Pion, kaon and nucleon structure using state-of-the-art lattice QCD simulations

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Phys. Rev. Lett. 134, 131902

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Why Analyse the Structure of Hadrons?

There has been always interest in the properties of hadrons. For example, the first attempts to measure the proton spin by the European Muon Collaboration.

ABSTRACT

The spin asymmetry in deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons has been measured in the range $0.01 < x < 0.7$. The spin dependent structure function $g_1(x)$ for the proton has been determined and, combining the data with earlier SLAC measurements, its integral over x found to be $0.126 \pm 0.010(stat.) \pm 0.015(syst.)$, in disagreement with the Ellis-Jaffe sum rule. Assuming the validity of the Bjorken sum rule, this result implies a significant negative value for the integral of g_1 for the neutron. These integrals lead to the conclusion, in the naïve quark parton model, that the total quark spin constitutes a rather small fraction of the spin of the nucleon. Results are also presented on the asymmetries in inclusive hadron production which are consistent with the above picture.

J. Ashman et al. Nucl. Phys. B328, 1 (1989), [,351(1989)]

This work found that about half of the proton spin is carried by the valence quarks. This is known as the proton spin puzzle.

Recent LQCD calculations showed that not only the valence quarks contribute to the spin, but also the sea quarks and gluons.

To compute the spin, it is required to evaluate the QCD Energy Momentum Tensor components.

$$\langle N(p',s')|T^{\mu\nu;q,g}|N(p,s)\rangle = \bar{u}_N(p',s')\left[\boxed{A_{20}^{q,g}(q^2)}\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q,g}(q^2)\frac{i\sigma^{\{\mu\rho}q_\rho P^{\nu\}}}{2m_N} + C_{20}^{q,g}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m_N}\right]u_N(p,s)$$

Average momentum fraction when $q \rightarrow 0$

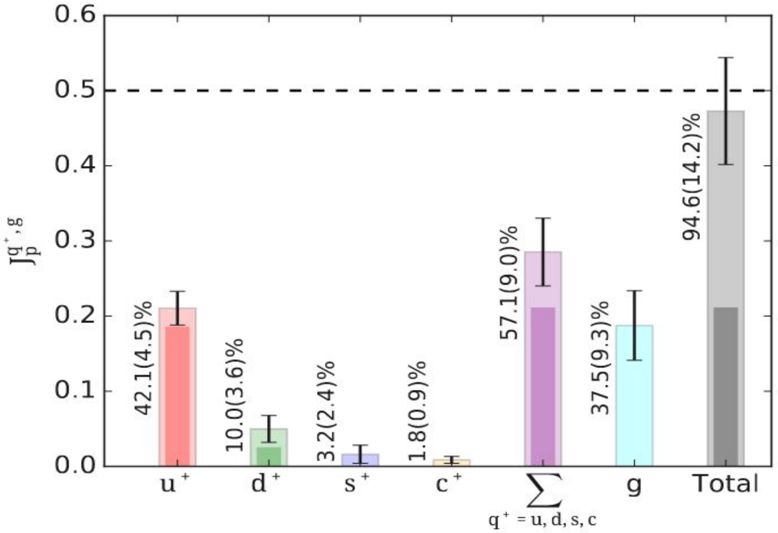
They decompose into three generalized form factors, being one of them the average momentum fraction.

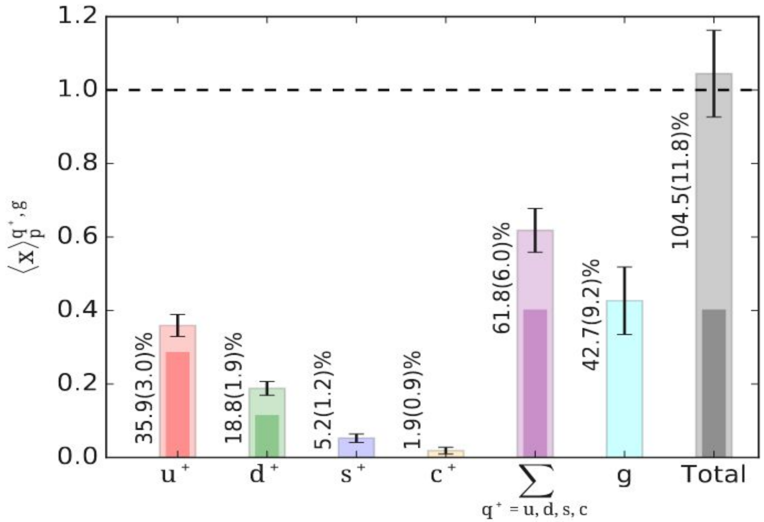
X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249

Why Analyse the Structure of Hadrons?

Complete flavor decomposition of the spin and momentum fraction of the proton using lattice QCD simulations at physical pion mass

C. Alexandrou et al. Phys. Rev. D 101, 094513





Studied one lattice spacing for the ensemble cB211.072.64

Why Analyse the Structure of Hadrons?

Unlike the proton with three valence quarks, pion and kaon have quark-antiquark pair structures. In the kaon case having a strange quark.

Pion and kaon are more experimentally challenging, leading this to a lack in studies.

Experimental data is limited to studies from decades ago and have been included in modern analysis.

Pion: J. S. Conway et al. Phys. Rev.D 39, 92–122 (1989)
Kaon: J. Badier et al. Phys. Lett.B 93, 354–356 (1980)
Analysis: P. C. Barry et al. Phys. Rev. Lett. 121, 152001(2018)
C. Chen et al. Phys. Rev. D 93, 074021(2016)
C. Shi et al. Phys. Rev. D 98, 054029 (2018)
K. D. Bednar et al. Phys. Rev. Lett. 124, 042002(2020)

Future experiments aim to quantify quark and gluon contributions to the mass of both pion and kaon.

EIC: arXiv:1907.08218 [nucl-ex]
arXiv:2103.05419 [physics.ins-det]
EicC: arXiv:2109.08483 [hep-ph]

Experimental and phenomenological studies are needed.

Average Momentum Extraction for pion and kaon

The average momentum fraction is related to the matrix elements of the EMT.

$$\langle h(\mathbf{p})|\bar{T}^{X,f}_{\mu\nu}|h(\mathbf{p})\rangle = 2\langle x\rangle^{X,f}\left(p_\mu p_\nu - \delta_{\mu\nu}\frac{p^2}{4}\right)$$

In order to extract the matrix elements, we can calculate the ratio between the three- and two-point function. In the large enough time limit is proportional to the matrix elements up to a kinematic factor.

