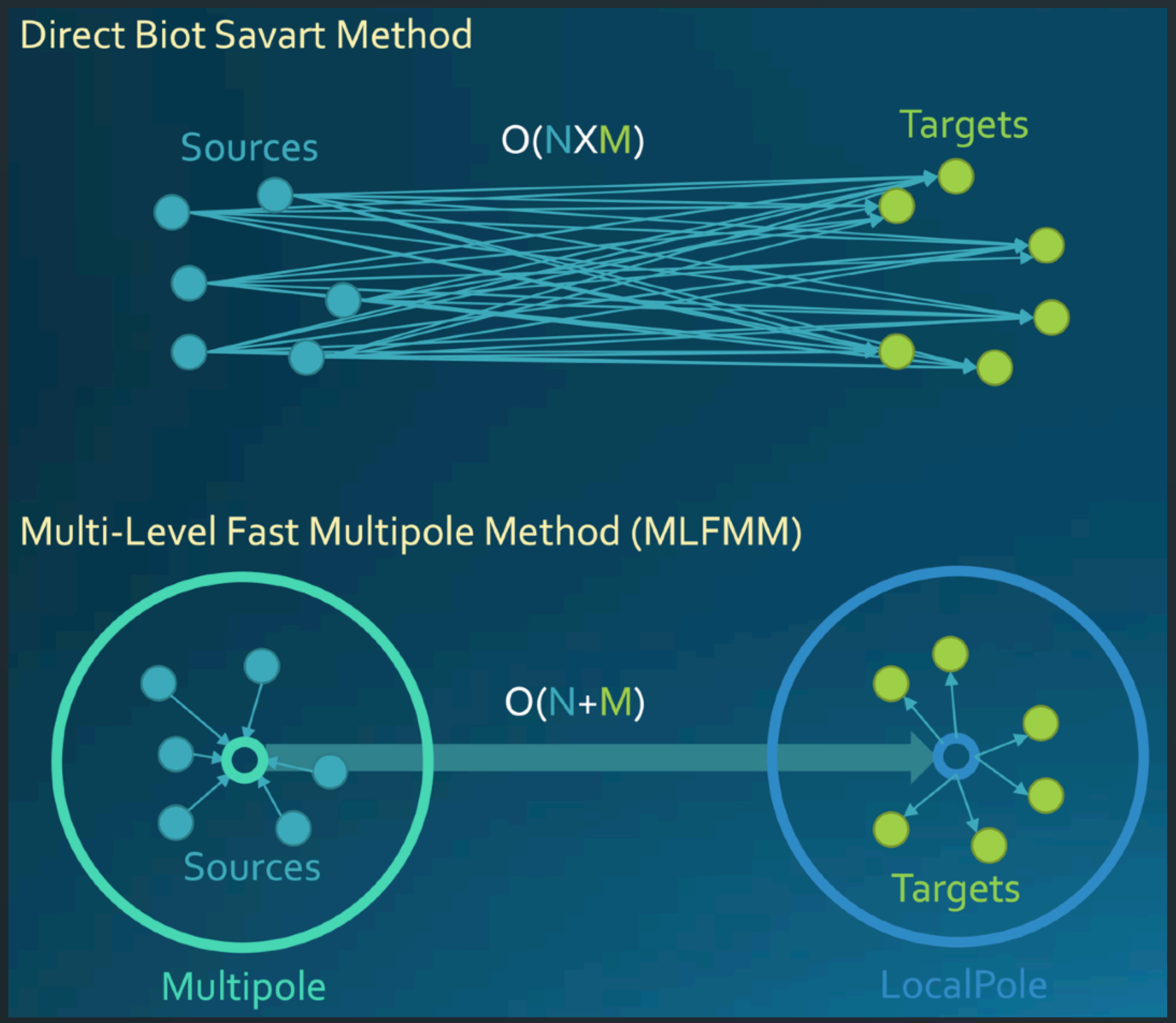
- Magnetic fields are computed from source to target interactions
- In Rat: using the vector potential version of Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\mathbf{l}' \times \mathbf{R}}{|\mathbf{R}|^3}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\mathbf{l}'}{|\mathbf{r} - \mathbf{r}'|}$$

## Direct interaction evaluation means:

- Every source $n_i$ interacts with every target $m_j$
- Computational complexity: $O(NM)$
- Large magnet systems quickly become computationally expensive




*Rat accelerates Biot-Savart integration using the Multi-Level Fast Multipole Method (MLFMM) such that the computational complexity becomes $O(N + M)$*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# MLFMM applied to Biot-Savart

- The Multi-Level Fast Multipole Method (MLFMM) is a highly efficient algorithm for evaluating long-range interactions in large systems
  - Electrostatics
  - Gravitational potential
- It was originally developed for problems involving $1/|\vec{r} - \vec{r'}|$, Green's functions, such as:

Electrostatics

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r})}{|\vec{r} - \vec{r}'|} dV'$$

Gravitational potential

$$\phi(\vec{r}) = -G \int \frac{\rho(\vec{r})}{|\vec{r} - \vec{r}'|} dV'$$

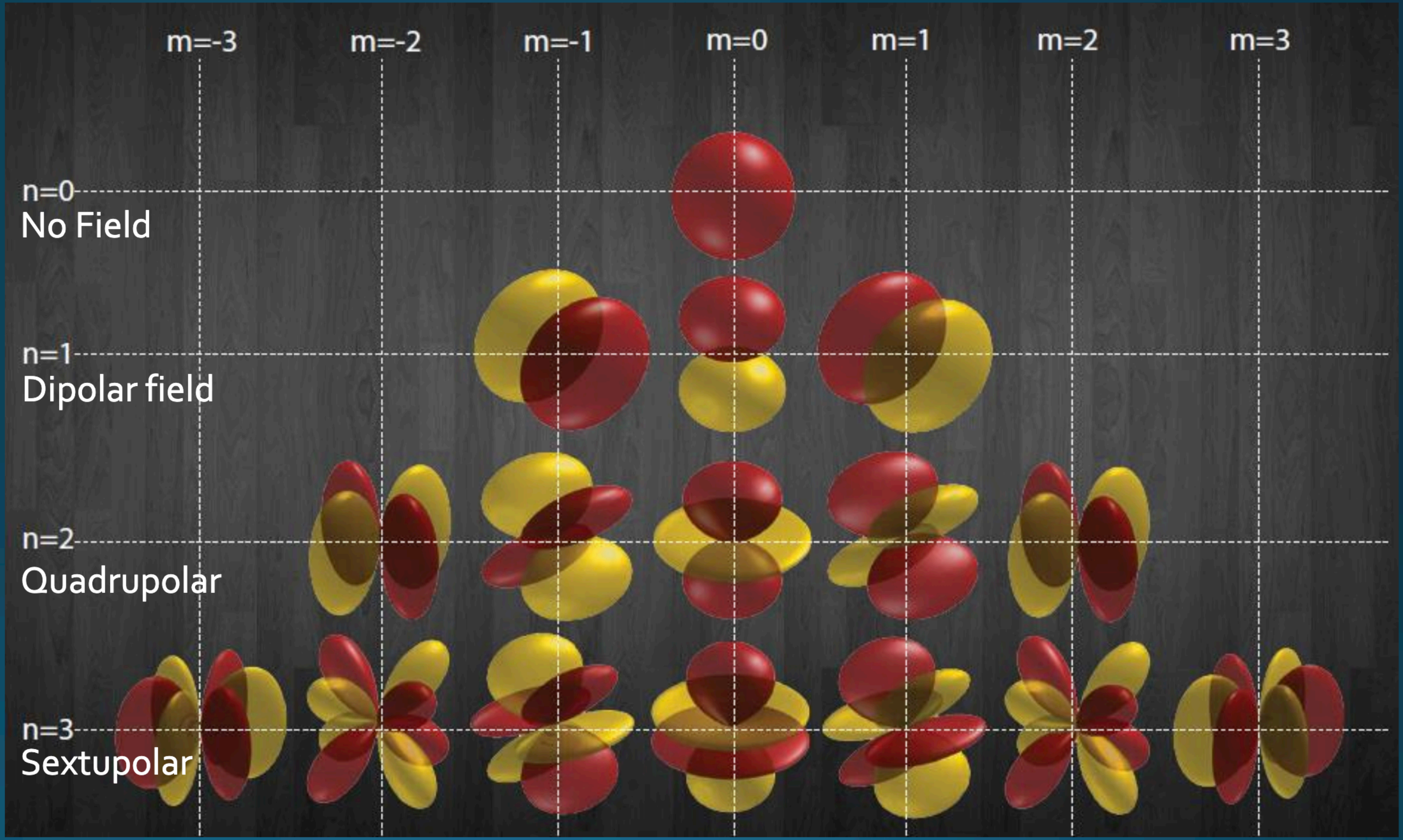
- For magnetostatics one can use "Biot-Savart" for the vector potential: $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l}'}{|\vec{r} - \vec{r}'|}$

# Spherical Harmonics

- Spherical harmonics are all possible solutions of the Laplace equation

$$Y_n^{-m}(\alpha,\beta)=\sqrt{\frac{(n-|m|)!}{(n+|m|)!}}\, e^{im\beta} P_n^{|m|}(\cos\alpha).$$

- By using a set of harmonics for each component of the vector potential $(A_x, A_y, A_z)$ it is possible to represent any possible magnetic field
- Where $P_n^m$ are the Legendre polynomials calculated recursively





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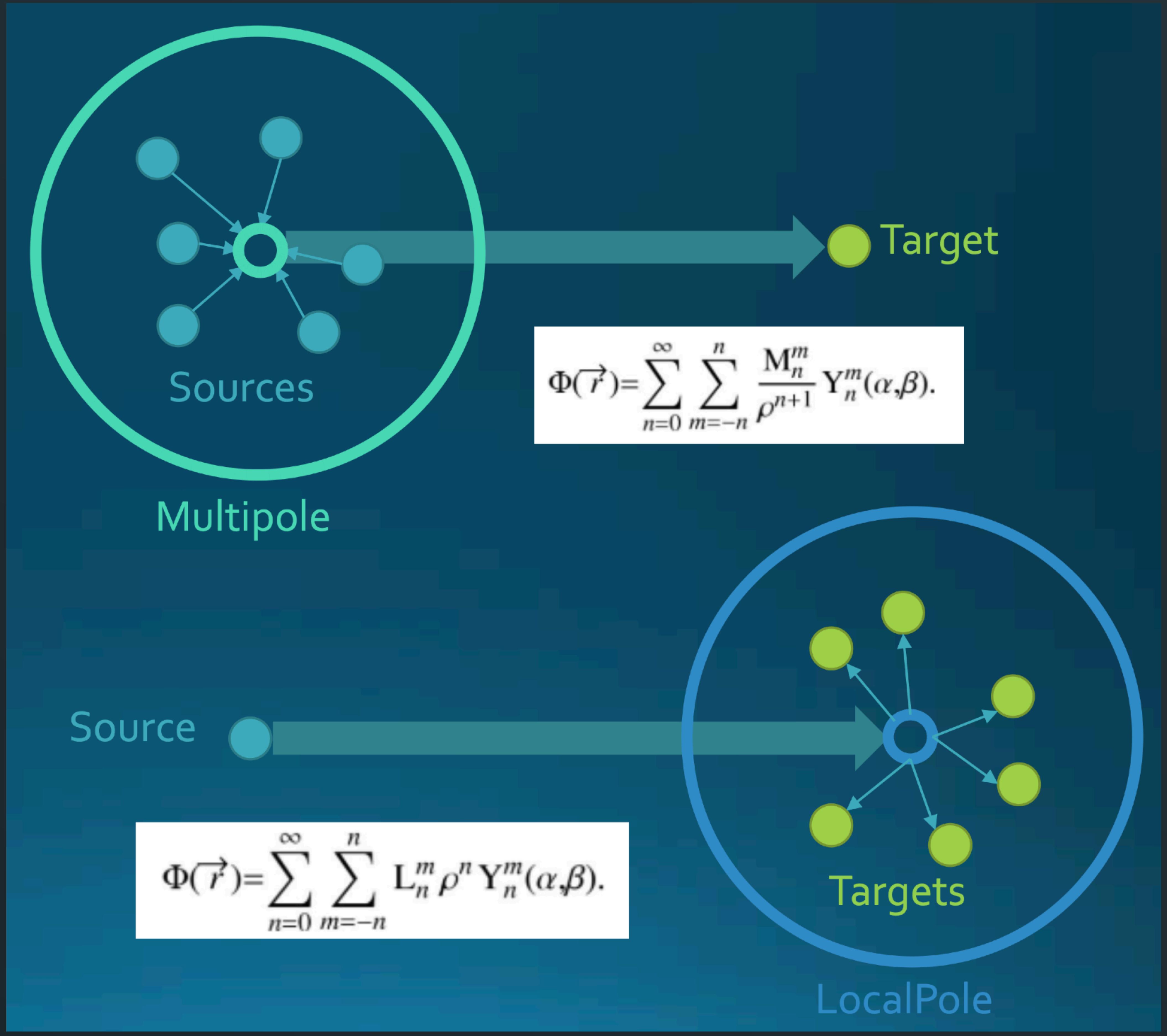
# MLFMM: Multipoles and Localpoles

- MLFMM use multipoles and localpoles to represent the field of the sources
- Multipoles represent the field of sources inside a sphere at any target point outside the sphere. It compresses the field to a few components of the Legendre polynomials
- Localpoles represent the field of sources outside a sphere at any target point inside the sphere
- Multipoles and localpoles are therefore essentially opposites
- Both multipoles and localpoles are described by spherical harmonics




For more information: https://www.youtube.com/watch?v=n1_HFxnG-8c

# Rat for non-linear materials: VIM

## Volume Integral Method (VIM):

- Only magnetic regions are discretized
- Open-boundary conditions are inherently satisfied
- No surrounding air mesh required

