Meson thermalization with a hot medium in the open Schwinger model

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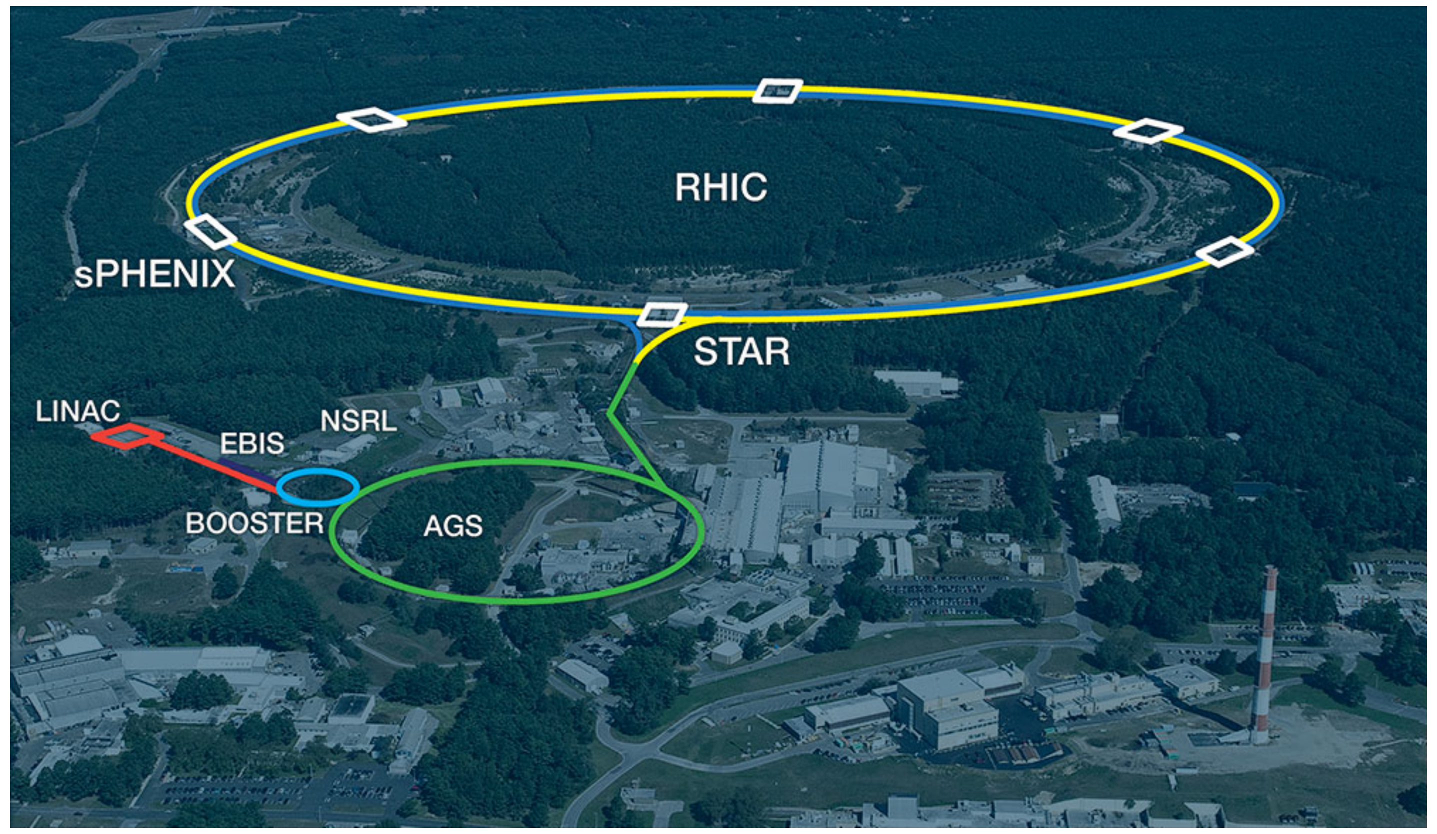
Collaborators: Yibin Guo, Karl Jansen, Stefan Kühn, Giuseppe Magnifico