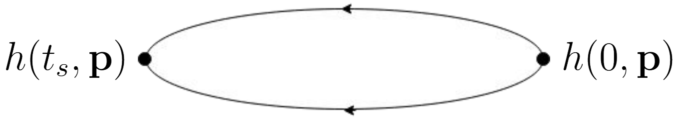
$$R^{X,f}_{\mu\nu}(t_s, t_{ins}; \mathbf{p}) = \frac{C^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p})}{C^X(t_s, \mathbf{p})} \xrightarrow[t_s - t_{ins} \to \infty]{t_s \to \infty} \frac{1}{2E_h} \frac{\langle h(\mathbf{p})|\bar{T}^{X,f}_{\mu\nu}|h(\mathbf{p})\rangle}{1 + \exp(-E_h(T - 2t_s))}$$

Average Momentum Extraction

Two-point function

$$C^h(t_s, \mathbf{p}) = \langle h(t_s, \mathbf{p}) h(0, \mathbf{p}) \rangle$$

$h(t_s, \mathbf{p})$: hadron state

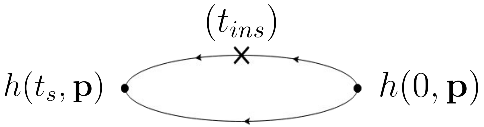


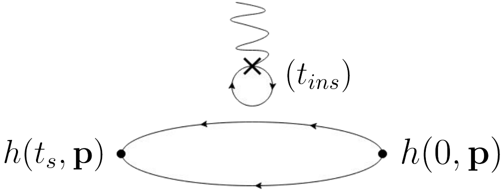
Three-point function

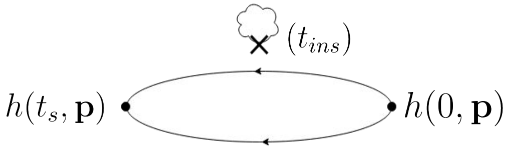
$$C^{f,h}_{\mu\nu}(t_s, t_{ins}, \mathbf{p}) = \langle h(t_s, \mathbf{p}) T^f_{\mu\nu}(t_{ins}) h(0, \mathbf{p}) \rangle$$

$T^f_{\mu\nu}$ = Euclidian energy momentum tensor

$f \in u, d, s, c$ and g



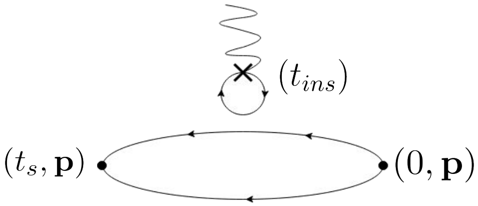


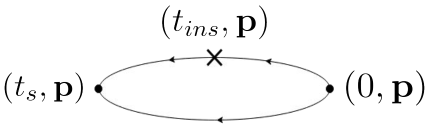


Average Momentum Extraction

The quark EMT

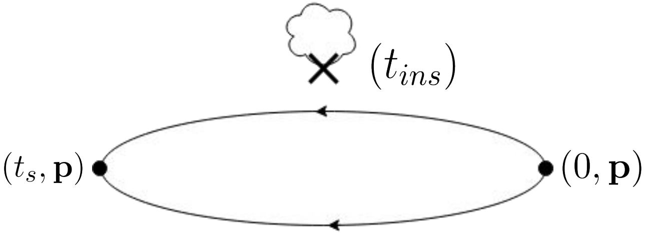
$$\bar{T}^q_{\mu\nu} = -\frac{(i)^{\kappa_{\mu\nu}}}{4}\,\bar{q}\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu - \delta_{\mu\nu}\frac{1}{2}\gamma_\rho \overleftrightarrow{D}_\rho\right)q$$





The gluon EMT

$$\bar{T}^g_{\mu\nu} = (i)^{\kappa_{\mu\nu}}\left(F_{\mu\rho}F_{\nu\rho} + F_{\nu\rho}F_{\mu\rho} - \delta_{\mu\nu}\frac{1}{2}F_{\rho\sigma}\,F_{\rho\sigma}\right)$$



Lattice Setup

$$R_{44}^{X,f}(t_s, t_{ins}; \mathbf{0}) \xrightarrow[t_s - t_{ins} \to \infty]{t_s \to \infty} -\frac{3m_h}{4}\langle x\rangle^{X,f} \qquad R_{4k}^{X,f}(t_s, t_{ins}; \mathbf{p}) \xrightarrow[t_s - t_{ins} \to \infty]{t_s \to \infty} p_k\langle x\rangle^{X,f}$$

We use $\bar{T}_{44}$ at zero momentum for connected. For disconnected contributions, $\bar{T}_{4k}, k = 1, 2, 3$ is used with $|\mathbf{p}| = \frac{2\pi}{L}$.

Stout smearing is used for the gluon loops, averaging $5 \leq N_{stout} \leq 10$.

Ensemble name	n_{conf}	n_{2pt}	stats	n_{3pt}	stats
cB211.072.64	755	400	302,000	8	6040
cC211.060.80	400	240	96,000	8	3200
cD211.054.96	499	600	299,400	8	3992

Statistics for each ensemble three-point(connected) and two-point functions.

Lattice Setup

Three ensembles with $N_f = 2 + 1 + 1$ Wilson twisted-mass fermions reproducing physical pion mass.

Ensemble	a [fm]	L [fm]	$M_{\pi^\pm}$ [MeV]	$M_{K^\pm}$ [MeV]
cB211.072.64 (B)	0.0796(1)	5.09	140.40(22)	498.41(11)
cC211.060.80 (C)	0.0682(1)	5.46	136.05(30)	495.27(14)
cD211.054.96 (D)	0.0569(1)	5.46	141.01(22)	494.77(11)

Extended Twisted Mass Collaboration

Pion, kaon and nucleon are studied, decomposing in all quark and gluon contributions for the three ensembles.

Jackknife resampling is done, and all the results are shown using model averaging with Akaike Information Criterion. William I. Jay and Ethan T. Neil, Phys. Rev. D 103, 114502

Renormalization

The renormalized results are given in the MS scheme at scale 2 GeV.