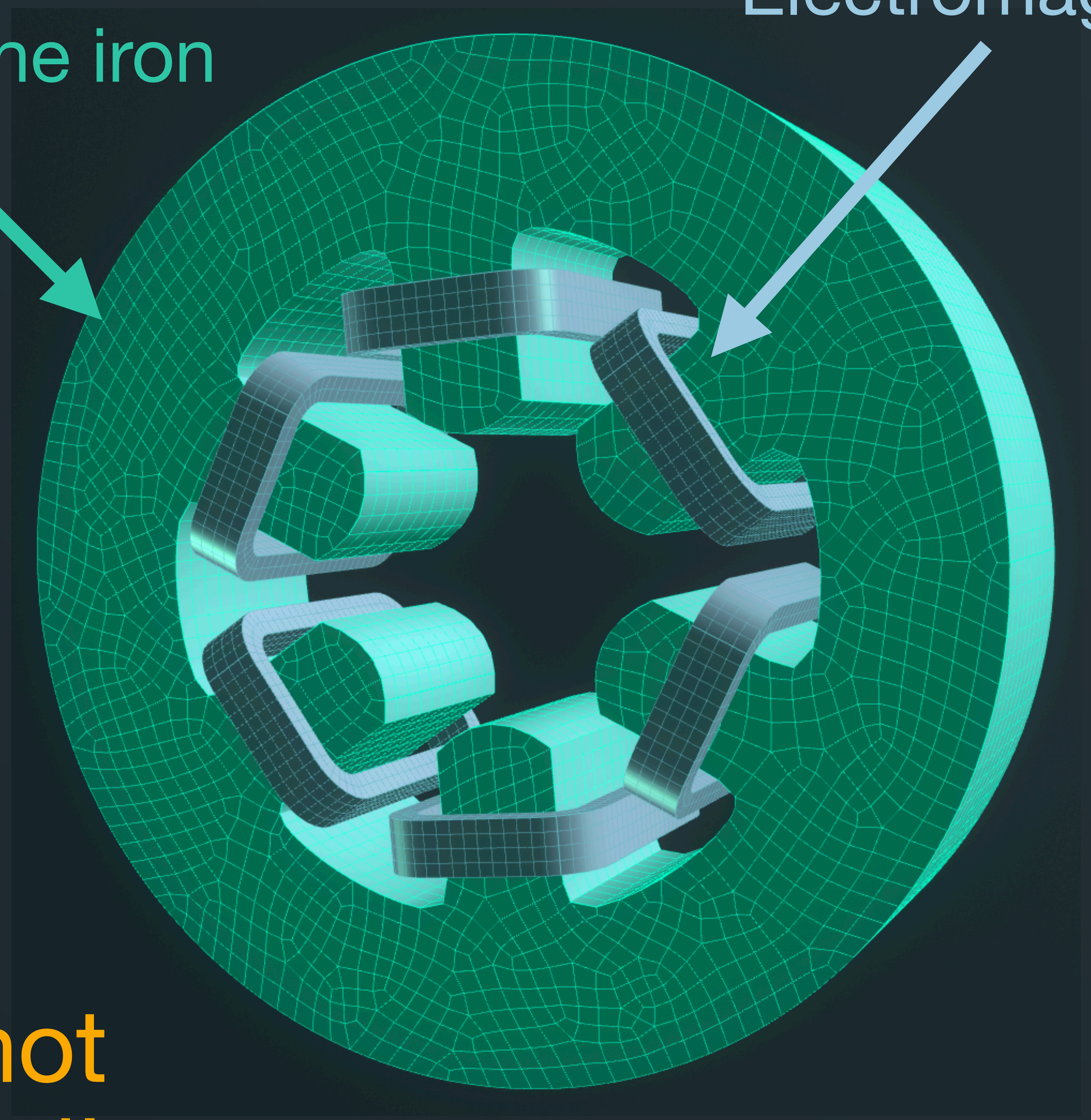
## Nonlinear magnetization

The magnetic flux density depends on the local magnetic field:

- $\mathbf{B} = \mu_0 \mu_r(\mathbf{H})\mathbf{H}$

## Resulting problem:

- Dense long-range interaction system
- Nonlinear material response
- Solved iteratively



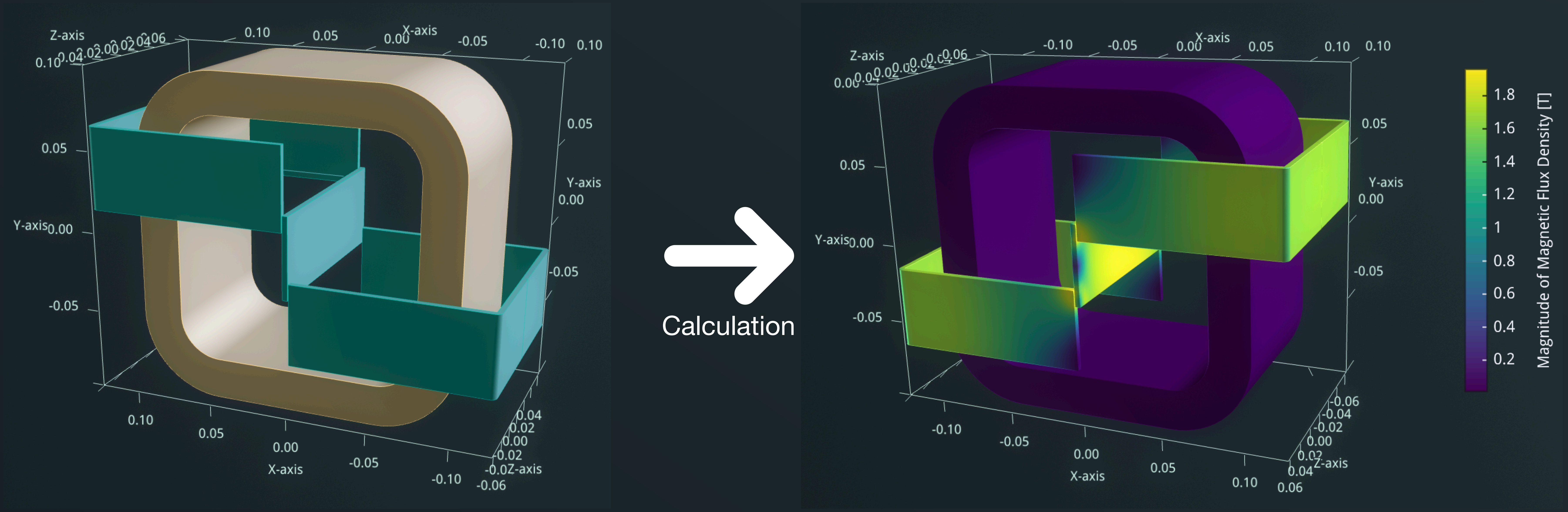
Iron-based Sextupole

For more information: https://www.youtube.com/watch?v=k87lxGyjOmY and https://www.youtube.com/watch?v=ISeok_3EsLE

# Validating VIM with magnetic measurements

## Using the TEAMs (Testing Electromagnetic Analysis Methods) problems

- Validate Rat against magnetic measurements
- TEAM 13 problem: Calculate magnetic field at different points for two steel channel exited by a coil
- HB-curve provided in the problem statement, as well as measure data
  https://www.compumag.org/wp/wp-content/uploads/2018/06/problem13.pdf
- V. Le-Van, G. Meunier, O. Chadebec, and J.-M. Guichon, “A Magnetic Vector Potential Volume Integral Formulation for Nonlinear Magnetostatic Problems,” *IEEE Transactions on Magnetics*, vol. 52, no. 3, pp. 1–4, Mar. 2016, doi: 10.1109/TMAG.2015.2490627.



# Validating Rat with magnetic measurements

## Using the TEAMs (Testing Electromagnetic Analysis Methods) problems

- V. Le-Van, G. Meunier, O. Chadebec, and J.-M. Guichon, “A Magnetic Vector Potential Volume Integral Formulation for Nonlinear Magnetostatic Problems,” *IEEE Transactions on Magnetics*, vol. 52, no. 3, pp. 1–4, Mar. 2016, doi: 10.1109/TMAG.2015.2490627.
- Comparing @ 3000AT and plotting the field along a line in x, we find a match with measured data within 2%

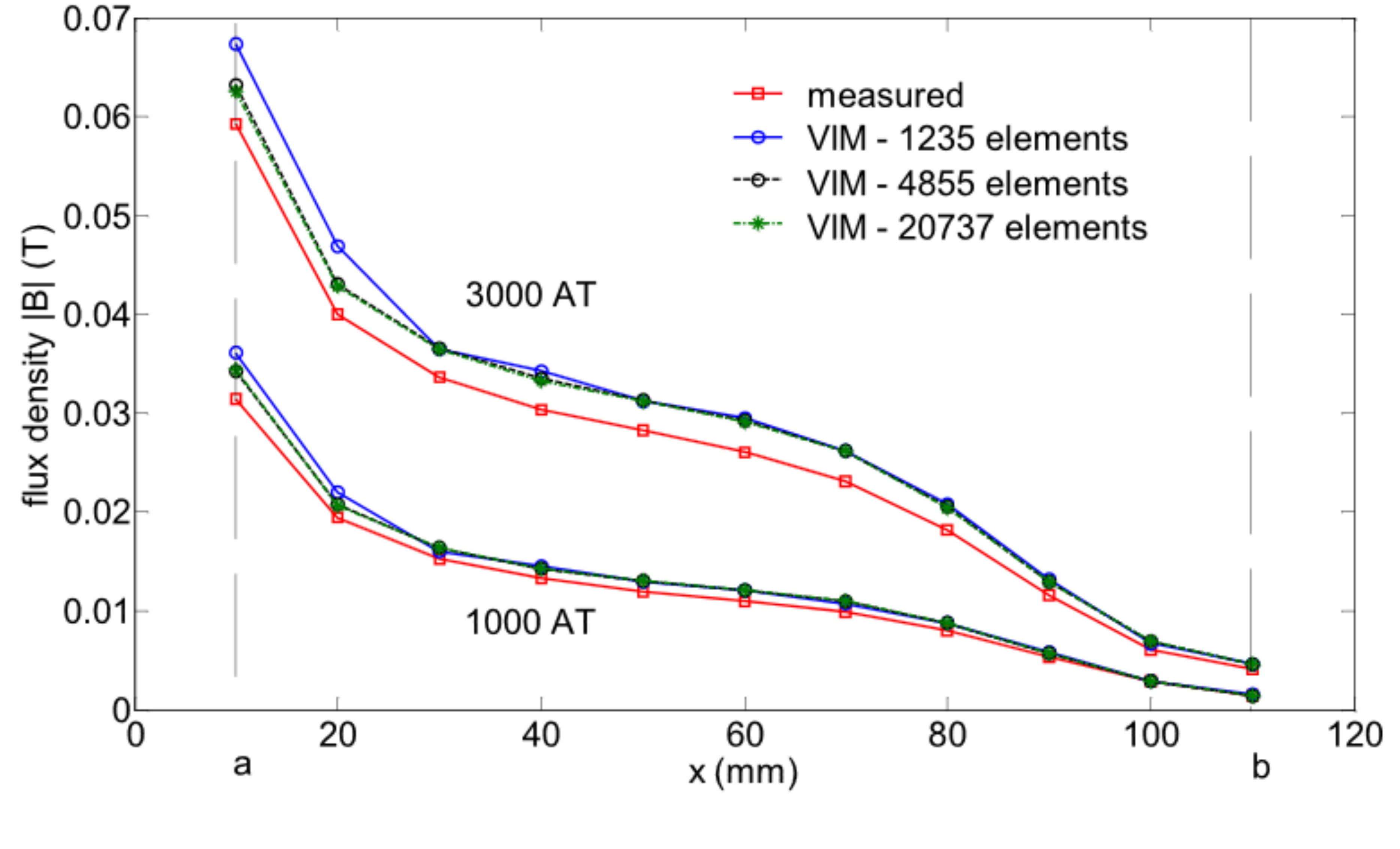
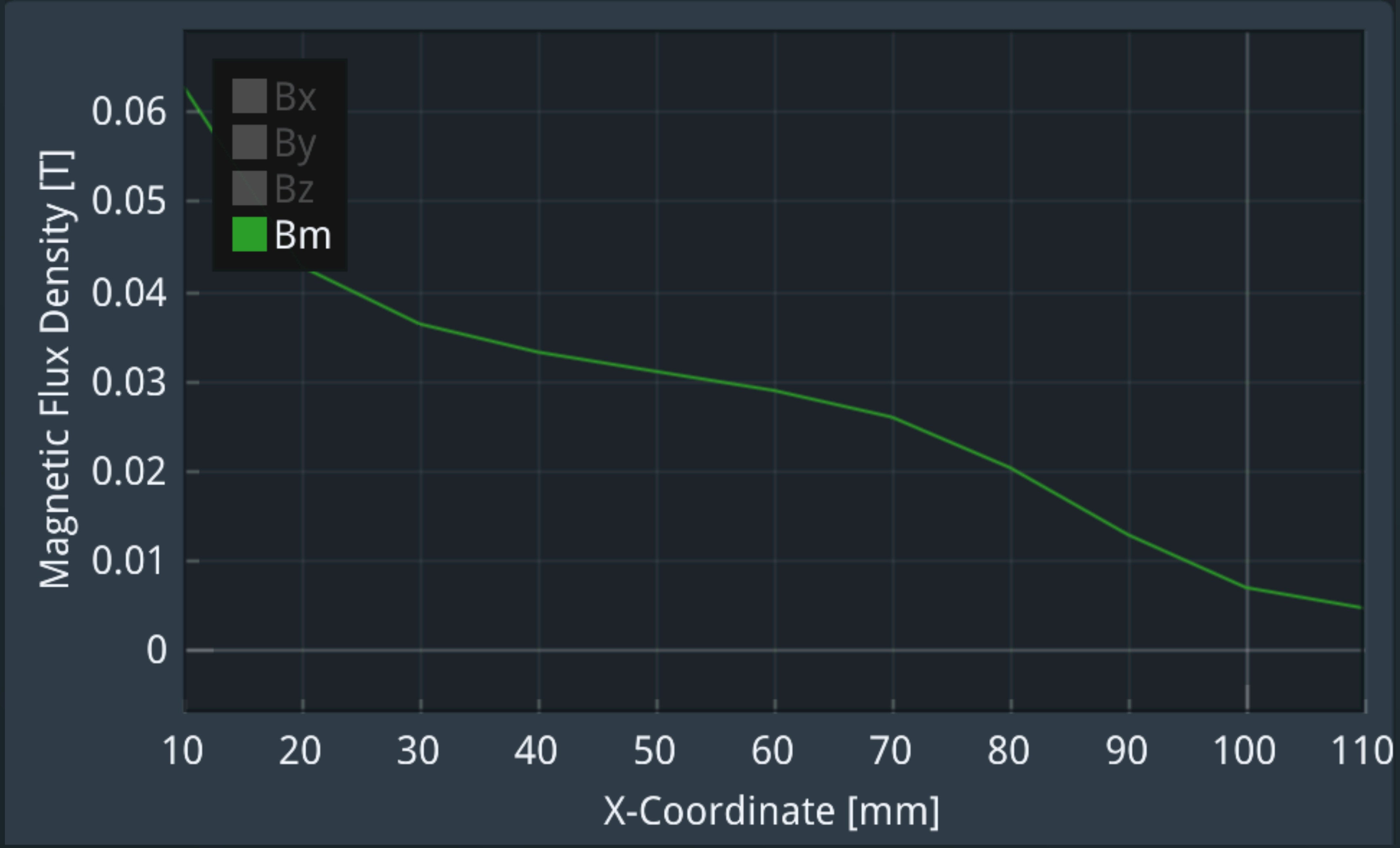