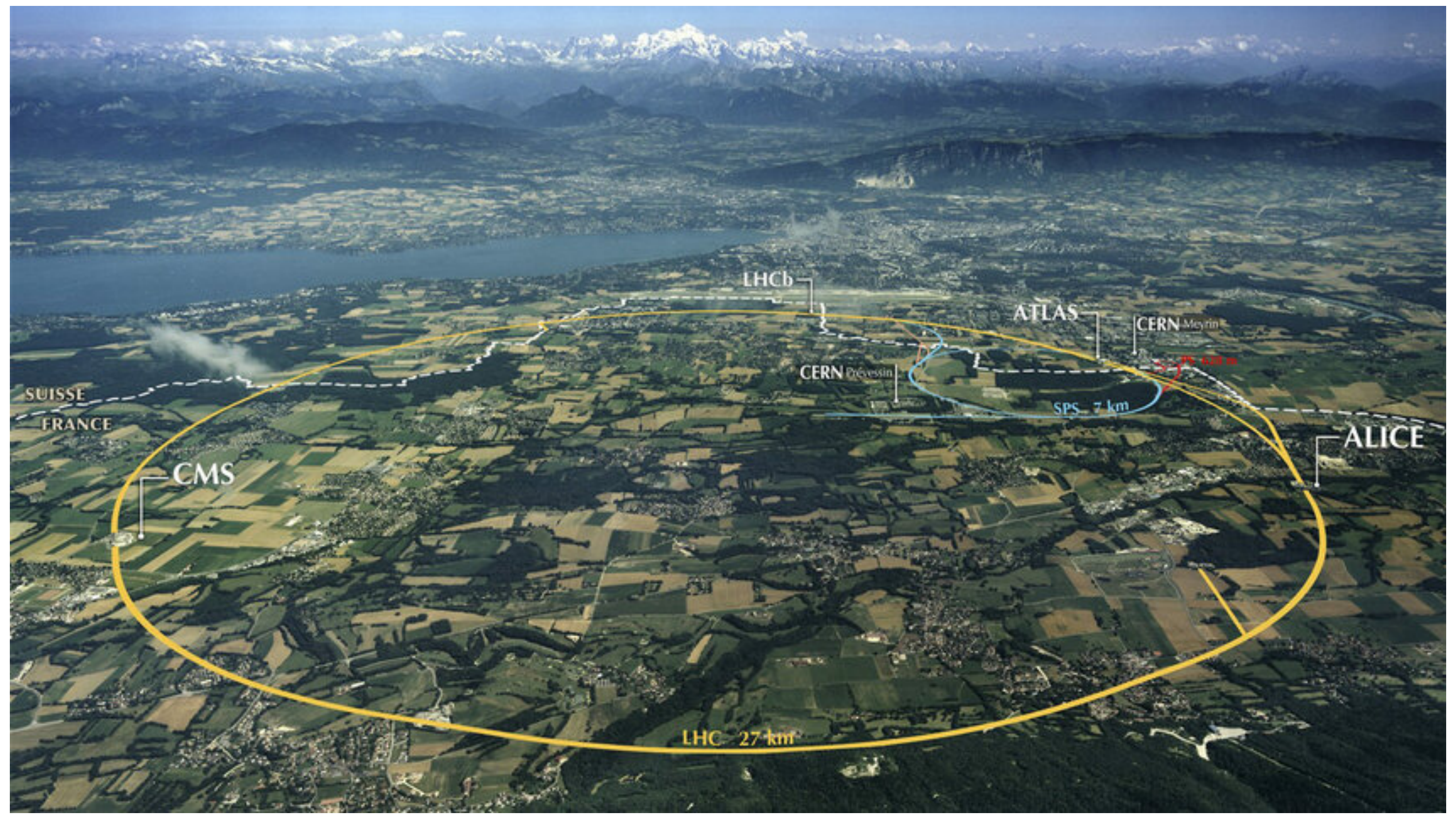
Takis Angelides

Motivation

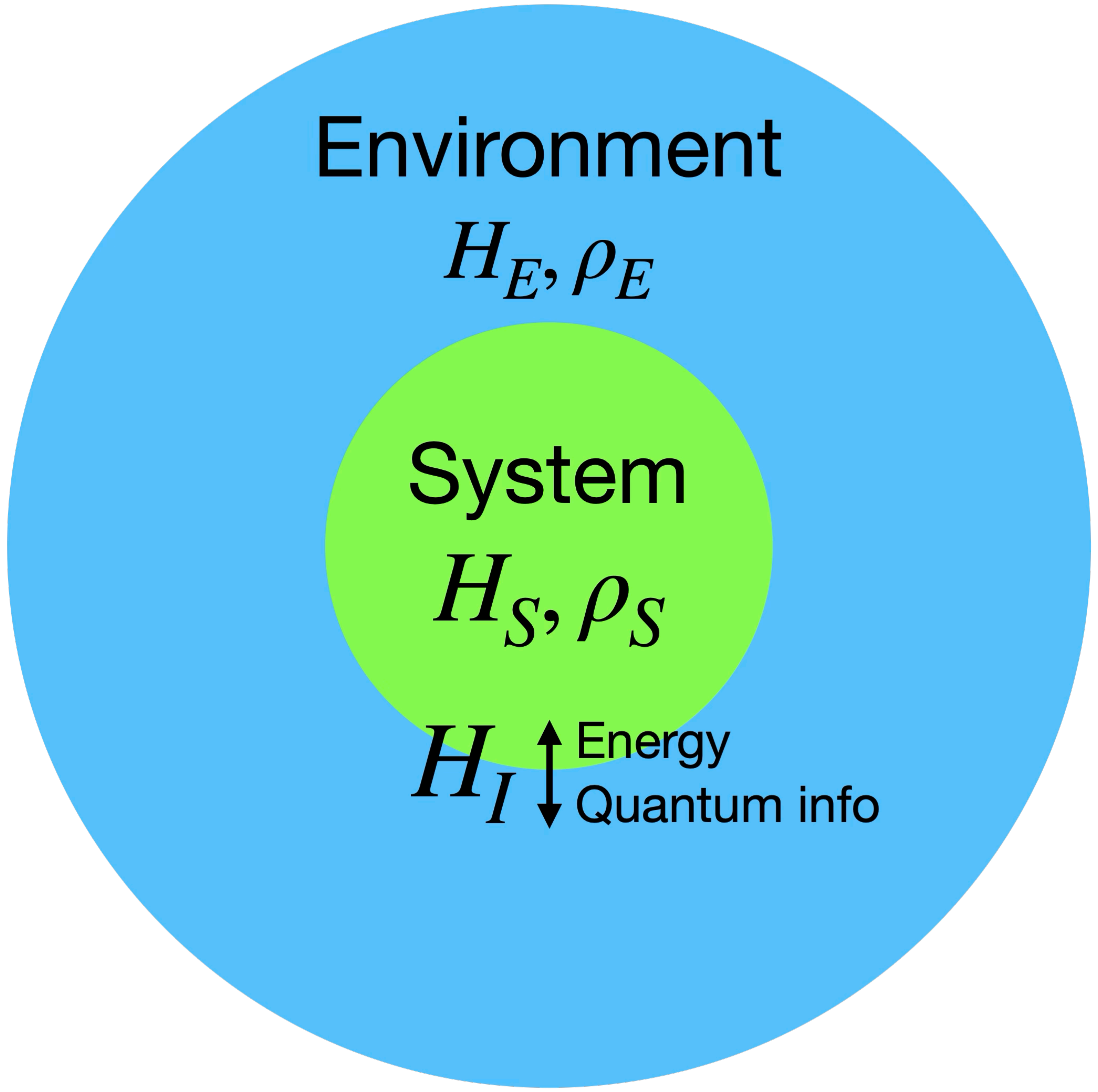
- Open quantum systems - quarkonia ($c\bar{c}$ or $b\bar{b}$) in quark-gluon plasma
- Schwinger model as a benchmark model for QCD
- Tensor networks for open quantum field theory