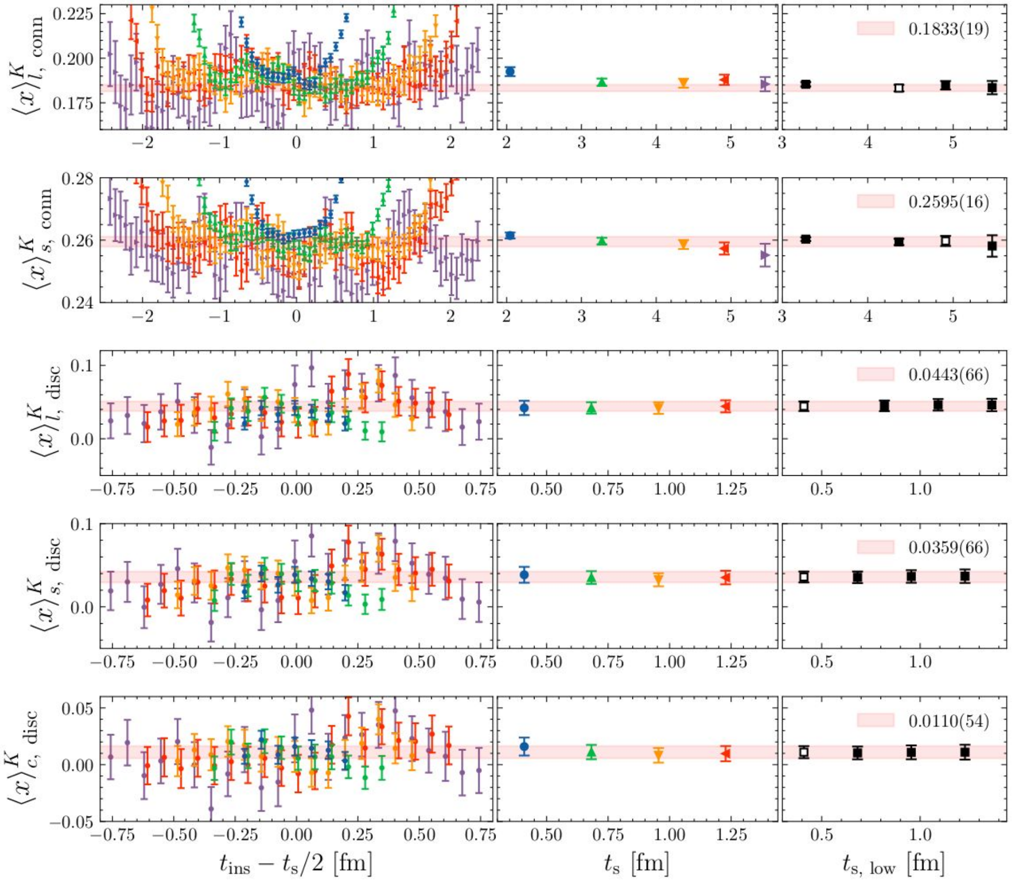
$$\langle x\rangle_{f,\mathrm{R}} = Z_{qq}\,\langle x\rangle_f + \frac{\delta Z_{qq}}{N_f}\sum_{f'}\langle x\rangle_{f'} + \frac{\boxed{Z_{qg}}}{N_f}\,\langle x\rangle_g\,,$$

$$\langle x\rangle_{g,\mathrm{R}} = Z_{gg}\,\langle x\rangle_g + \boxed{Z_{gq}}\sum_{f'}\langle x\rangle_{f'}\,.$$

Mixing between quark-singlet and gluon components

Diagonal elements of the mixing matrix non-perturbatively, while for the off-diagonal elements we employ one-loop lattice perturbation theory.

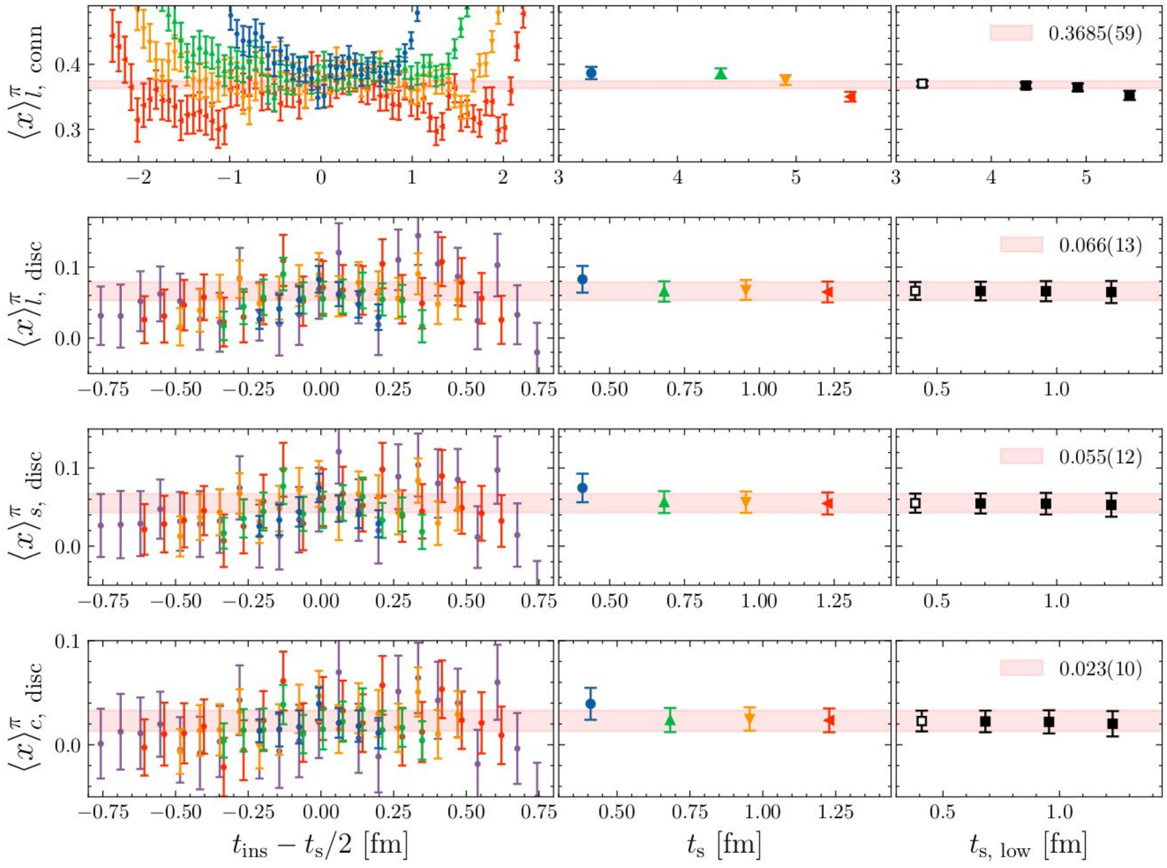
Full Quark Decomposition



Results for cC211.060.80 ensemble kaon.