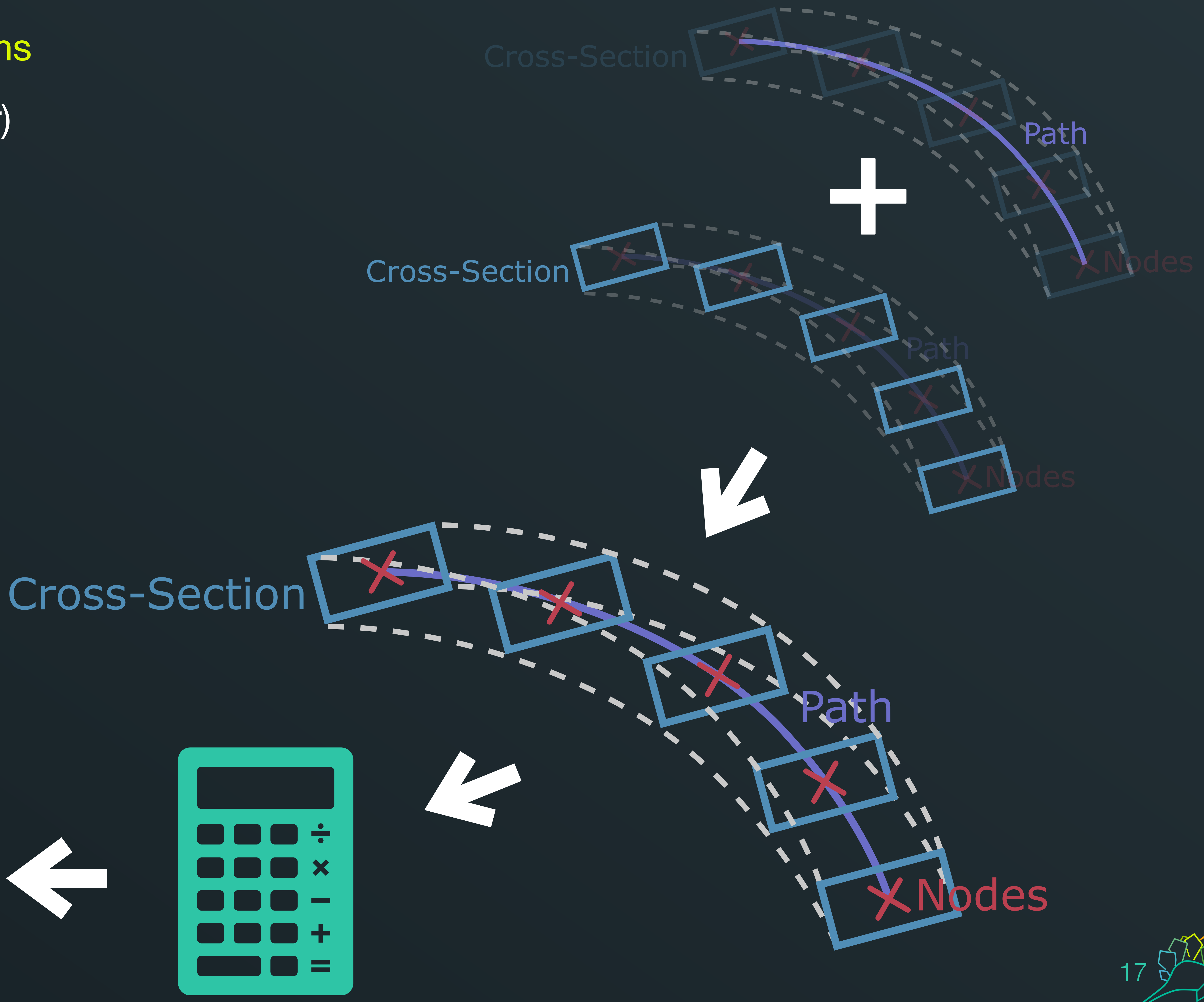

Fig. 7. Spatial distribution of magnetic flux density in air.

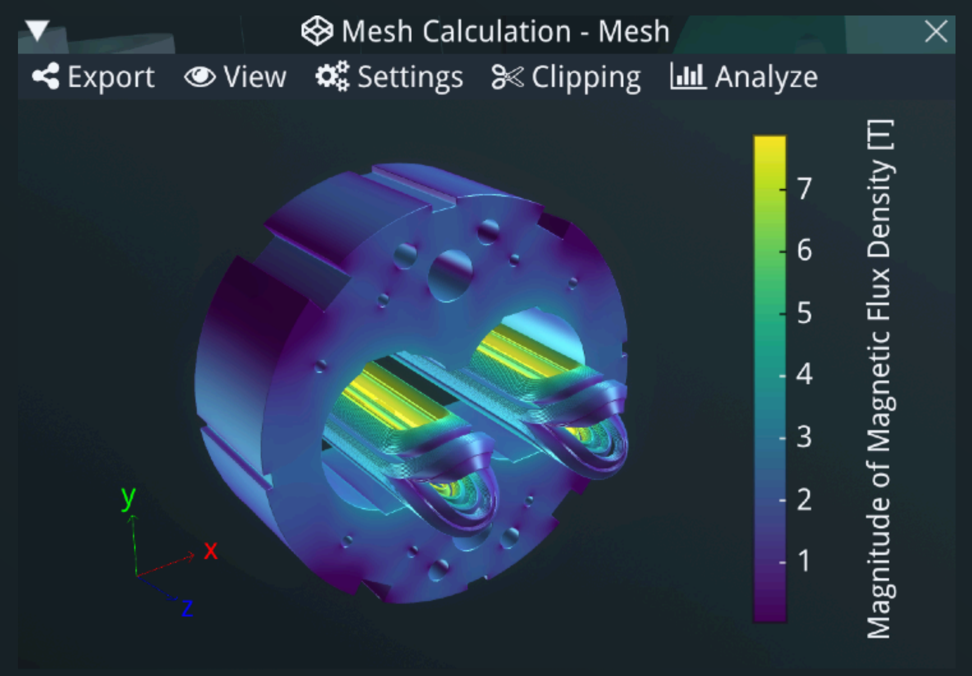


# More than just the mathematical method

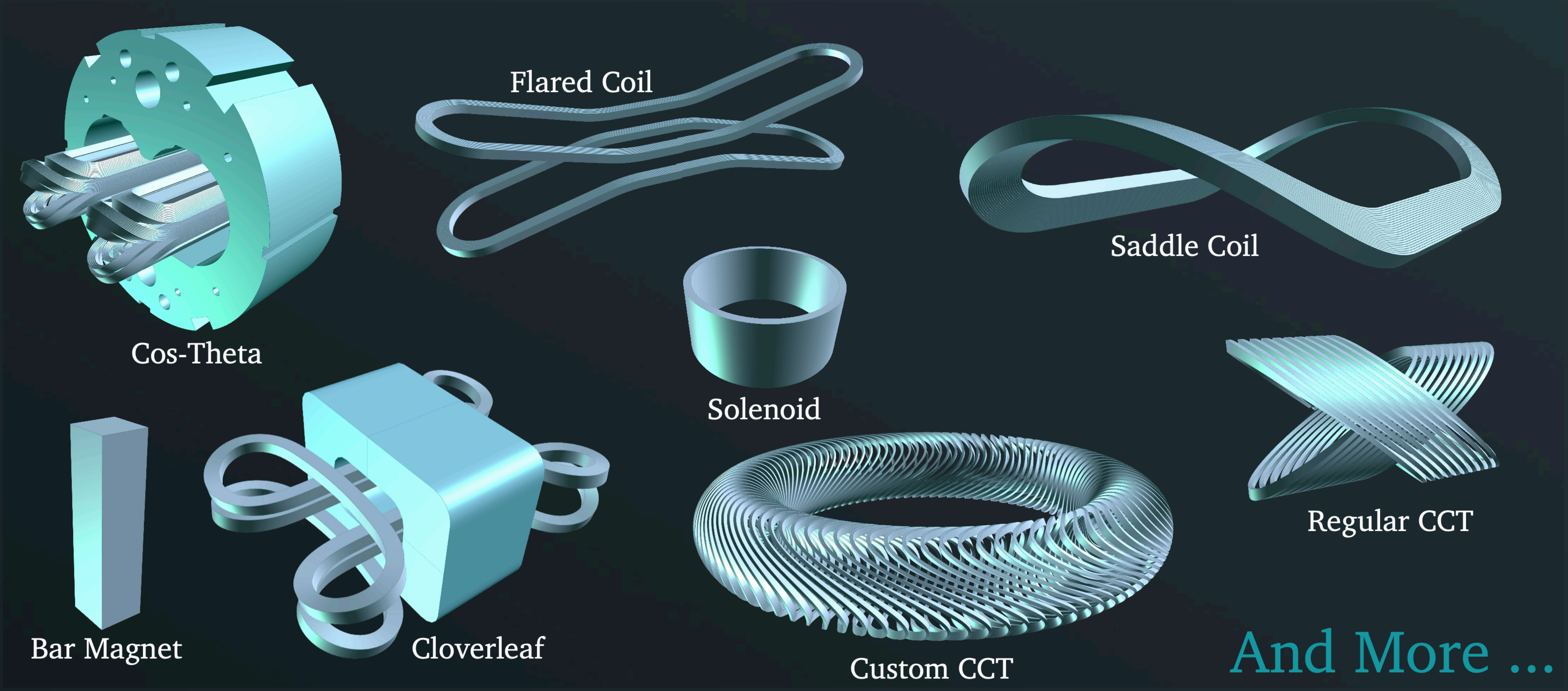
# The building blocks of Rat

- First you model the coil/magnet in three dimensions
    1. Take a path (windings of you conductor)
    2. Extrude a cross-section
- This model is fed to the calculator to obtain:
    - Field maps in 1D, 2D and 3D
    - Harmonics
    - Quench for superconductors
    - Particle tracking
- Results can be analysed in Rat or exported