Brookhaven national lab



Large hadron collider



Theory

Schwinger model staggered Hamiltonian

$$H_S = x \sum_{n=0}^{N-2} \left(S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+ \right)$$

Kinetic hopping term $x = 1/(ag)^2$

$$+ \frac{1}{2} \sum_{n=0}^{N-2} \sum_{k=n+1}^{N-1} (N - k - 1) Z_n Z_k$$

Long range Coulomb interaction

$$+ \sum_{n=0}^{N-2} \left(\frac{N}{4} - \frac{1}{2} \left\lceil \frac{n}{2} \right\rceil + l_0 (N - n - 1) \right) Z_n$$

Electric field term

$$+ \frac{m_{lat}}{g} \sqrt{x} \sum_{n=0}^{N-1} (-1)^n Z_n$$

Mass term

$$+ l_0^2 (N - 1) + \frac{1}{2} l_0 N + \frac{1}{8} N^2$$

Constants

Gauss's law

$$L_n = \sum_{k=0}^{n-1} Q_k$$

$$Q_n = (Z_n + (-1)^n)/2$$

Theory

Total Hamiltonian

$$H = H_S + H_E + H_I$$

Total Hamiltonian of both system and environment

$$H_E = \int dx \left[\frac{1}{2}\Pi^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{4!}g\phi^4\right]$$

Environment Hamiltonian

ϕ^4 scalar field theory

$$H_I = \int dx \underbrace{\left(\lambda\phi(x)\right)}_{O_E(x)} \underbrace{\left(\bar{\psi}(x)\psi(x)\right)}_{O_S(x)}$$

Yukawa interaction between system and environment

PRD 108.094518

Theory

Quantum Brownian motion limit

$\tau_R \sim \dfrac{T}{\left(H_I^{(int)}\right)^2}$ Relaxation time of the system

$\tau_E \sim 1/T$ Environment correlation time

$\tau_S \sim 1/H_S$ Intrinsic time scale of the system

Separation of time scales

$\tau_R \gg \tau_E$ Markovian dynamics valid when H_I is weak

$\tau_S \gg \tau_E$ Valid when $T \gg H_S$

$\rho_E^{(int)}(t) = \rho_E = \dfrac{e^{-\beta H_E}}{\text{Tr}_E(e^{-\beta H_E})}$ Environment is assumed thermal at fixed temperature $T = 1/\beta$

Theory

Lindblad master equation

$$\frac{d\rho_S(t)}{dt} = -\,i\left[H_S, \rho_S(t)\right] + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2)\rho_S L^\dagger(x_1) - \frac{1}{2}\left\{L^\dagger(x_1)L(x_2), \rho_S\right\}\right)$$

Lindblad master equation

Theory

Lindblad master equation

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S, \rho_S(t)\right] + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2)\rho_S L^\dagger(x_1) - \frac{1}{2}\left\{L^\dagger(x_1)L(x_2), \rho_S\right\}\right)$$

Lindblad master equation

$$D(x_1 - x_2) = \lambda^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \mathrm{Tr}_E\left[\phi^{(int)}(t_1, x_1)\phi^{(int)}(t_2, x_2)\rho_E\right]$$

Theory

Lindblad master equation

$$L(na) = O(n) - \frac{1}{4T}\left[O(n), H_S\right]$$

Lindblad jump operators

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S, \rho_S(t)\right] + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2)\rho_S L^\dagger(x_1) - \frac{1}{2}\left\{L^\dagger(x_1)L(x_2), \rho_S\right\}\right)$$

Lindblad master equation

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Theory

Lindblad master equation

$$O(n) = (-1)^n \frac{Z_n + 1}{2a}$$

$$L(na) = O(n) - \frac{1}{4T}\left[O(n), H_S\right]$$

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Theory

Lindblad master equation

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Lindblad master equation

$$D(x_1 - x_2) = \lambda^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \mathrm{Tr}_E\left[\phi^{(int)}(t_1, x_1)\phi^{(int)}(t_2, x_2)\rho_E\right] = D\delta_{x_1,x_2}$$

Methods

Time evolution

$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

Lindblad master equation

$$\rho_S(t) = e^{\mathscr{L}t}\rho_S(t=0) = e^{\mathscr{L}_{odd}+\mathscr{L}_{even}+\mathscr{L}_{Taylor}}\rho_S(t=0)$$

Formal solution

$$e^{\mathscr{L}\tau} \approx e^{\mathscr{L}_{even}\frac{\tau}{2}}e^{\mathscr{L}_{Taylor}\frac{\tau}{2}}e^{\mathscr{L}_{odd}\tau}e^{\mathscr{L}_{Taylor}\frac{\tau}{2}}e^{\mathscr{L}_{even}\frac{\tau}{2}} + \mathcal{O}\left(\tau^2\right)$$