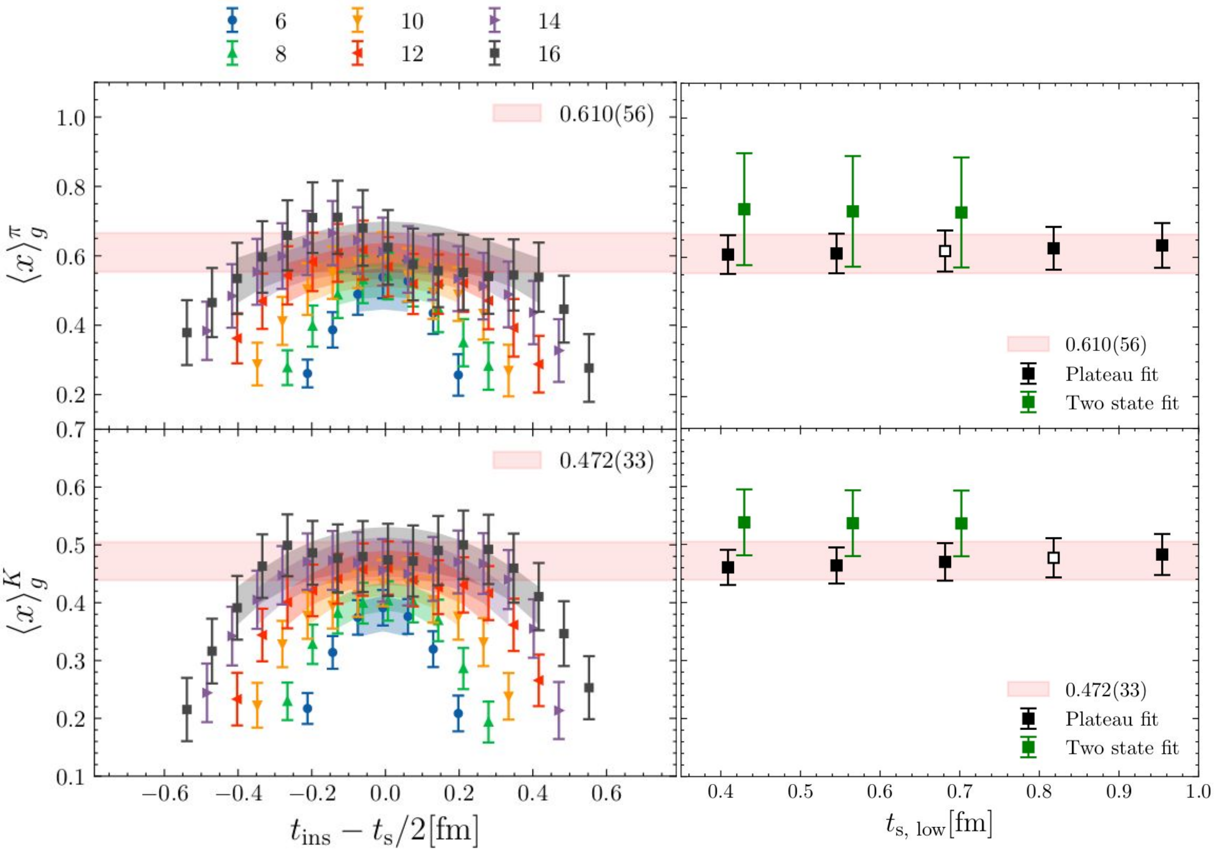
Plateau fit for different combinations of source-sink separations and insertions.

Full Quark Decomposition



Results for cC211.060.80 ensemble pion.

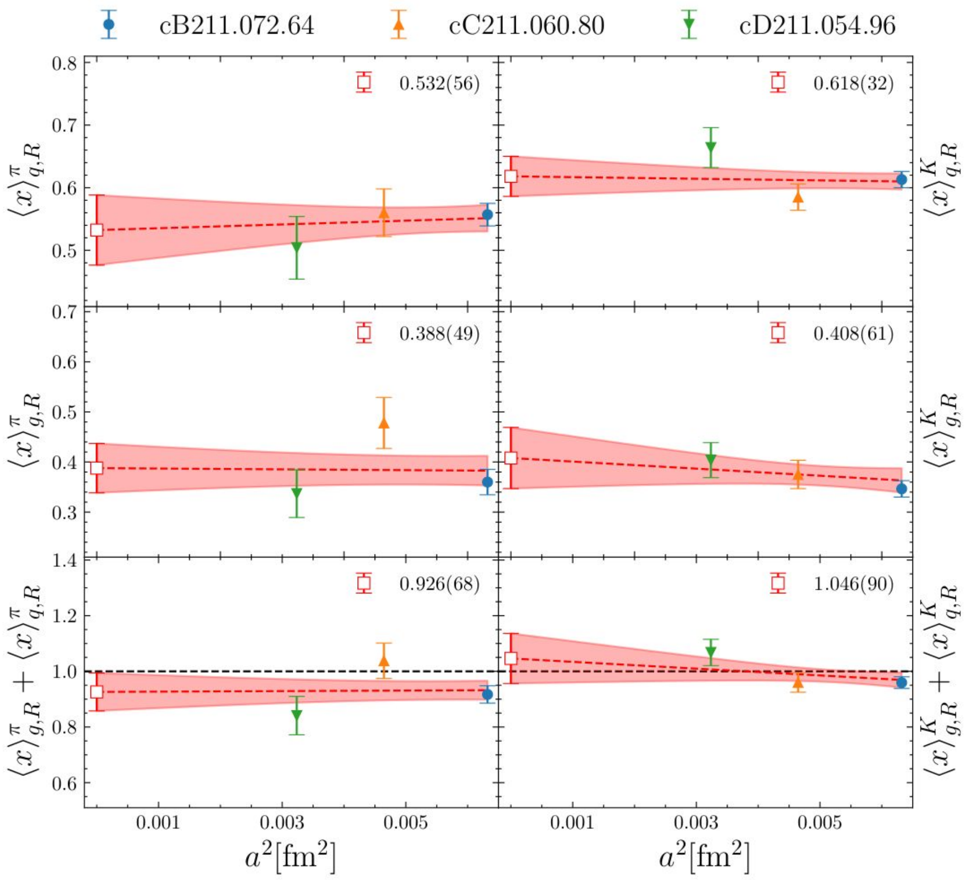
Gluon Decomposition



Results for gluon cC211.060.80 ensemble at 10 stout smearing steps.

Plateau fit and two-state fit were performed.

Continuum Limit

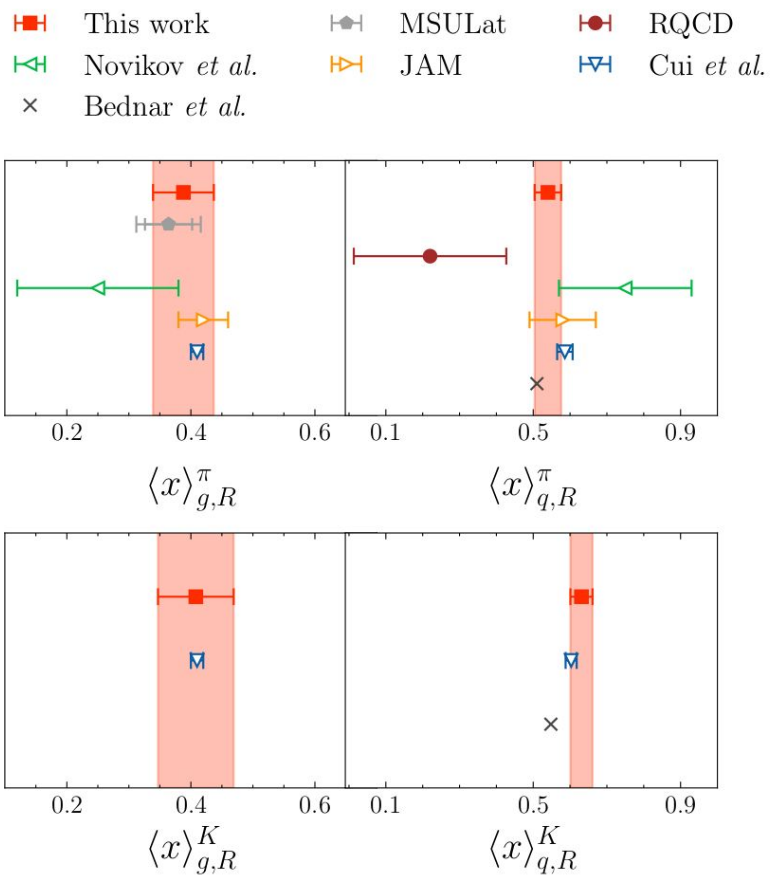


Results in $\overline{\mathrm{MS}}$ scheme at 2 GeV.