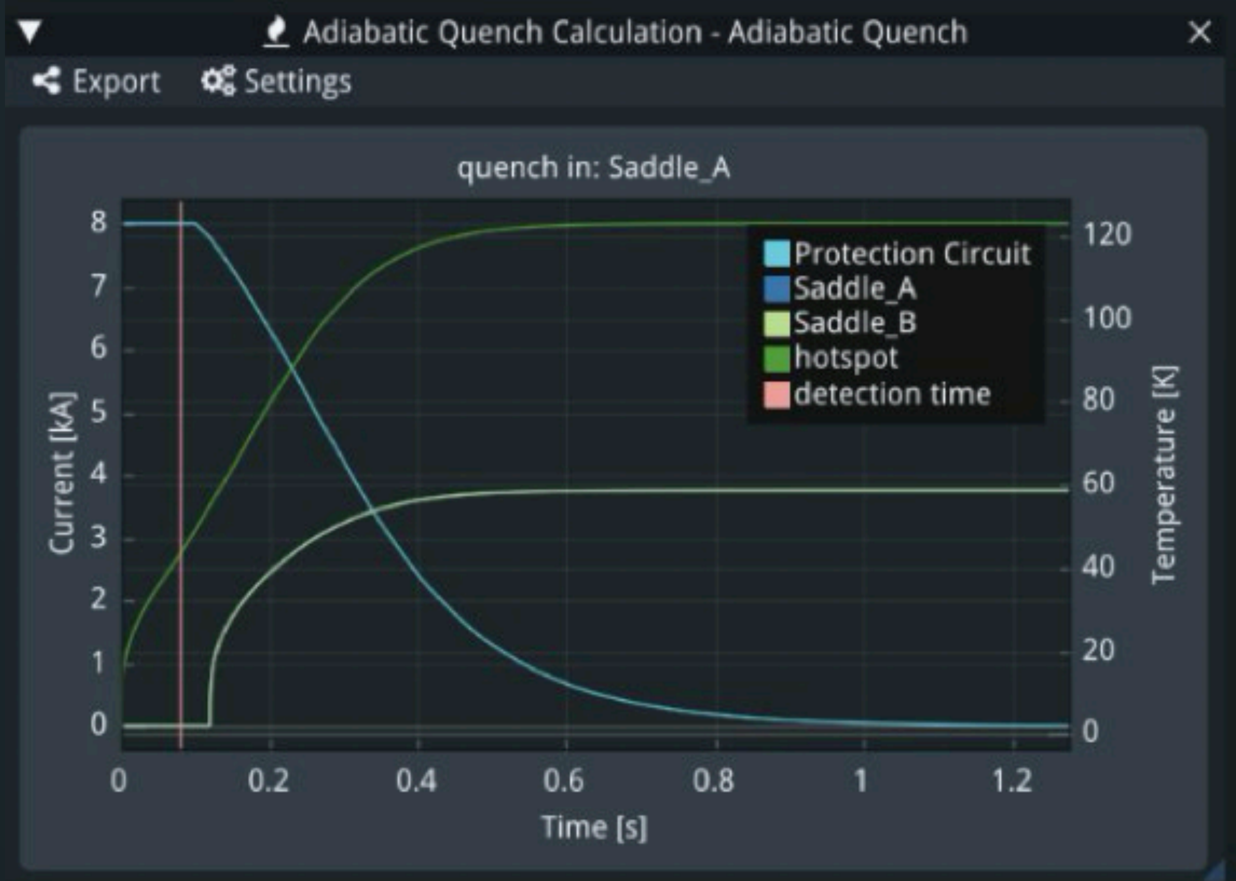


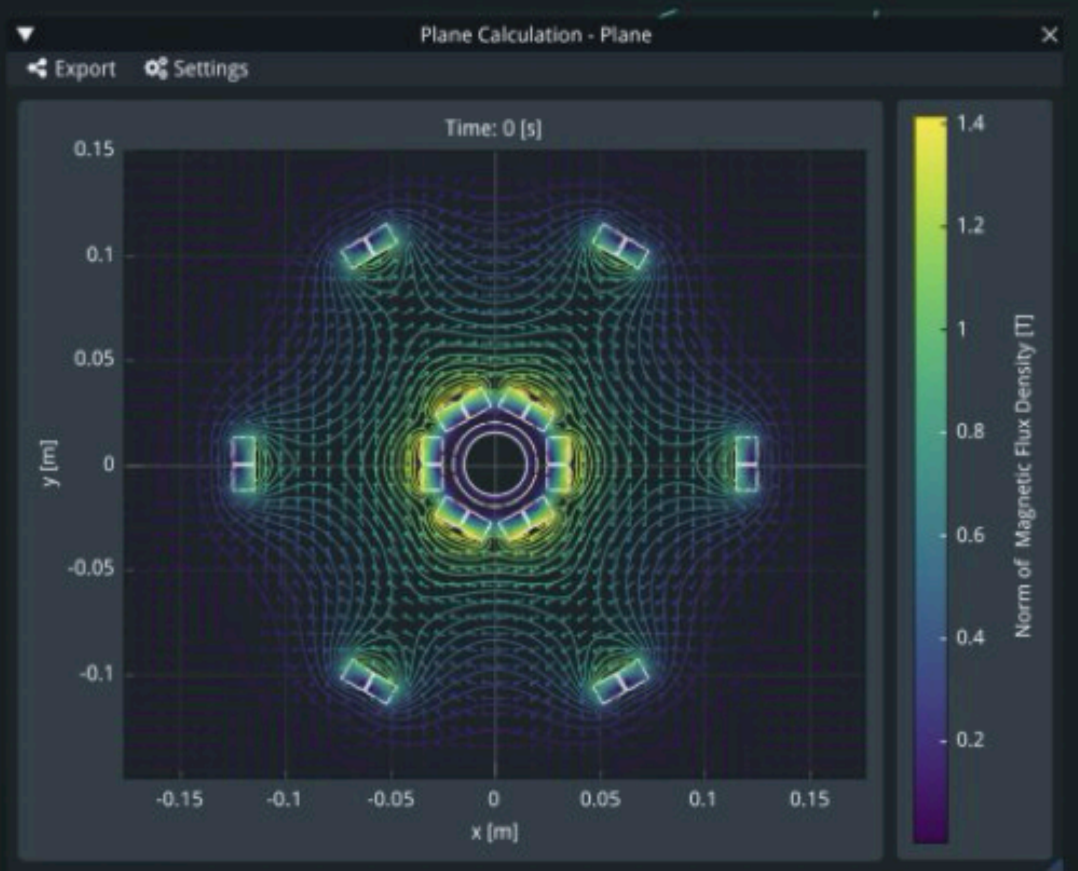
# Many build in shapes



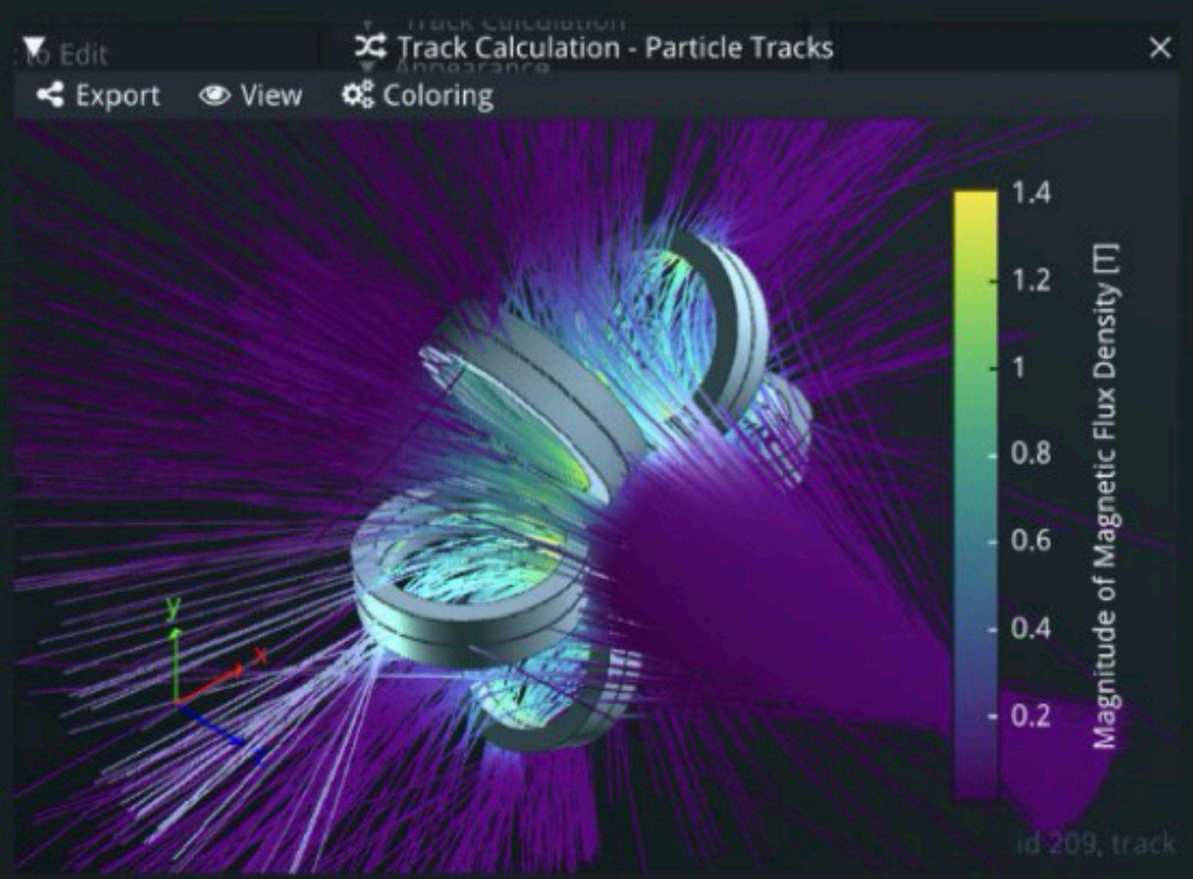
# Many build in calculations



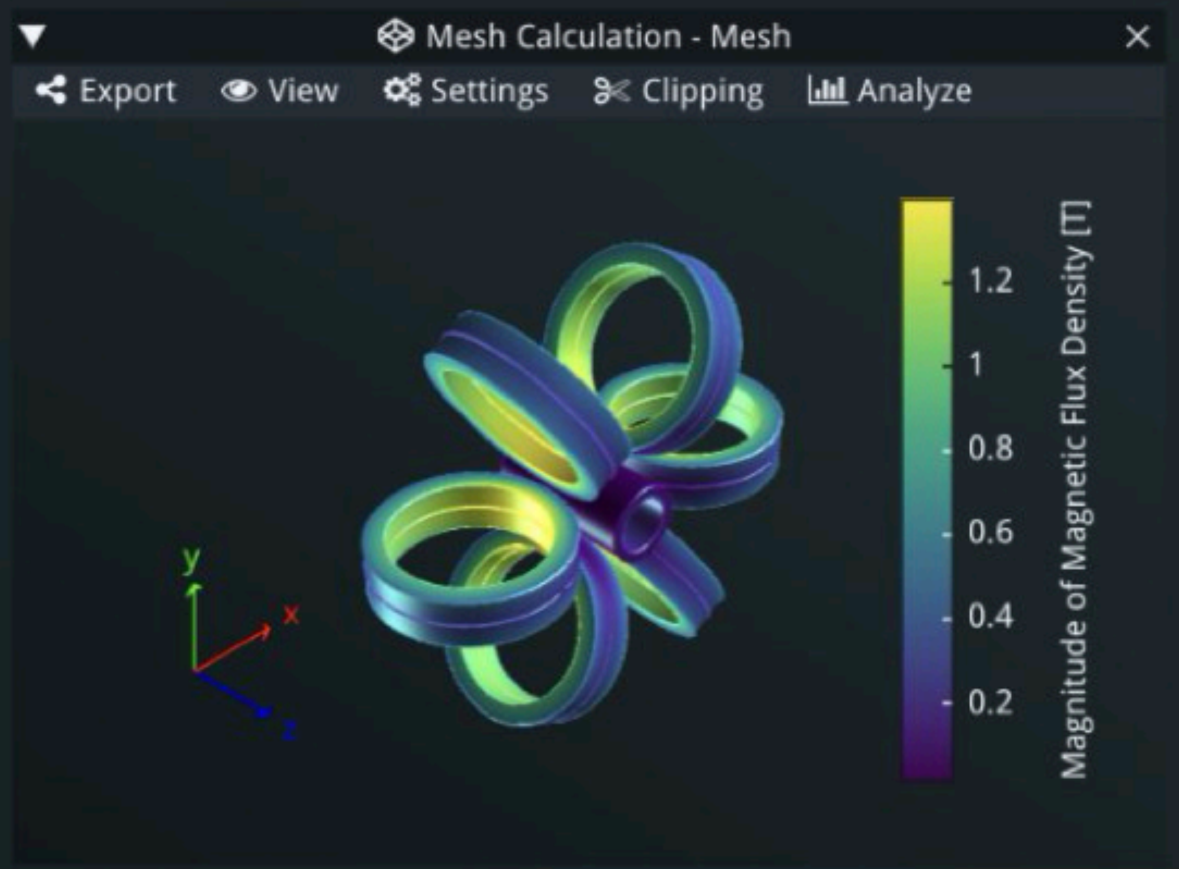

Adiabatic Quench (new)