Integration scheme

Results

String state

- Dirac vacuum eigenstate in infinite mass limit $|\psi\rangle = |10..10\rangle, \quad \rho = |\psi\rangle\langle\psi|$
- Absence of electrical charge $Q_n = (Z_n + (-1)^n)/2, \quad n \in [0, N-1]$
- Create positive/negative charge pair by spin flipping gives string state
- Evolve both states and track subtracted observables
- Subtracted electric field (SEF) from Gauss's law

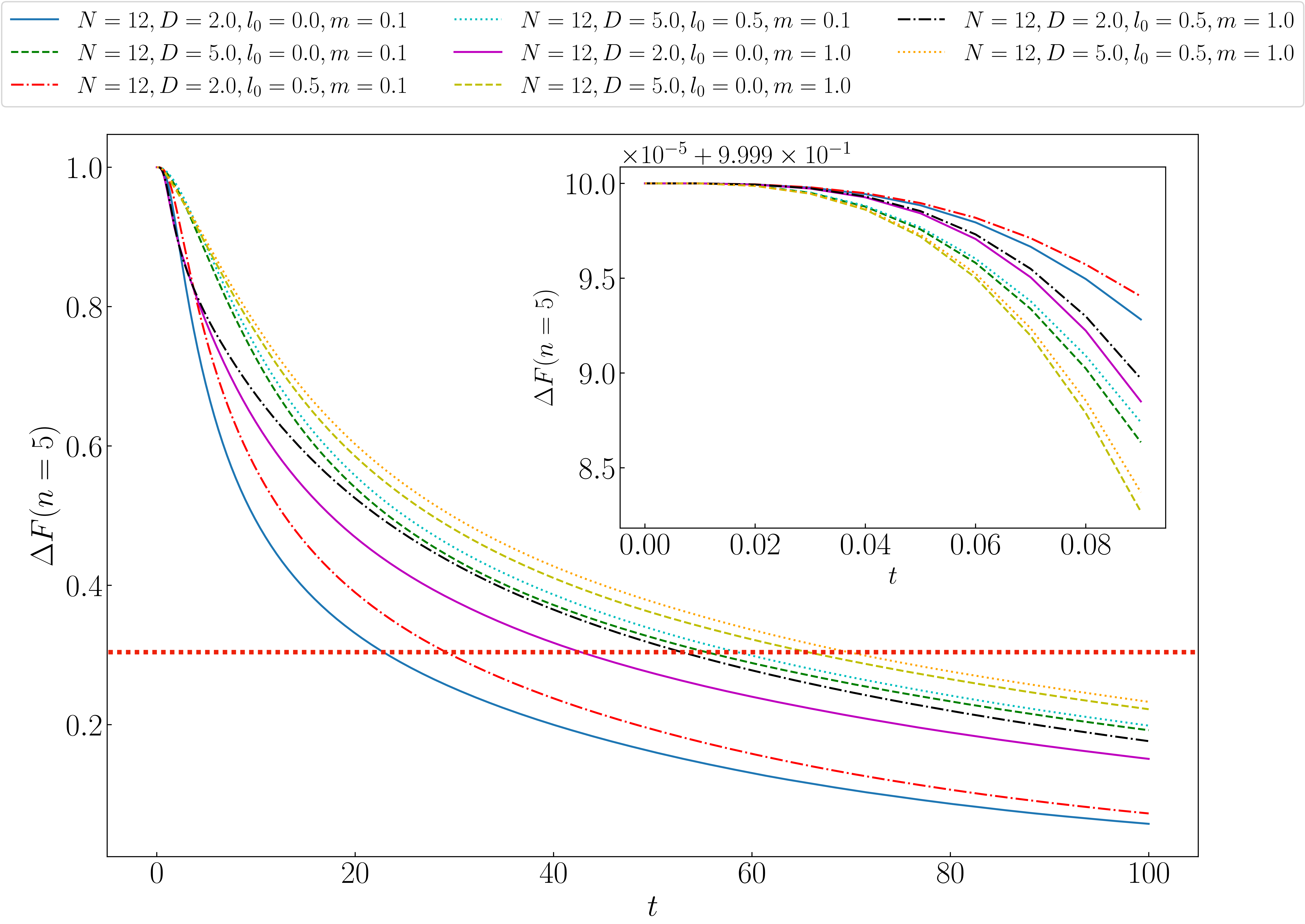
$$L_n = \sum_{k=0}^{n-1} Q_k \qquad \Delta F(n) = L_n^{(String)} - L_n^{(Dirac)}$$


(a) : ΔF(n)
0.00
0.25
0.50
0.75
1.00
n
t
0
5
10
15
20
25
30
0
1
2
3
4
5
6
7
8
9
10
(b) : ΔQ(n)
−1.0
−0.5
0.0
0.5
1.0
n
t
0
1
2
3
4
5
6
7
0
1
2
3
4
5
6
7
8
9
10
11

Results

Thermalization time definition

- Both states evolve to the same steady state
- Middle link has largest thermalization time to steady state
- Define thermalization time as time taken by the SEF of middle link to reach 30% of its initial value