Continuum limit using a constant fit and a linear fit in the square lattice spacing. Results from them were model averaged.

Continuum limit taken at the physical point, confirming the momentum sum rule!

Comparison

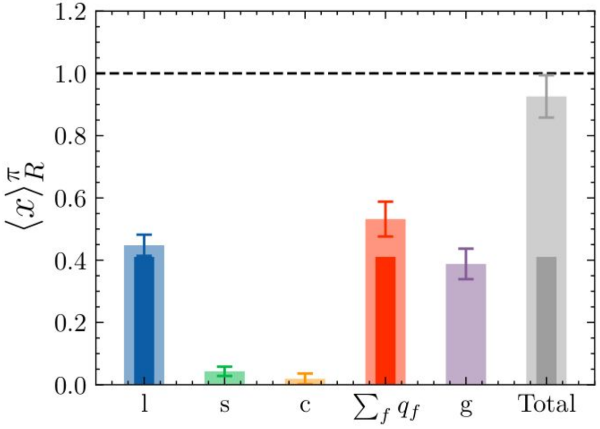


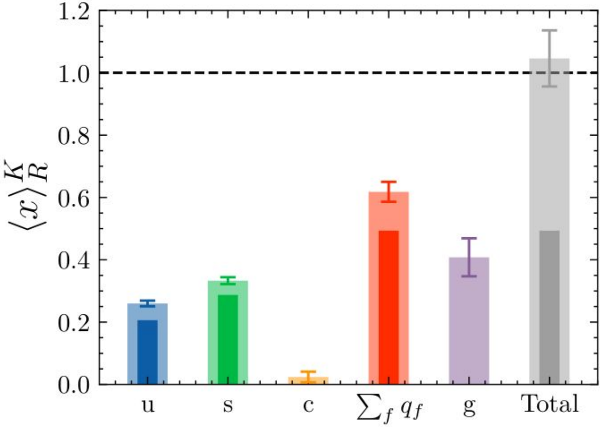
Phenomenological analyses of PDFs data are given by open symbols.

LQCD results are given by filled symbols.

Results using DSE are given by the black cross, where no error is provided.

Momentum Fractions





Contribution to the total momentum for each quark and gluon at 2 GeV.

	π	K
$\langle x \rangle_{l,\mathrm{R}}$	0.448(34)	0.260(09)
$\langle x \rangle_{s,\mathrm{R}}$	0.043(15)	0.333(11)
$\langle x \rangle_{c,\mathrm{R}}$	0.019(17)	0.024(17)
$\langle x \rangle_{g,\mathrm{R}}$	0.388(49)	0.408(61)
$\langle x \rangle_{q,\mathrm{R}}$	0.532(56)	0.618(32)
$\langle x \rangle_{u+d-2s,\mathrm{R}}$	0.382(17)	−0.409(16)
$\langle x \rangle_{u+d+s-3c,\mathrm{R}}$	0.445(48)	0.487(39)

Higher Moments in the Pion and Kaon

Instead of inserting a one-derivative operator, we can use a two- or three- derivative operator as:

$$\mathcal{O}^{\mu\nu\rho} = \bar{q}\gamma^{\{\mu}D^{\nu}D^{\rho\}}q \qquad\qquad \mathcal{O}^{\mu\nu\rho\sigma} = \bar{q}\gamma^{\{\mu}D^{\nu}D^{\rho}D^{\sigma\}}q$$

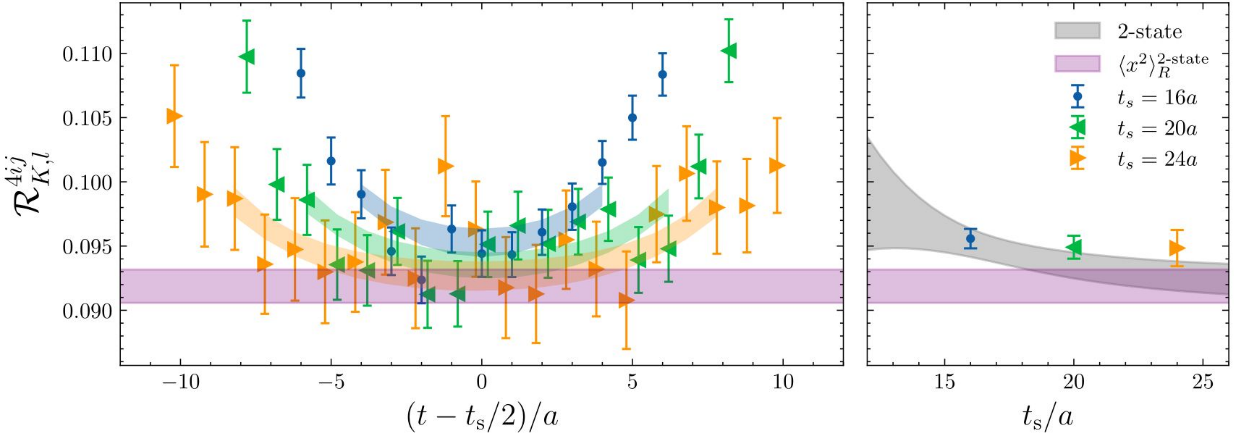
Their insertion leads to the so-called higher moments.