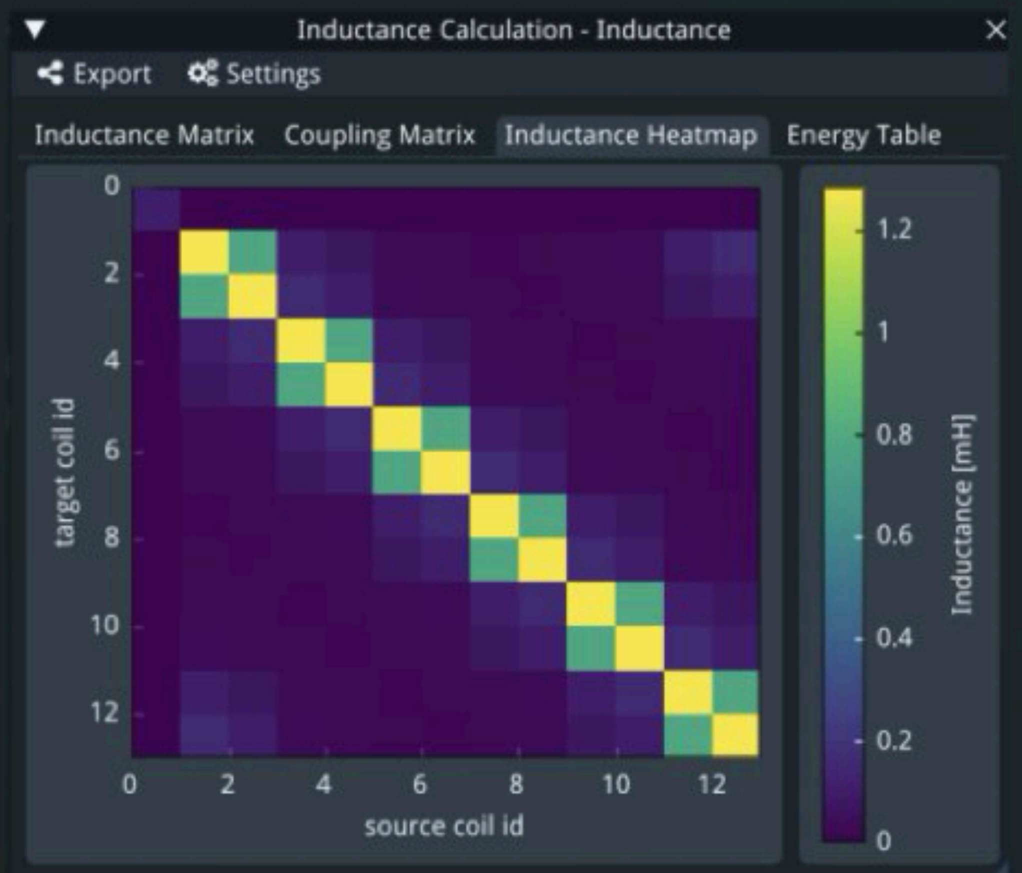

Plane Calculation




Particle Tracks




Mesh Calculation



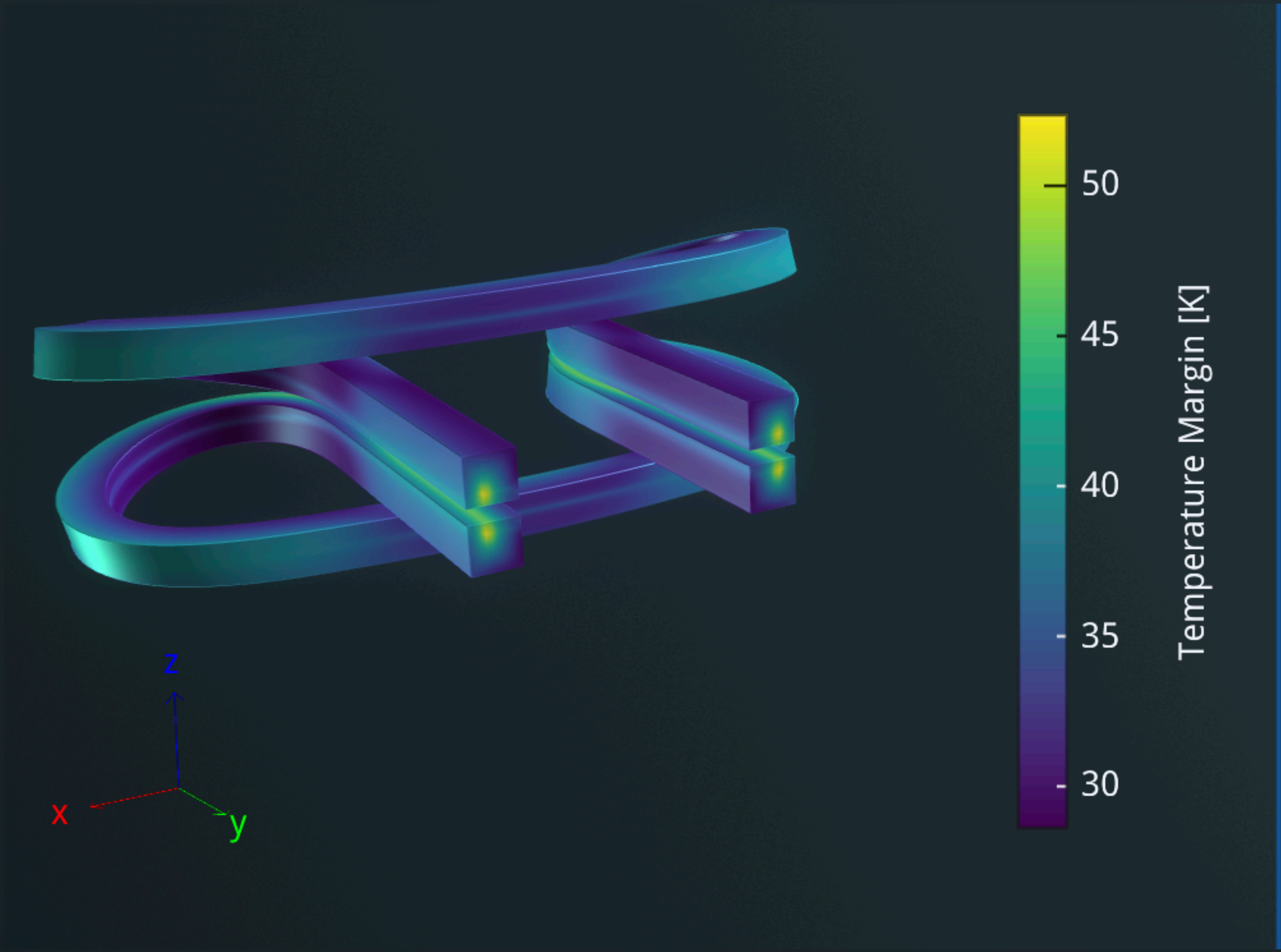

Inductance Calculation

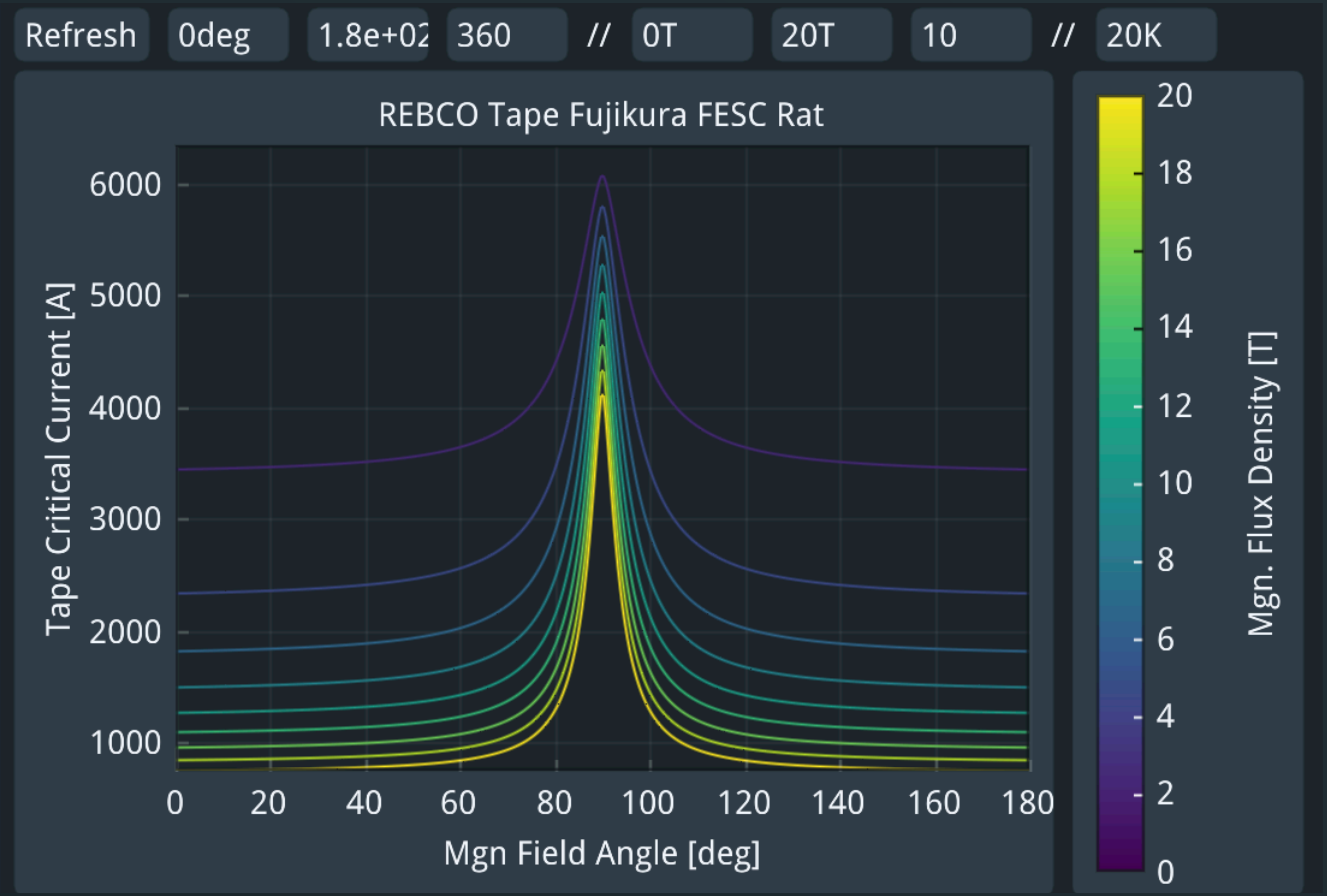

Line Calculation

- 1, 2, 3D field
- Inductance
- Particle tracking
- Quench
- Thermal margin, current density, relative permeability, force density on a mesh
- Harmonics

# Many HTS Related Features

- For example angular dependent critical current fit allows plotting the temperature margin on the surface of the coil.



# Rat for IDs:

# Extending VIM for Permanent Magnets

# Extending VIM for Permanent Magnets

- ESRF pointed out that the VIM method could be extended to permanent magnets, and it turns out it can:
  - Added remanent flux density -> right hand side offset to the system of equations.
  - Susceptibility calculation includes offset flux.
  - Anisotropic Material model for Susceptibility in addition to the isotropic model.

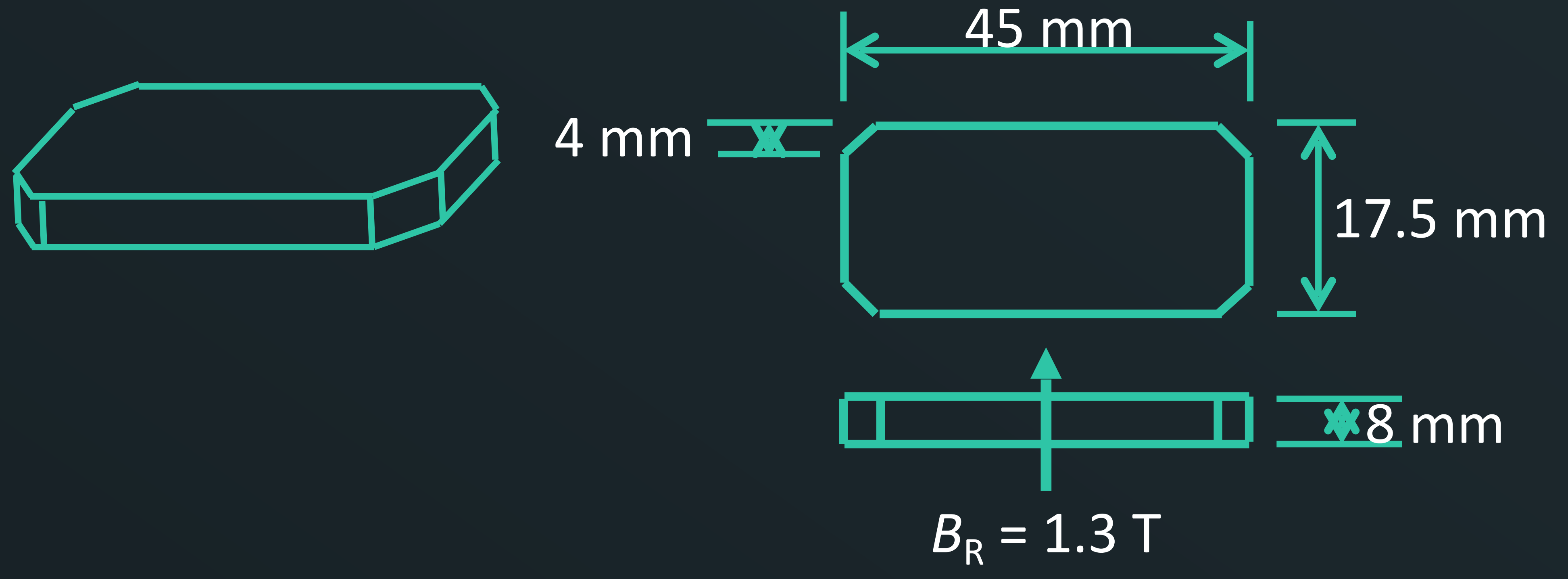
$$\mathbf{M} = \mathbf{M}_R + \chi \mathbf{H} \qquad \chi = \begin{pmatrix} \chi_{\parallel} & 0 & 0 \\ 0 & \chi_{\perp} & 0 \\ 0 & 0 & \chi_{\perp} \end{pmatrix}$$

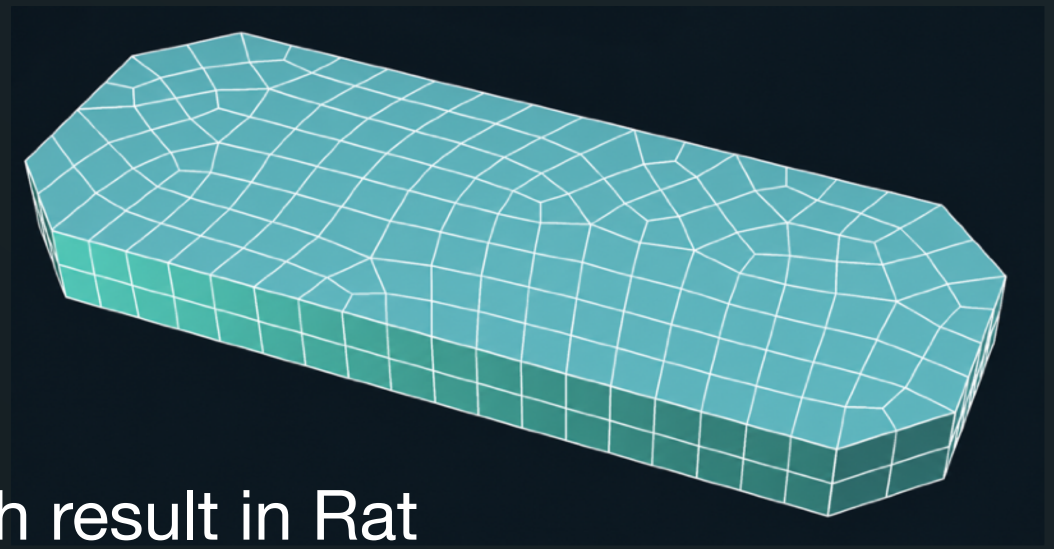
Thanks for G. Le Bec

# Validating Rat with a simple Undulator magnet

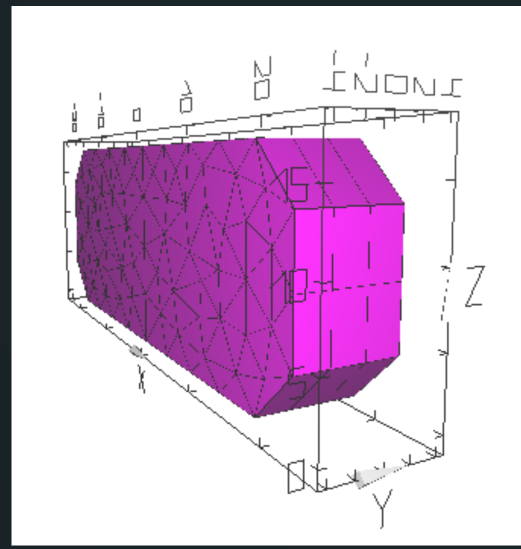
## Start from a single undulater magnet

- Reproduce the 3D shape in Rat, using Distmesh
- Set the 1.3T Br
- Assume anisotropic, constant magnetisation
- Compare results to Radia simulation of the magnetic field