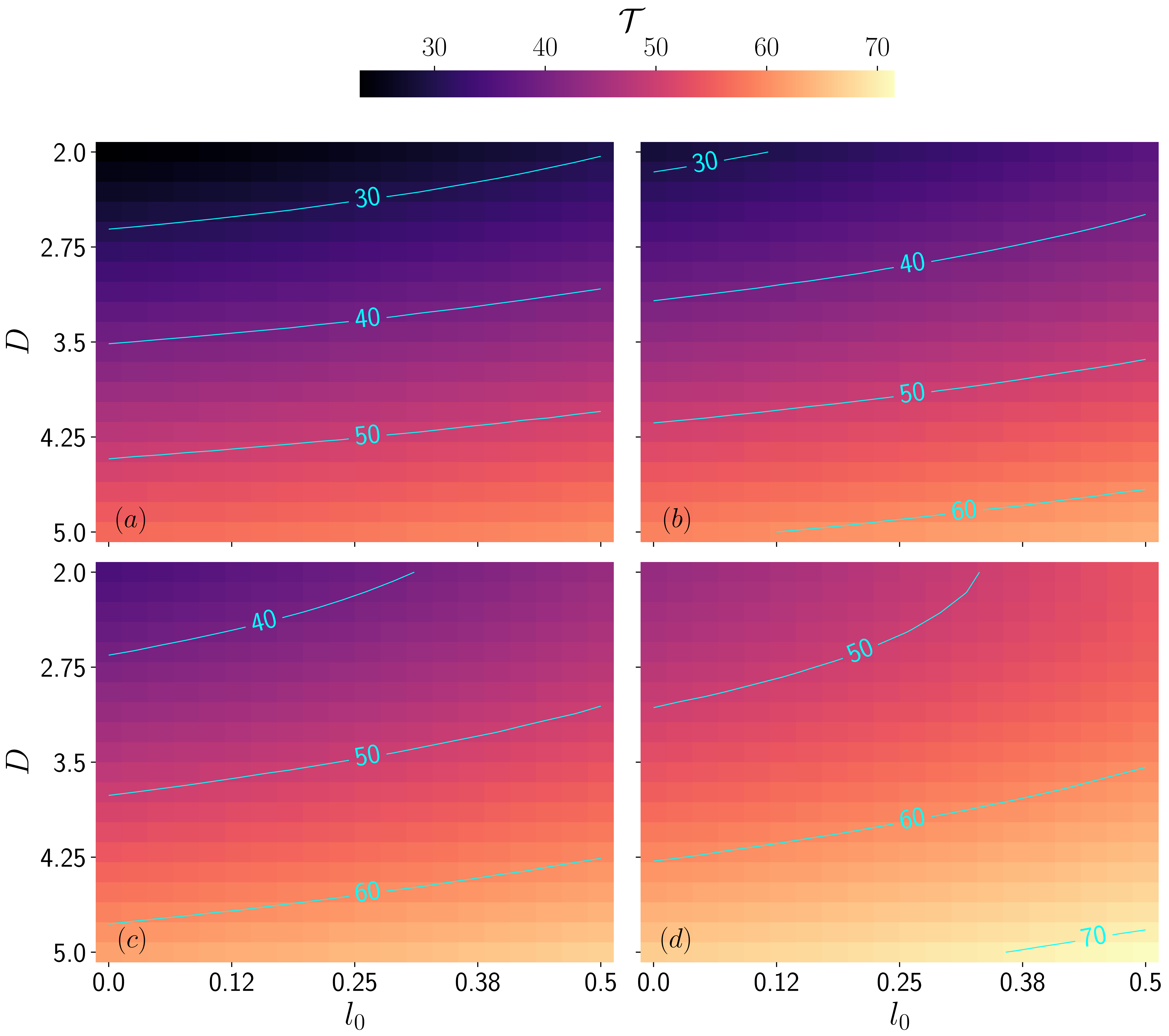


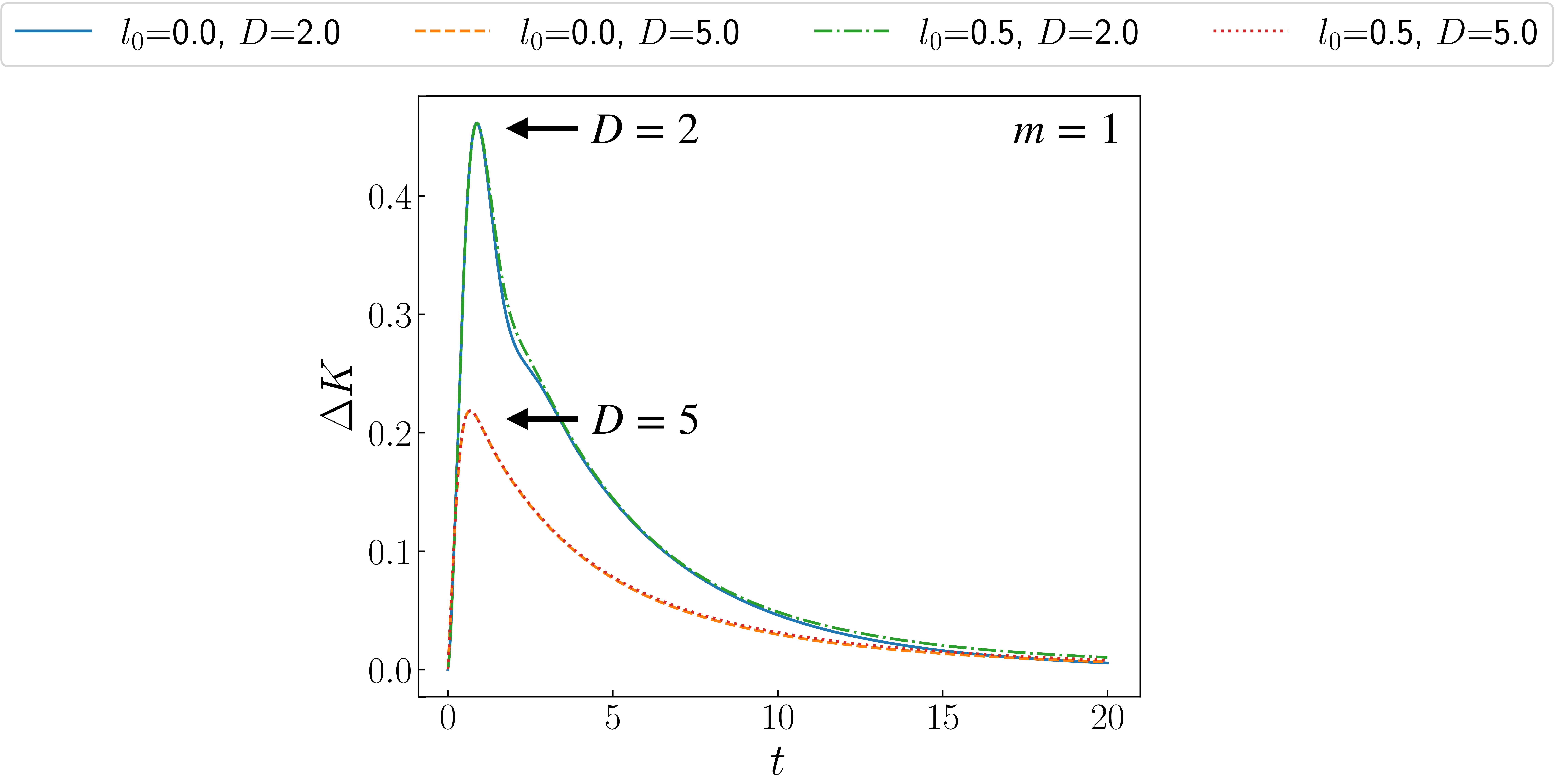
$N = 12, D = 2.0, l_0 = 0.0, m = 0.1$
$N = 12, D = 5.0, l_0 = 0.0, m = 0.1$
$N = 12, D = 2.0, l_0 = 0.5, m = 0.1$
$N = 12, D = 5.0, l_0 = 0.5, m = 0.1$
$N = 12, D = 2.0, l_0 = 0.0, m = 1.0$
$N = 12, D = 5.0, l_0 = 0.0, m = 1.0$
$N = 12, D = 2.0, l_0 = 0.5, m = 1.0$
$N = 12, D = 5.0, l_0 = 0.5, m = 1.0$
$\Delta F(n = 5)$
t
0 20 40 60 80 100
0.2 0.4 0.6 0.8 1.0