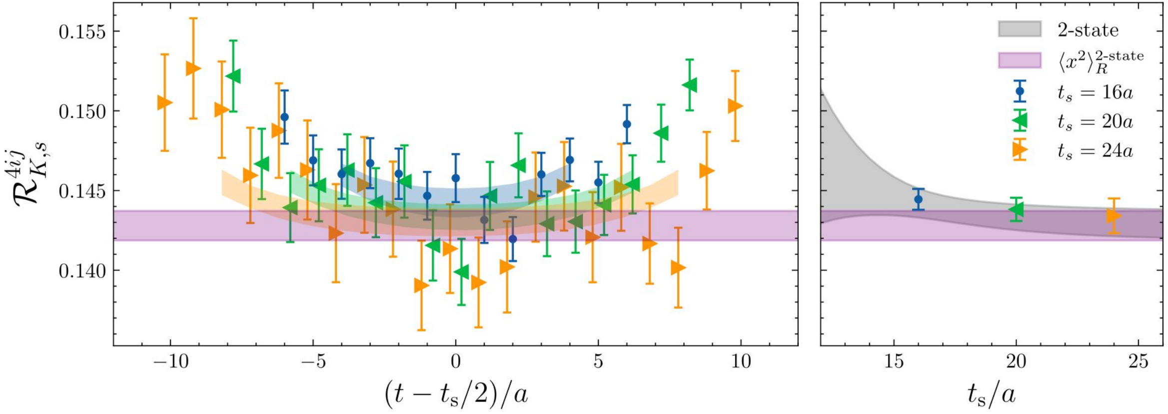
$$\langle h(\mathbf{p})|\mathcal{O}^{4\mu\nu}|h(\mathbf{p})\rangle = -p^{\mu}p^{\nu}\langle x^2\rangle \qquad \langle h(\mathbf{p})|\mathcal{O}^{\mu\nu\rho\sigma}|h(\mathbf{p})\rangle = -\frac{1}{E}p^{\{\mu}p^{\nu}p^{\rho}p^{\sigma\}}\langle x^3\rangle$$

p = 0 is not an option :(

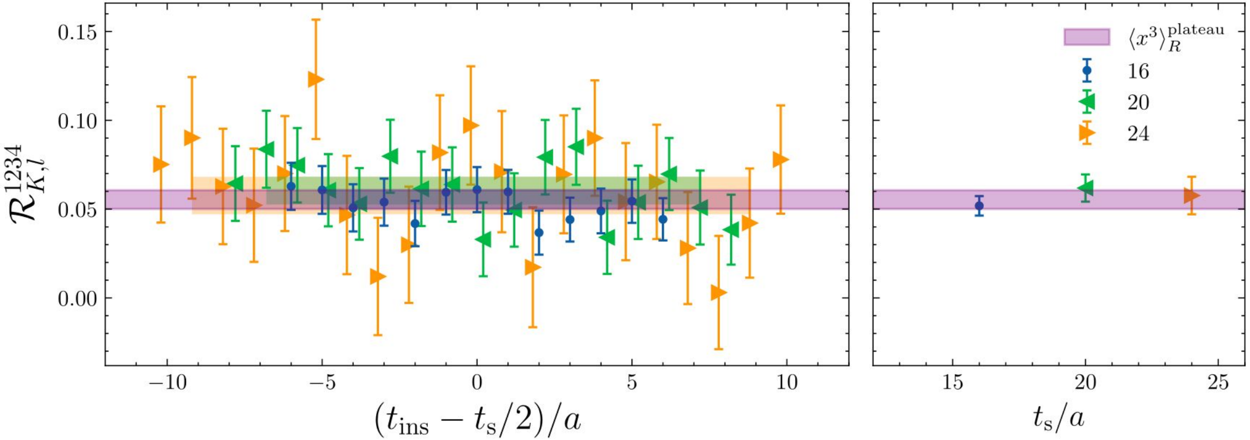
The operators with all indices different don’t have mixing with other operators.