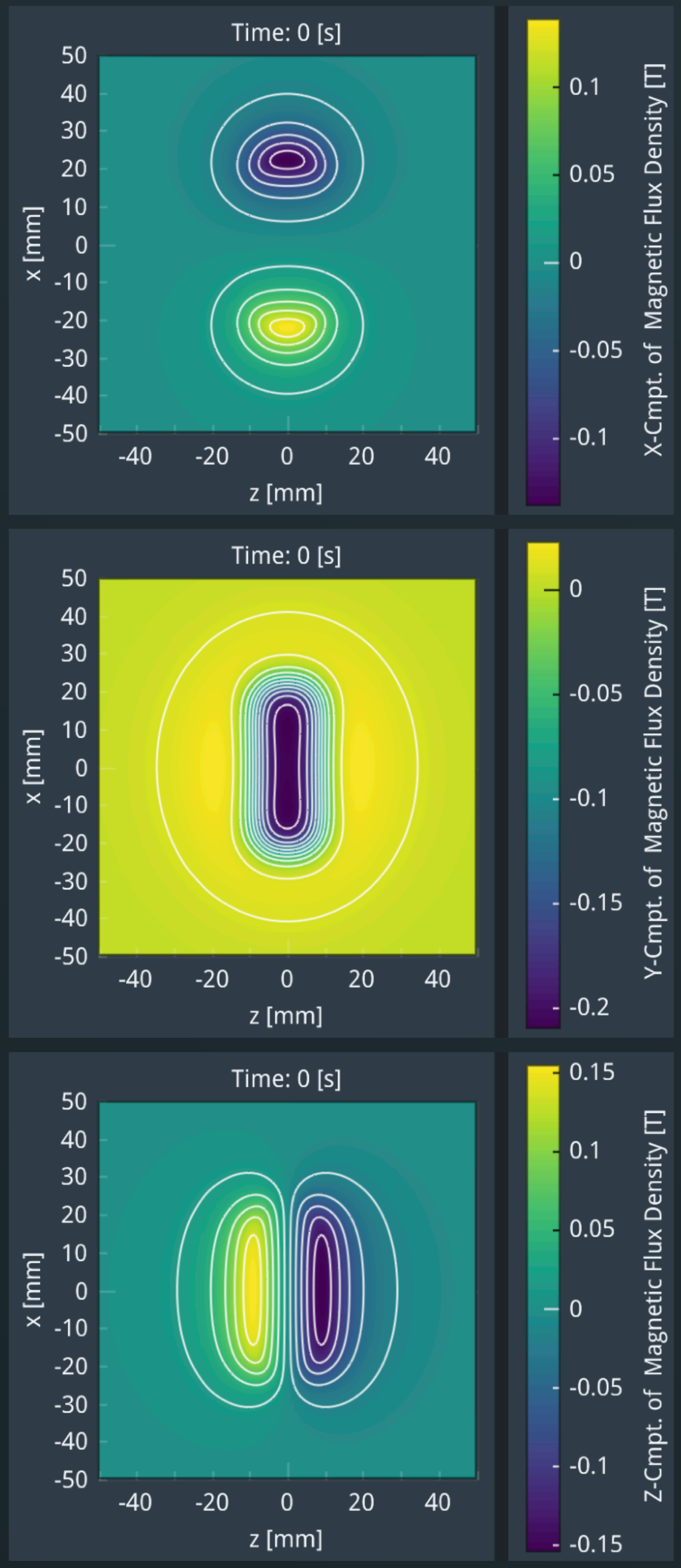


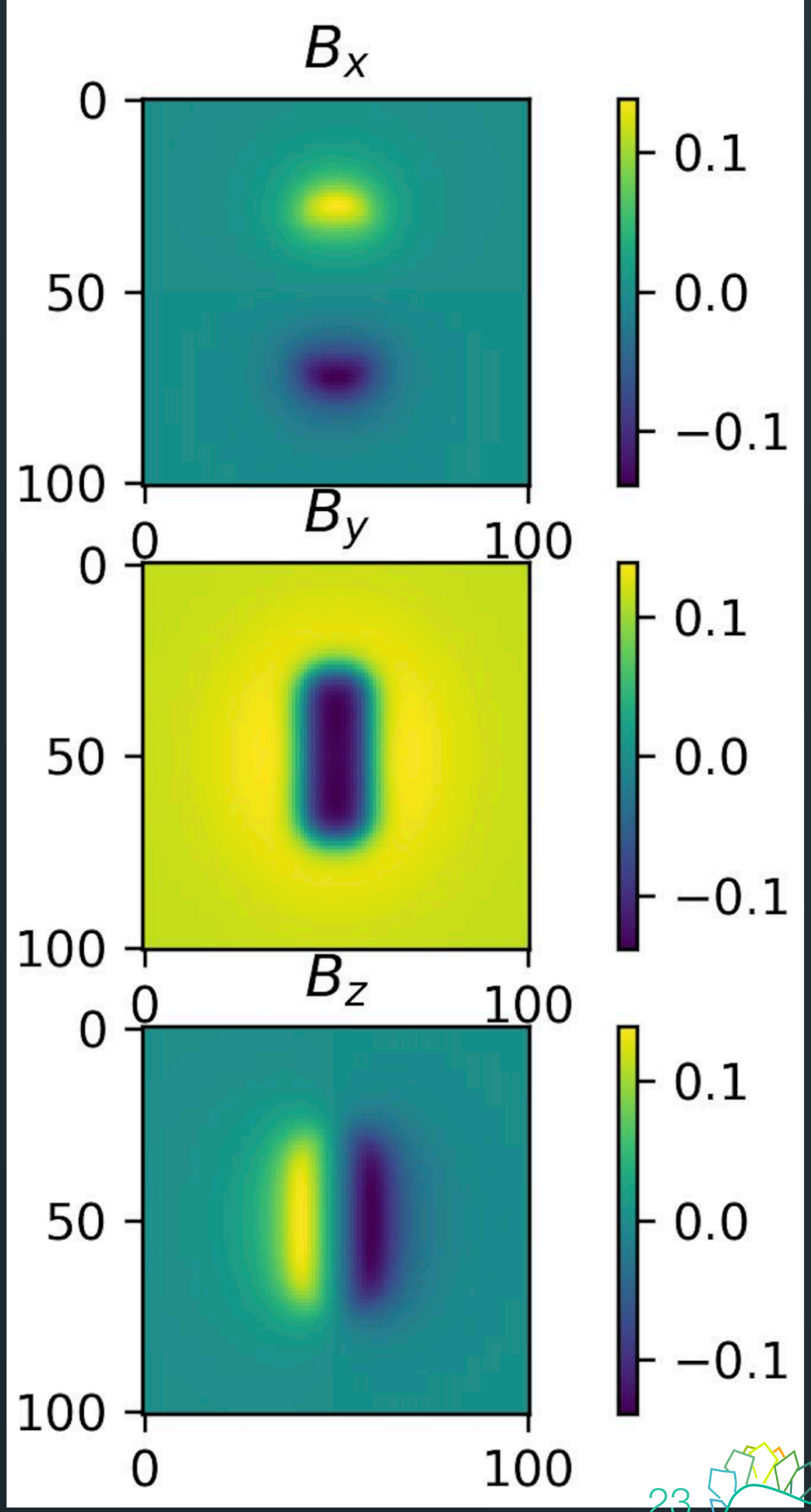
Distmesh result in Rat



Difference between Radia and Rat: 2mT




Rat




Radia



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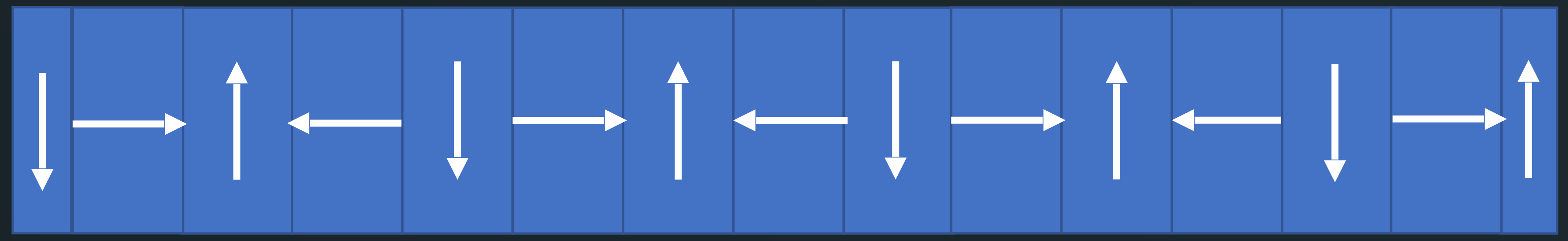
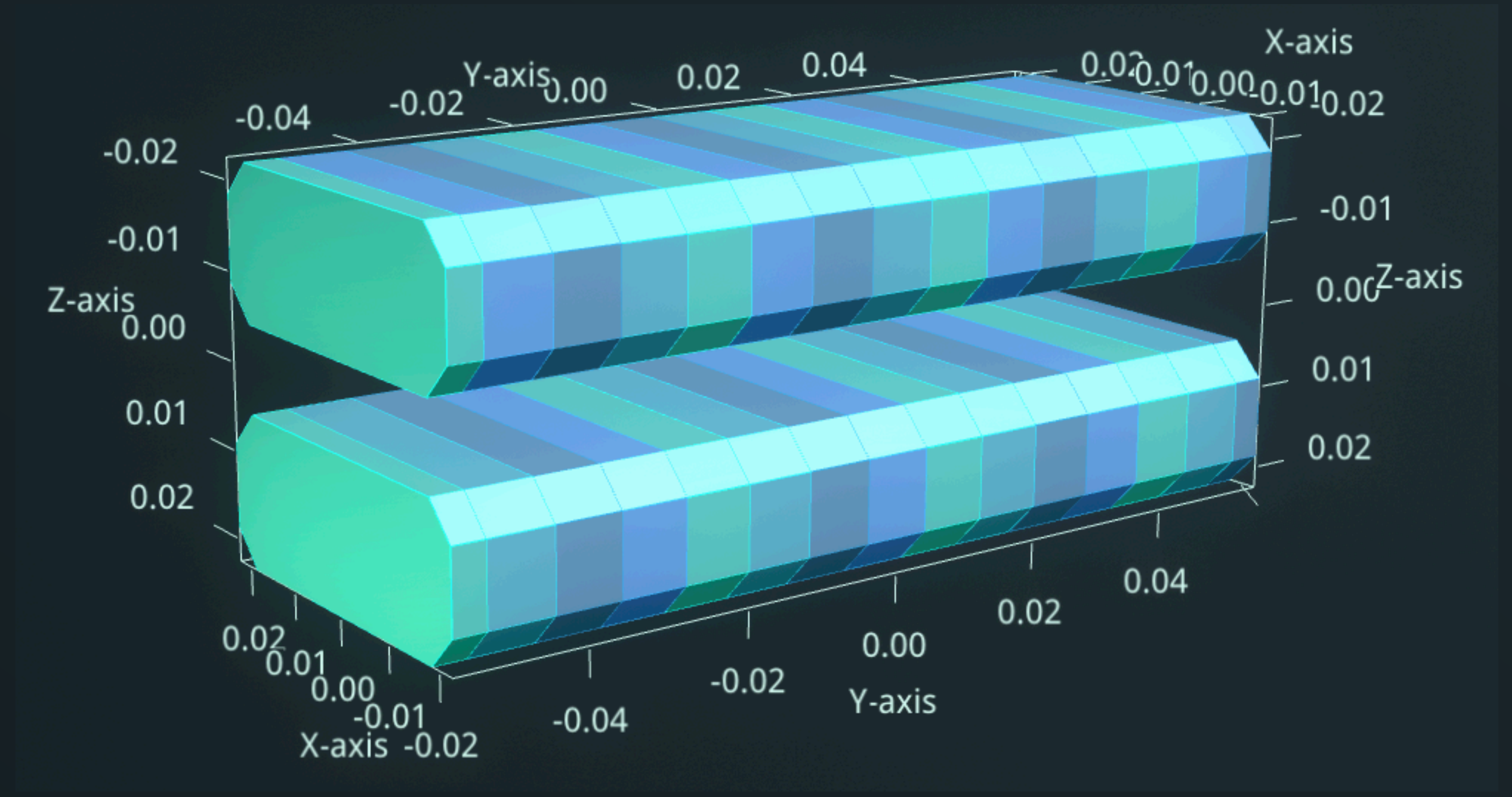
# Validating Rat with a simple Undulator model

## Repeat this for a simple undulator

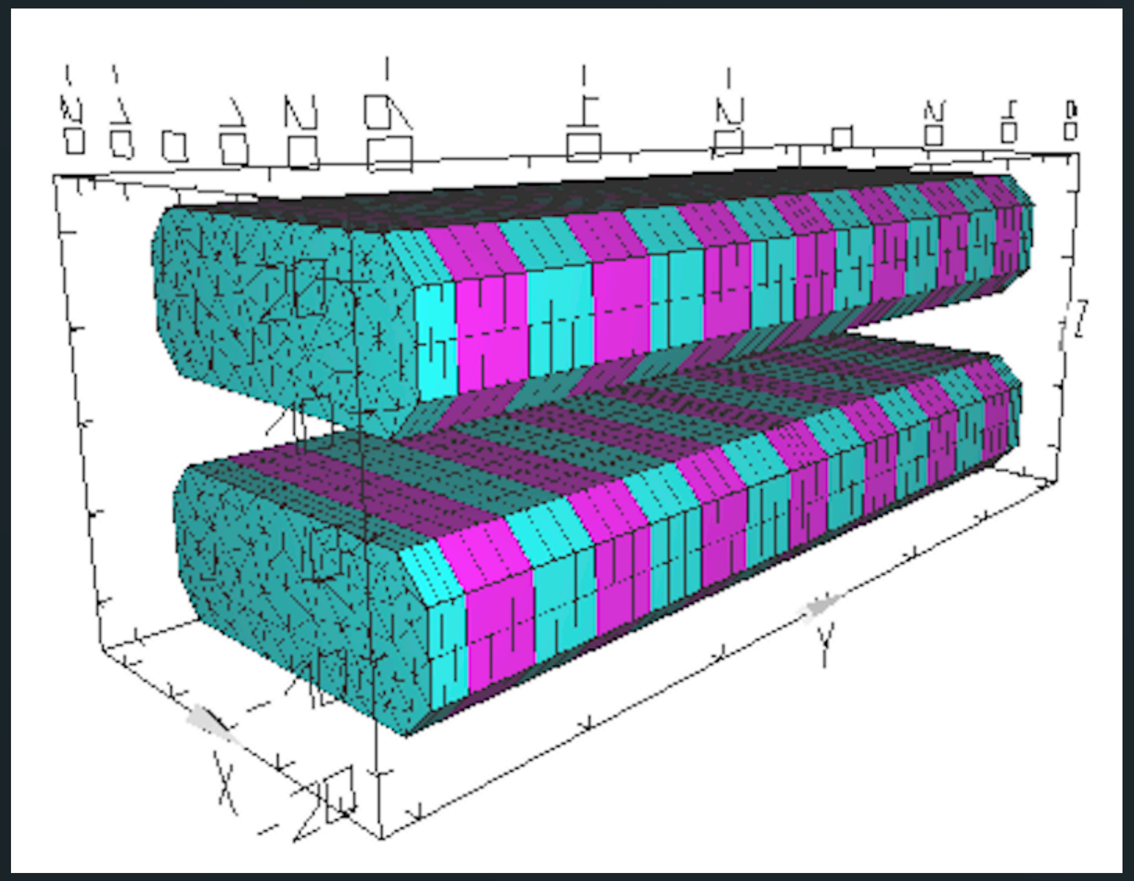
- Reproduce the 3D shape in Rat, using Distmesh
- Set the magnetisation separately for each magnet
- Compare results to Radia simulation of the magnetic field