$\times 10^{-5} + 9.999 \times 10^{-1}$
10.0 9.5 9.0 8.5
0.00 0.02 0.04 0.06 0.08

Results

TT vs D, m, l_0

- $N = 12, \quad T = 10$
- $\dfrac{m_{lat}}{g} = m = 0.1, 0.5, 0.75, 1$
- $x = 1, \tau = 0.01$





l_0=0.0, D=2.0
l_0=0.0, D=5.0
l_0=0.5, D=2.0
l_0=0.5, D=5.0
$D = 2$
$m = 1$
$D = 5$
ΔK
0.0
0.1
0.2
0.3
0.4
0
5
10
15
20
t

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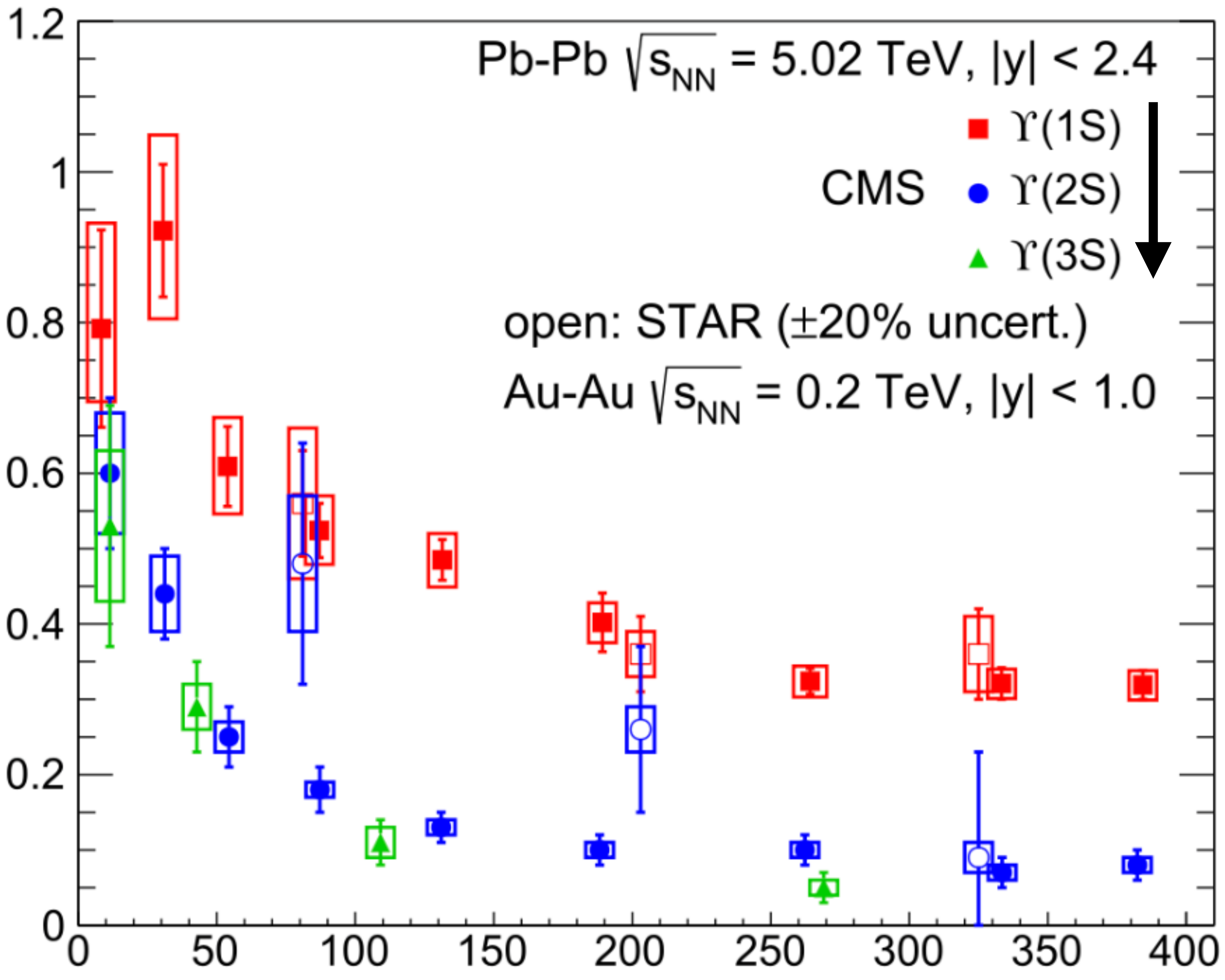
Nuclear modification factor (yield) = R_{AA}^{Υ}