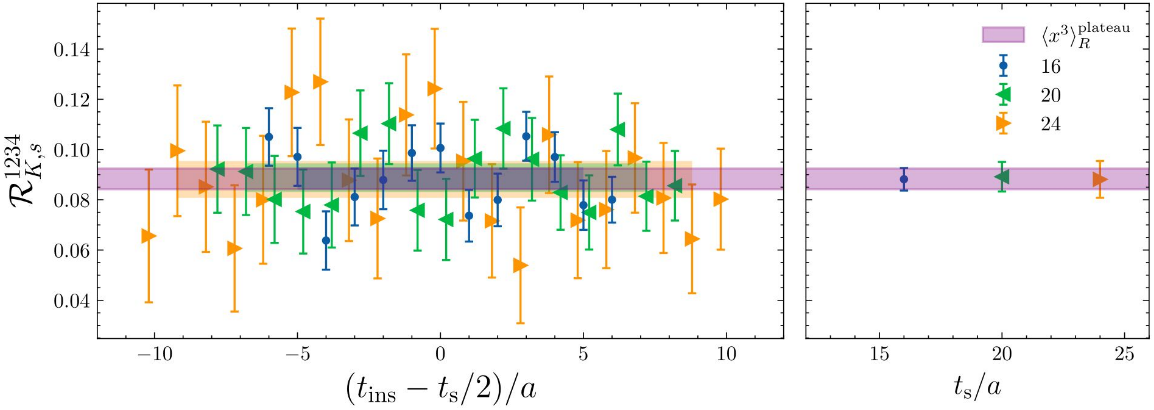
Higher Moments in the Kaon



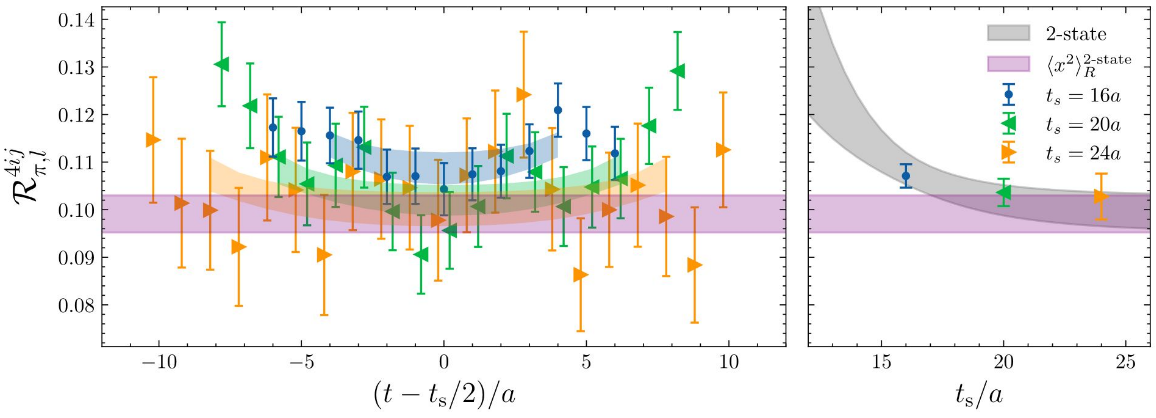


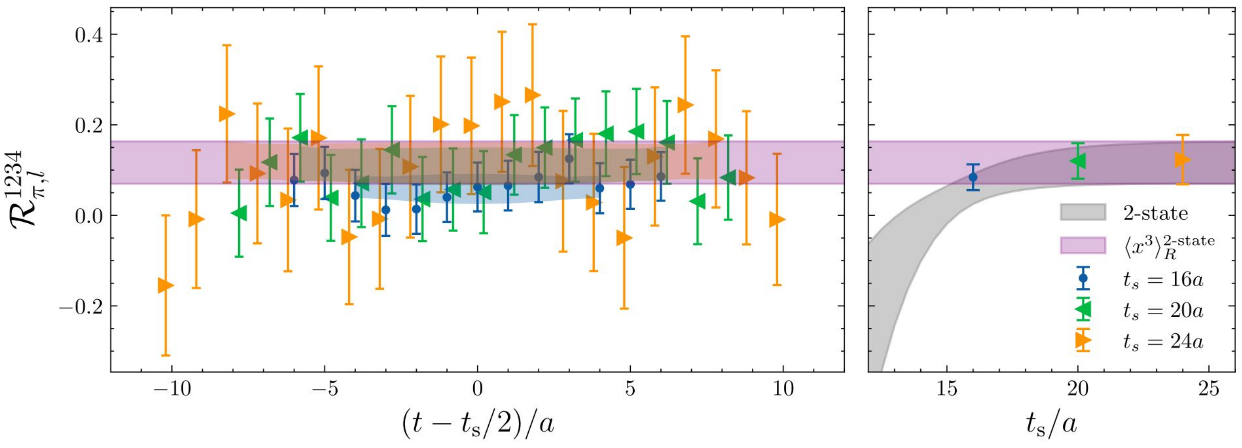
Higher Moments in the Kaon





Higher Moments in the Pion





Form Factors in the Nucleon

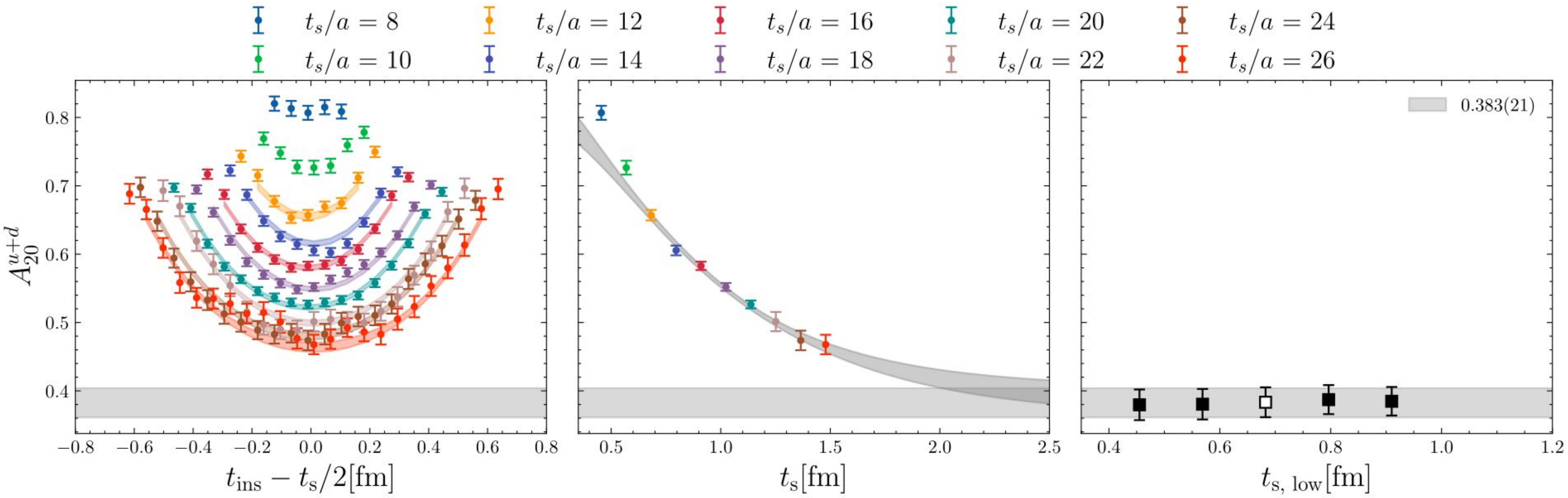
In the case of the nucleon, the matrix elements of the EMT can be decomposed in three GFFs.

$$\langle N(p', s')|T^{\mu\nu;q,g}|N(p,s)\rangle = \bar{u}_N(p', s') \times \left[A_{20}^{q,g}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q,g}(q^2)\frac{i\sigma^{\{\mu\rho}q_\rho P^{\nu\}}}{2m_N} + C_{20}^{q,g}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m_N}\right]u_N(p,s)$$

Only the average momentum fraction can be extracted directly at zero momentum transfer. We need to calculate ratios for non-zero momentum transfer:

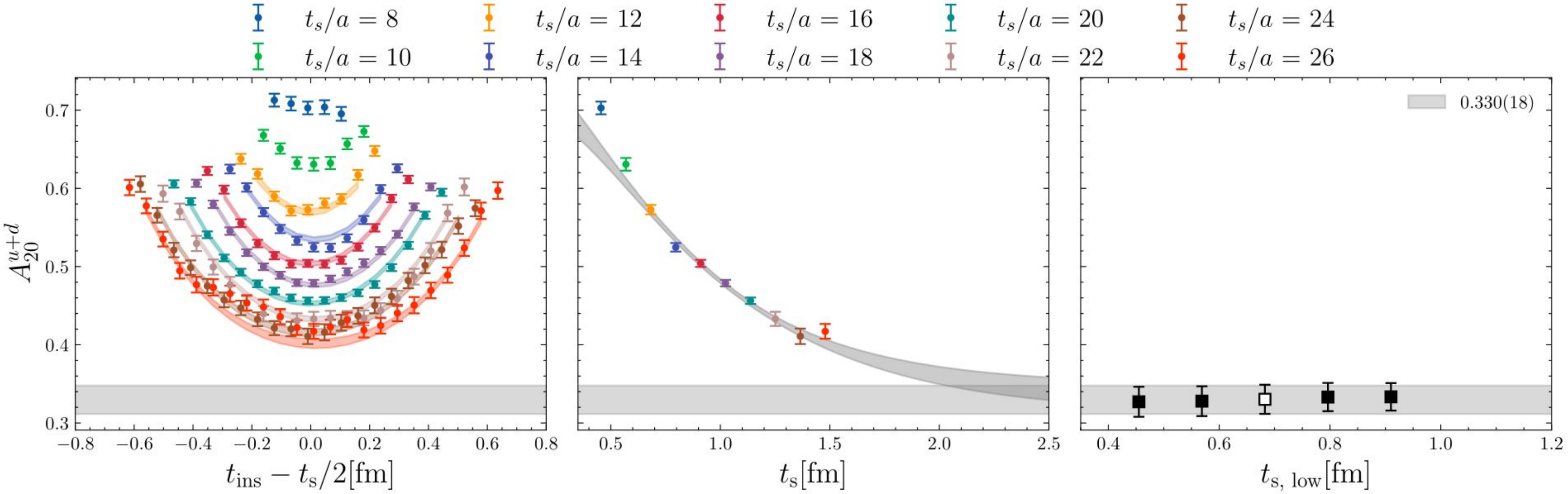
$$R^{\mu\nu}(\Gamma_\rho, \vec{p}\,', \vec{p}; t_s, t_{\text{ins}}) = \sqrt{\frac{C(\Gamma_0, \vec{p}; t_s - t_{\text{ins}})C(\Gamma_0, \vec{p}\,'; t_{\text{ins}})C(\Gamma_0, \vec{p}\,'; t_s)}{C(\Gamma_0, \vec{p}\,'; t_s - t_{\text{ins}})C(\Gamma_0, \vec{p}; t_{\text{ins}})C(\Gamma_0, \vec{p}; t_s)}} \times \frac{C^{\mu\nu}(\Gamma_\rho, \vec{p}\,', \vec{p}; t_s, t_{\text{ins}})}{C(\Gamma_0, \vec{p}\,'; t_s)}$$