Magnetisation direction for the magnets

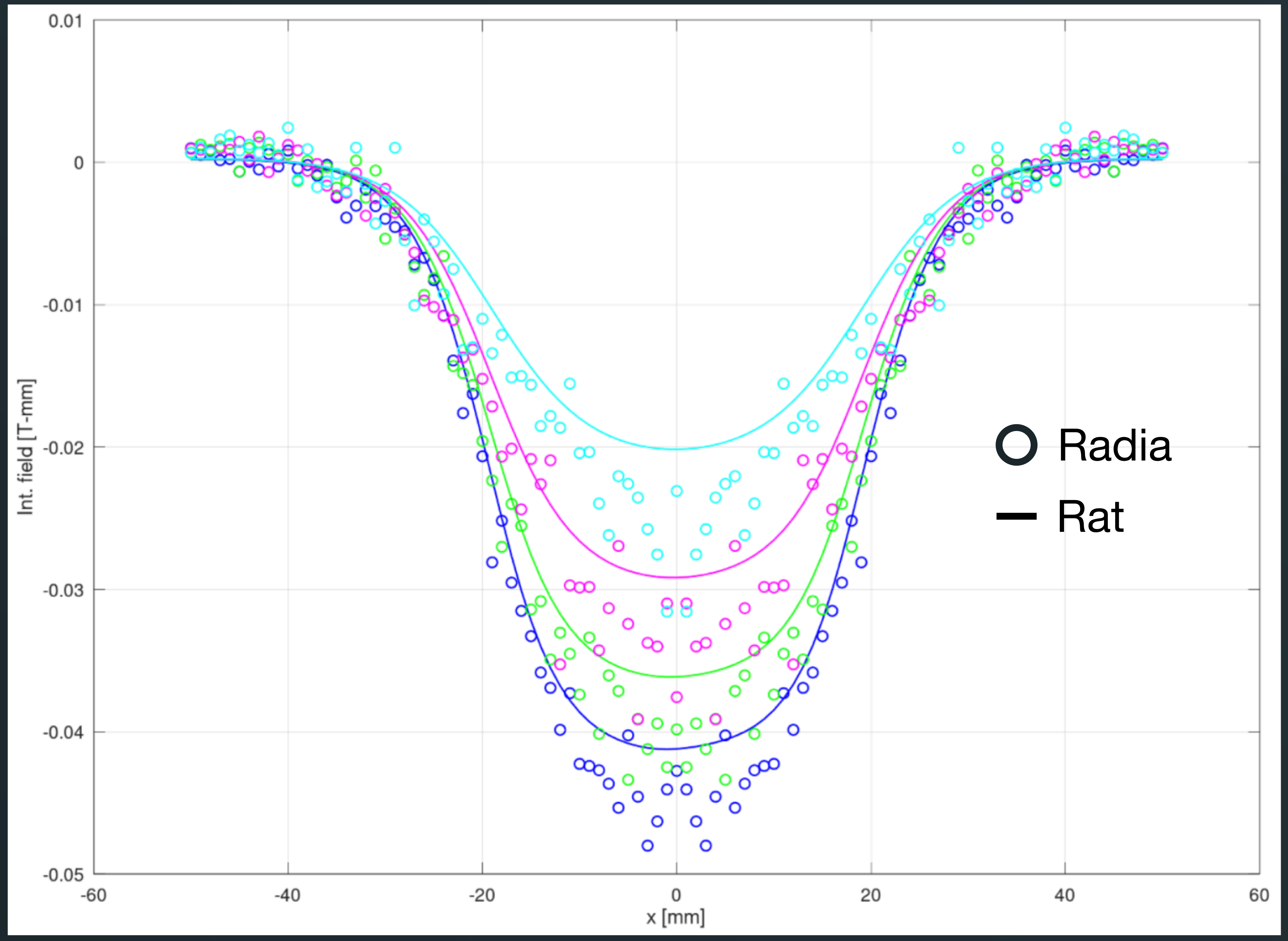



Simple undulator in Rat



Simple undulator in Radia

Integrated B-field along different x-lines

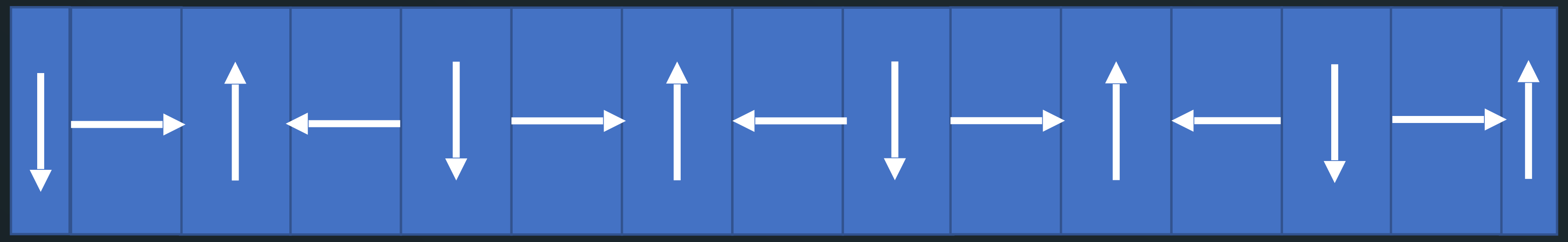



Difference between Radia and Rat in the order of mT

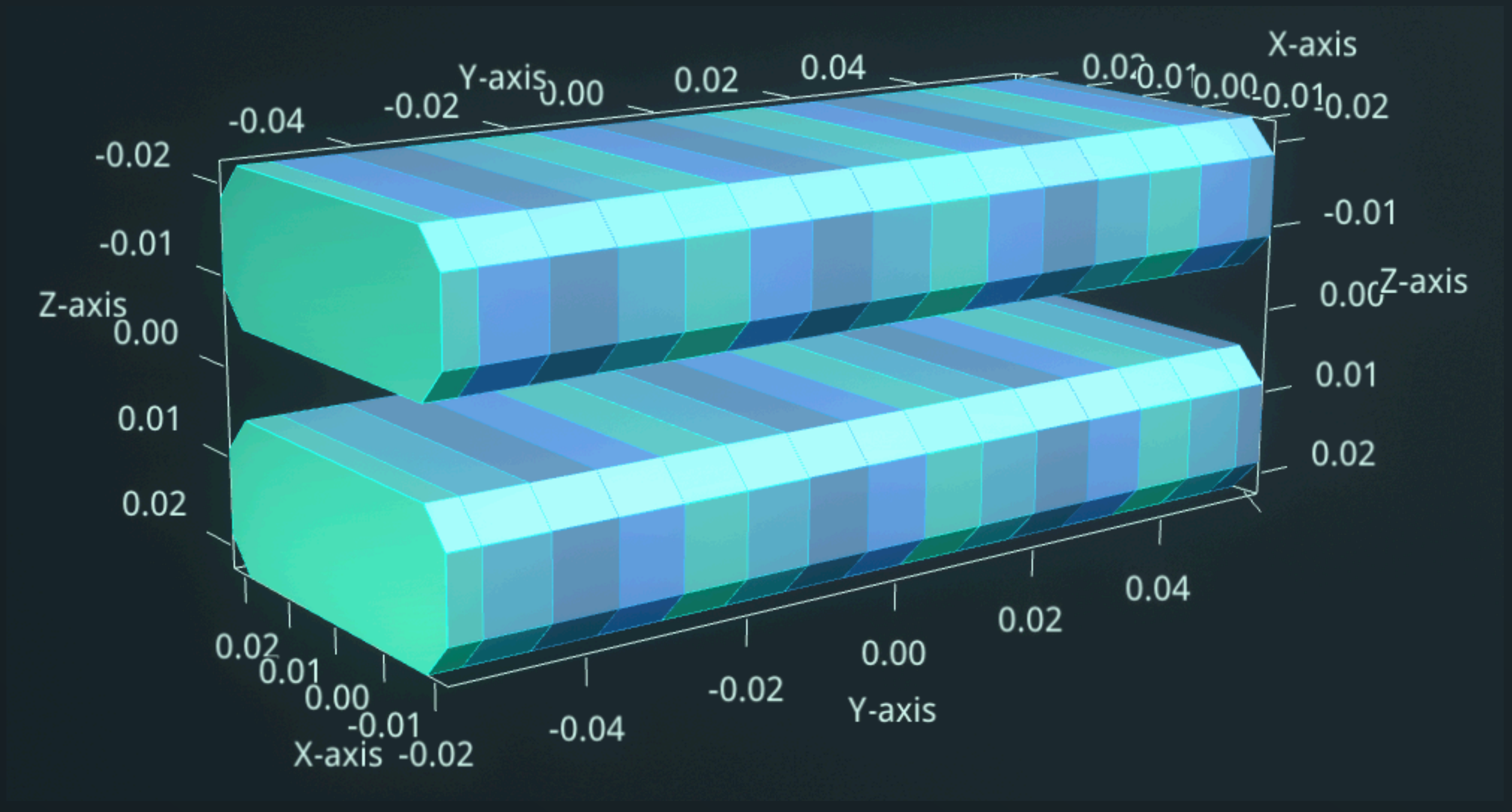
# Validating Rat with a simple Undulator model

## Repeat this for a simple undulator

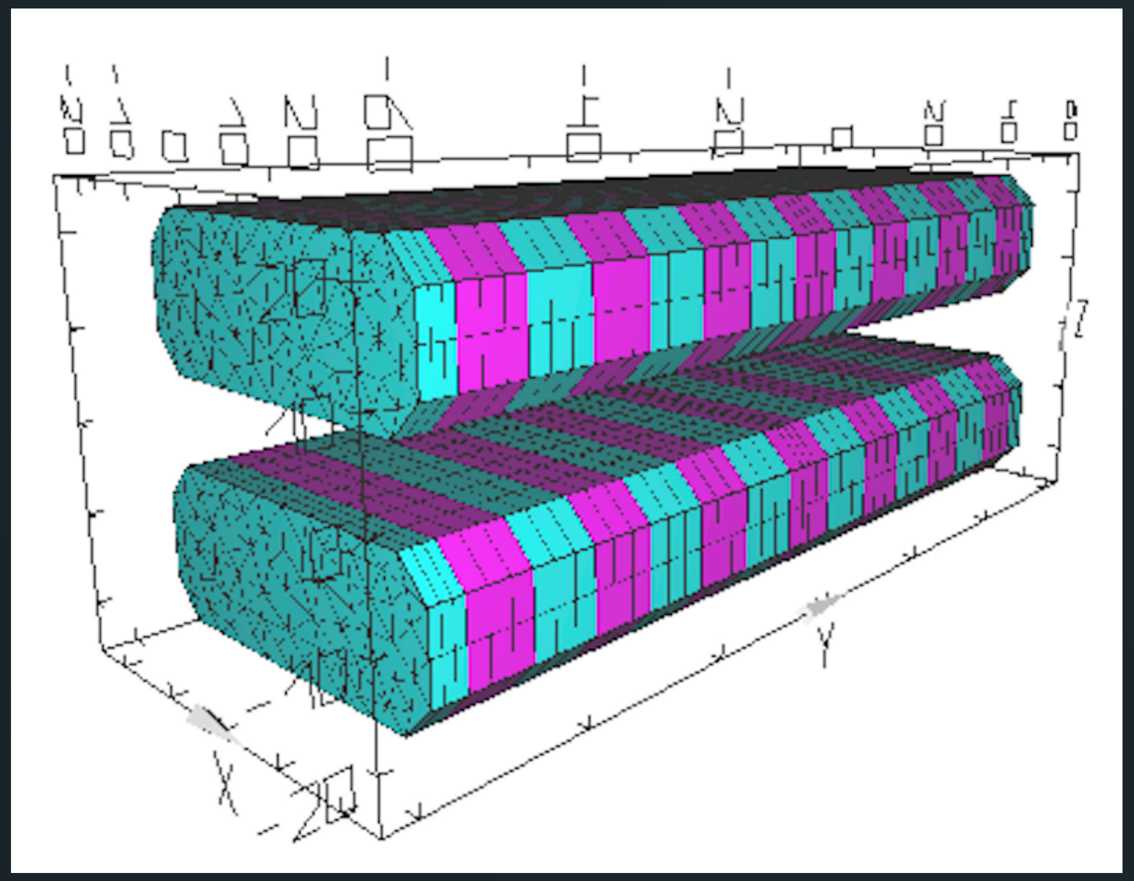
- Reproduce the 3D shape in Rat, using Distmesh
- Set the magnetisation separately for each magnet
- Compare results to Radia simulation of the magnetic field