Faster thermalization

More delocalization radially extended bottomonium states

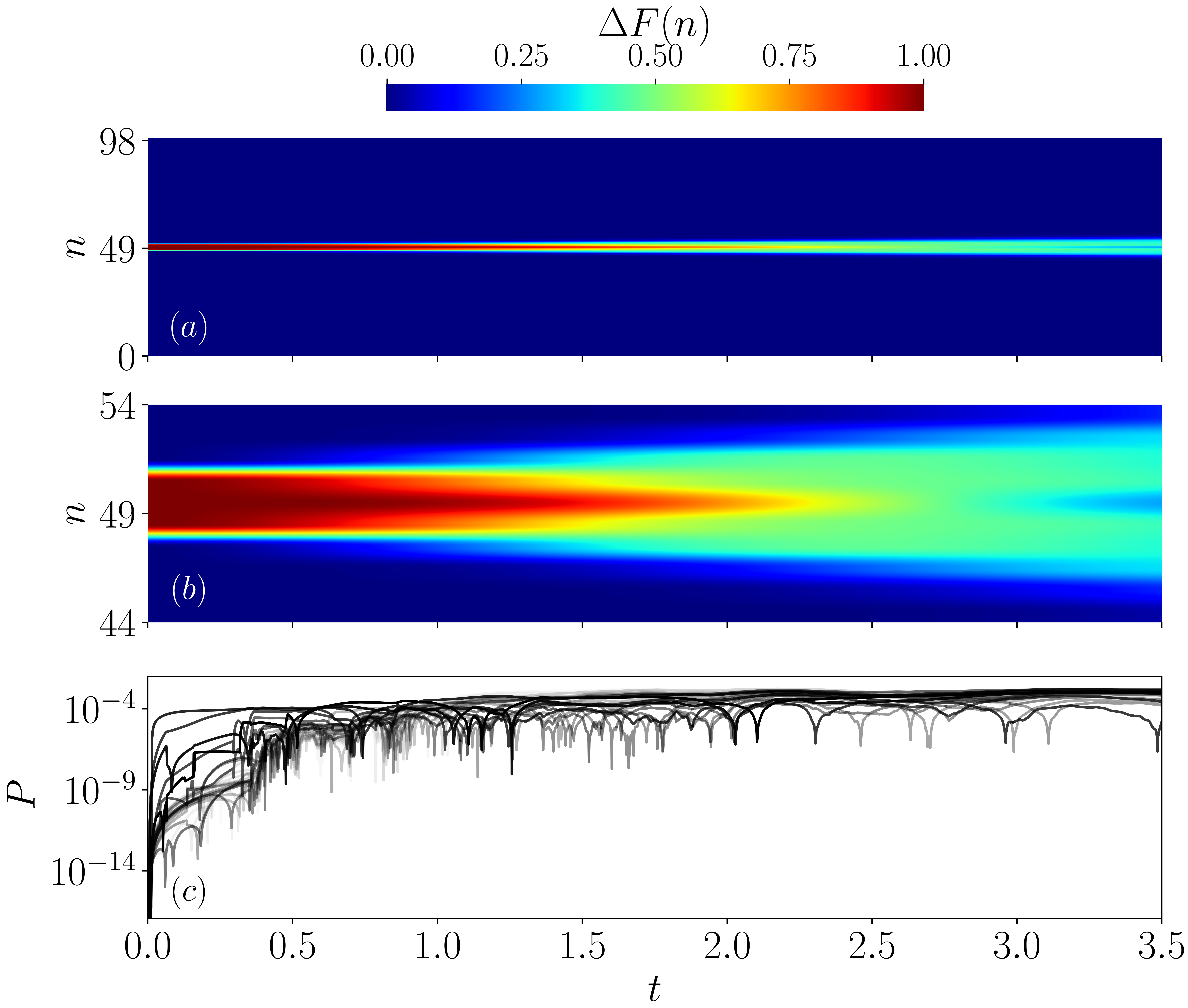
N_{part} = # of participating nucleons

Hotter and denser QGP

Results

Larger systems and symmetry preservation

- Thermalization time results agree to $\mathcal{O}(0.1)$ from $N = 12$ with $N = 24$
- The delta function dissipator $\Longrightarrow$ electric field parity symmetry
- $N = 100, D = 0.15, x = 4, m = 0, \tau = 0.001$
- Measure of symmetry preservation: $P = \left| \Delta F(n = i) - \Delta F(n = N - i - 2) \right|$
- Preserved symmetry to $\mathcal{O}\left(10^{-4}\right)$
- Demonstrates scalability