Form Factors in the Nucleon



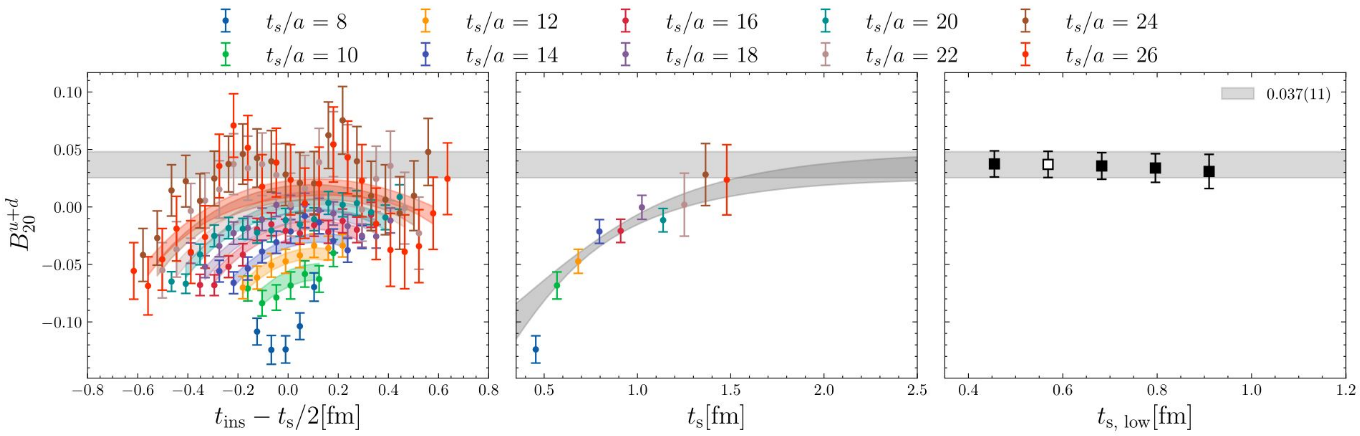
Average momentum fraction at zero momentum transfer

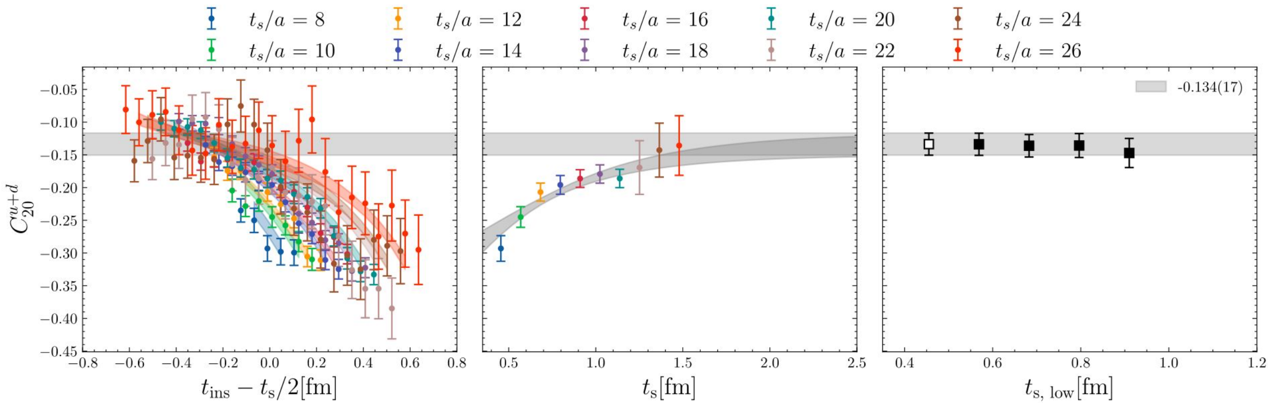
Form Factors in the Nucleon



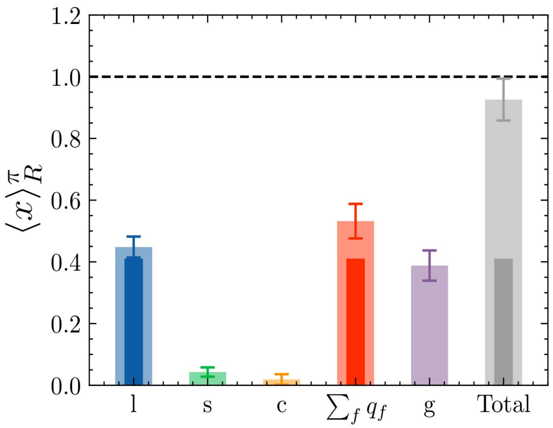
Average momentum fraction at two units of momentum transfer

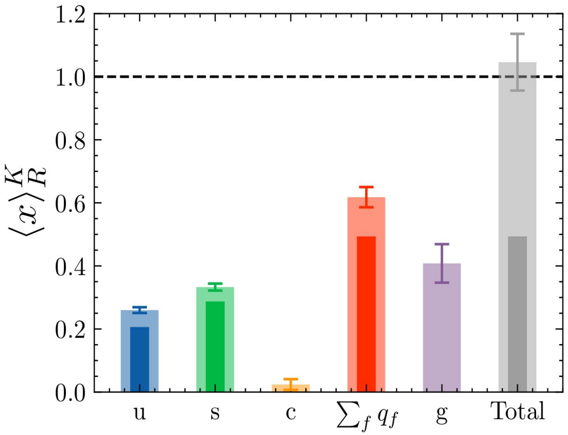
Form Factors in the Nucleon





Conclusions and Outlook





We have performed the first full decomposition of the pion and kaon average momentum fractions.

Manuscript for higher moments on preparation.

Ongoing analysis on the Q^2 dependence of the Form Factors for the nucleon to calculate the spin in the continuum limit.