Magnetisation direction for the magnets

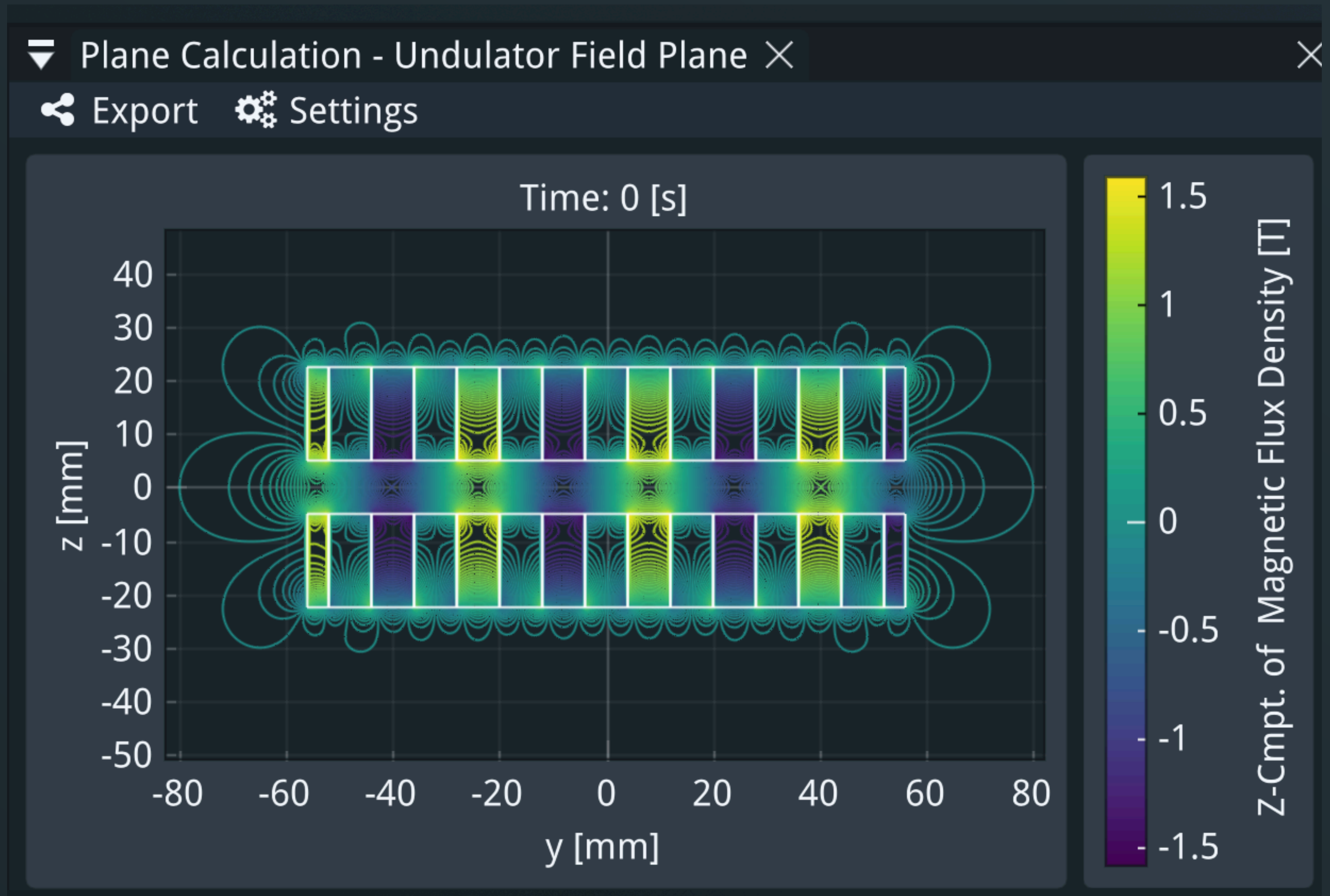


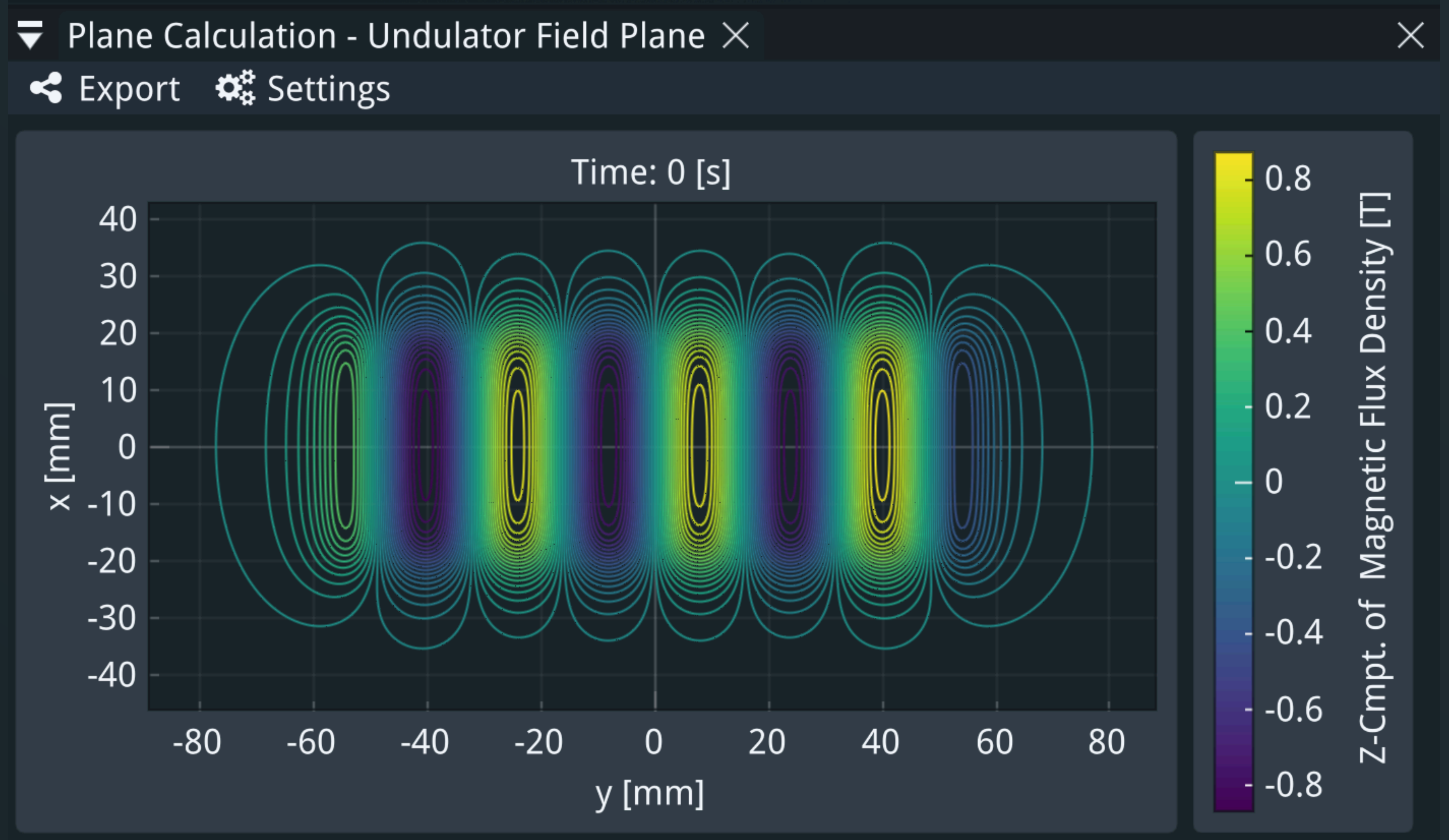

Simple undulator in Rat



Simple undulator in Radia

Magnetic field in planes (Rat)

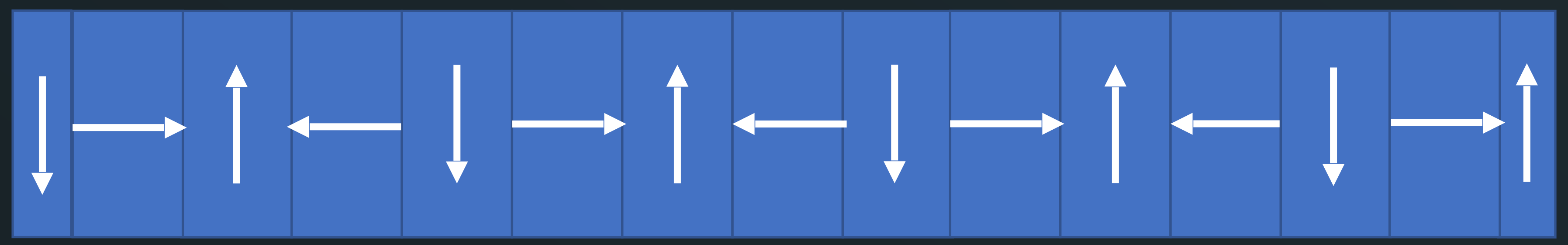


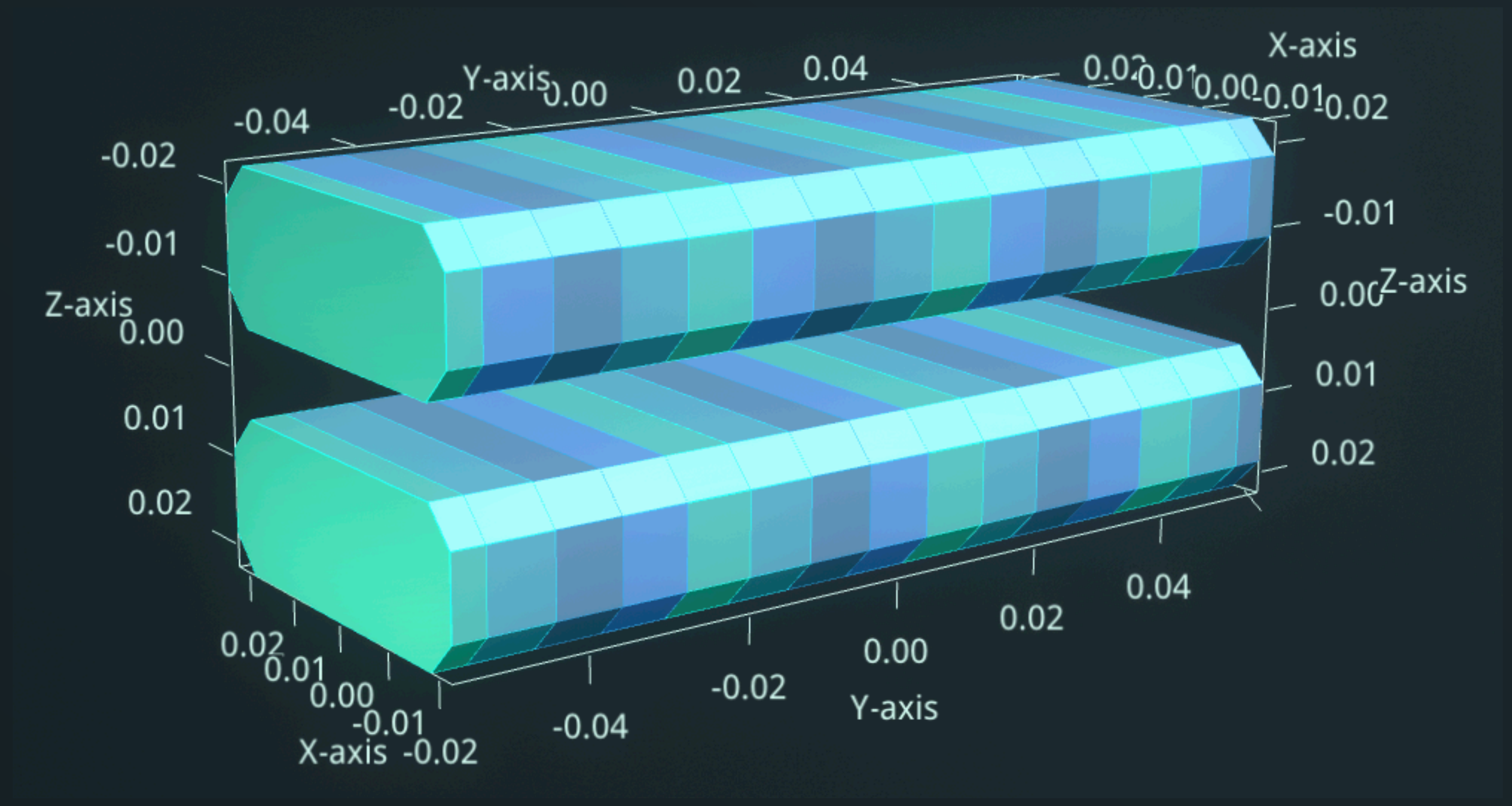
# Validating Rat with a simple Undulator model

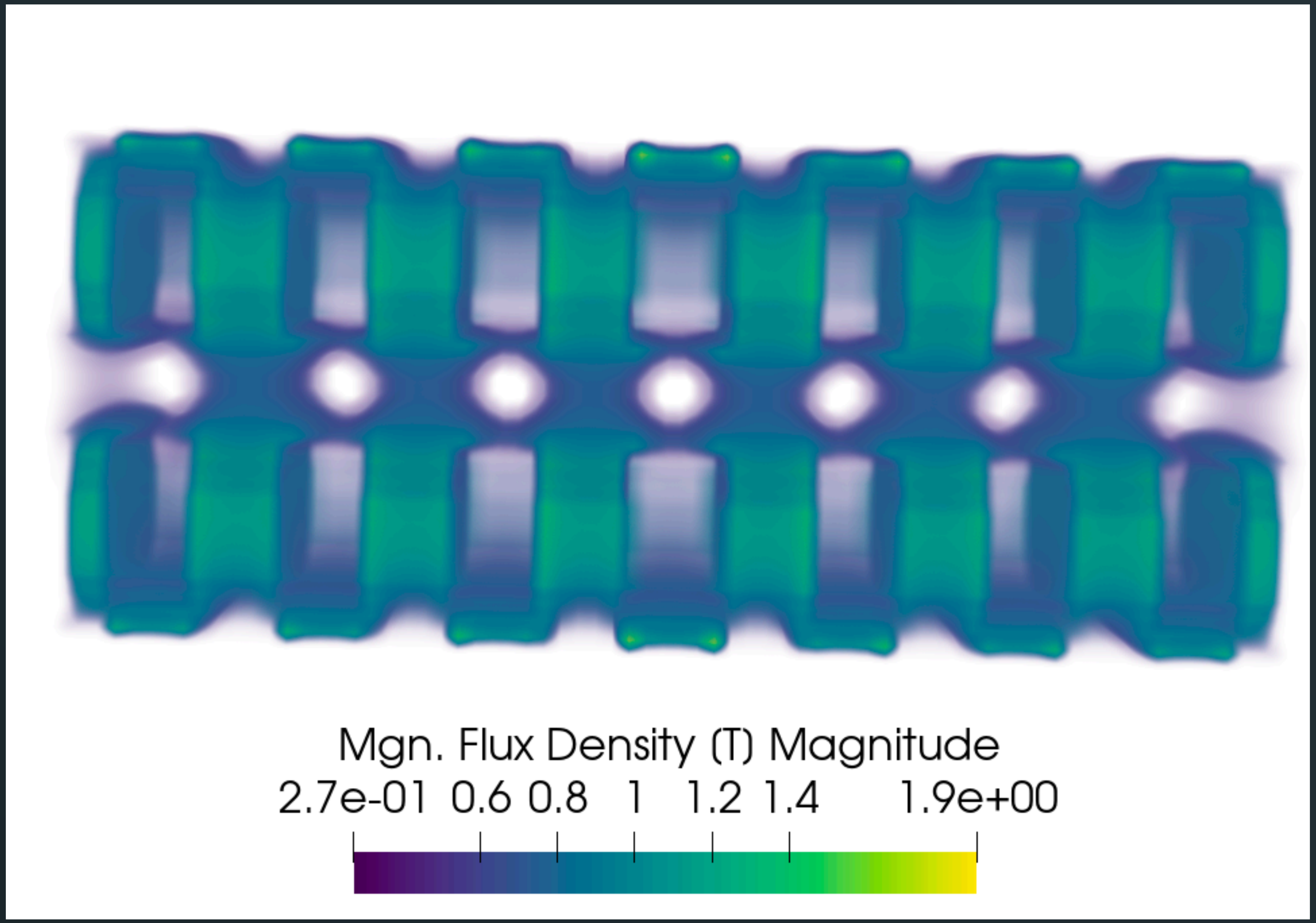
## Repeat this for a simple undulator

- Reproduce the 3D shape in Rat, using Distmesh
- Set the magnetisation separately for each magnet
- Compare results to Radia simulation of the magnetic field



Magnetisation direction for the magnets




Magnetic field in 3D (Rat)

# Summary

- Rat uses integral methods to solve different magnetic field calculation problems:
  - Biot-Savart with MLFMM
  - VIM for both iron and now also permanent magnets
- Rat includes many tools to quickly model magnets in 3D, and calculate their electro-magnetostatic properties
  - As well as quench, particle tracking, force density, etc