$\Delta F(n)$
0.00
0.25
0.50
0.75
1.00
98
49
0
n
(a)
54
49
44
n
(b)
10^{-4}
10^{-9}
10^{-14}
P
(c)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
t

Results

Schwinger boson

- $N = 14, T = 10, l_0 = 0, m = 0$
- First excited state of H_S minus its ground state
- Stable mesonic particle of H_S : Schwinger boson


$D = 2.0$
$D = 2.75$
$D = 3.5$
$D = 4.25$
$D = 5.0$
$\Delta F_B(n=6)$
t
ΔE_B
$\mathcal{T}_{\Delta F_B}$
$\mathcal{T}_{\Delta E_B}$
$\mathcal{T}_{\Delta F_B}, \mathcal{T}_{\Delta E_B}$
D

Conclusion

- First TN simulation of OQFT
- Investigated how $D,\ l_0,\ m,$ influence thermalization of mesonic states
- Identified similarities with quarkonia in QGP
- Demonstrated robustness and scalability
- Outlook: extend to higher-dimensional gauge theories

DESY.

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ENGAGE

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References

- [1]: Lee, Kyle and Mulligan, James and Ringer, Felix and Yao, Xiaojun, Liouvillian dynamics of the open Schwinger model: String breaking and kinetic dissipation in a thermal medium, DOI: 10.1103/PhysRevD.108.094518
- [2]: S. Caron-Huot and G.D. Moore, Heavy quark diffusion in perturbative qcd at next-to-leading order, Phys. Rev. Lett. 100 (2008) 052301

Appendices

- Schwinger model staggered fermions Hamiltonian
- Derivation of the Lindblad master equation
- Results of the thermalization time against individual system parameters
- Subtracted kinetic energy time evolution
- Subtracted electric field against D, m, l_0 for $N = 12,\ 24$
- Schwinger boson thermalization dynamics
- Thermalization time vs temperature in QCD

Theory

Schwinger model

$$\mathcal{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Temporal gauge $A_0 = 0$

Theory

Schwinger model

Two component Dirac spinor

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

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Kinetic term

Temporal gauge $A_0 = 0$

Theory

Schwinger model

Two component Dirac spinor

$$\mathcal{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Kinetic term

Coupling term between fermions and gauge fields

Temporal gauge $A_0 = 0$

Theory

Schwinger model

Mass term

Two component Dirac spinor

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Kinetic term

Coupling term between fermions and gauge fields

Temporal gauge $A_0 = 0$

Theory

Schwinger model

Two component Dirac spinor

Mass term

Electric field term

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Kinetic term

Coupling term between fermions and gauge fields

Temporal gauge $A_0 = 0$

Theory

Schwinger model

Electric field term

Topological theta term

Mass term

Two component Dirac spinor

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Kinetic term

Coupling term between fermions and gauge fields

Temporal gauge $A_0 = 0$

Theory

Schwinger model

Electric field term

Topological theta term

Mass term

Two component Dirac spinor

$$\mathscr{H} = - i\bar{\psi}(x)\gamma^1 \left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Kinetic term

Coupling term between fermions and gauge fields

Temporal gauge $A_0 = 0$

Gauss's law $-\partial_1 \dot{A}^1 = g\bar{\psi}\gamma^0\psi$

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Temporal gauge $A_0 = 0$

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$$H_S = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger U_n\phi_{n+1} - \phi_{n+1}^\dagger U_n^\dagger\phi_n\right) + m_{lat}\sum_{n=0}^{N-1}(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_{n=0}^{N-2}L_n^2$$

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Temporal gauge $A_0 = 0$

Gauss's law $-\partial_1\dot{A}^1 = g\bar{\psi}\gamma^0\psi$

Number of lattice sites ↑

$$H_S = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger U_n \phi_{n+1} - \phi_{n+1}^\dagger U_n^\dagger \phi_n\right) + m_{lat}\sum_{n=0}^{N-1}(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_{n=0}^{N-2}L_n^2$$

Theory

Schwinger model

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Lattice site number

Theory
Schwinger model

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Lattice site number

Link operator on the right of site n

Theory

Schwinger model

$$\mathcal{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

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Number of lattice sites

Lattice mass

Lattice site number

Link operator on the right of site n

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Temporal gauge $A_0 = 0$

Gauss's law $-\partial_1\dot{A}^1 = g\bar{\psi}\gamma^0\psi$

Number of lattice sites

Lattice mass

Single component Dirac spinor

$$H_S = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger U_n \phi_{n+1} - \phi_{n+1}^\dagger U_n^\dagger \phi_n\right) + m_{lat}\sum_{n=0}^{N-1}(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_{n=0}^{N-2}L_n^2$$

Lattice site number

Link operator on the right of site n

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

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- Number of lattice sites (N)
- Lattice site number (n)
- Link operator on the right of site n (U_n)
- Lattice mass (m_{lat})
- Single component Dirac spinor (ϕ_n)
- Electric field operator on link to the right of site n (L_n)

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

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$$L_n - L_{n-1} = Q_n$$

Number of lattice sites

Lattice site number

Link operator on the right of site n

Lattice mass

Single component Dirac spinor

Electric field operator on link to the right of site n

Theory

Schwinger model

$$\mathscr{H} = - i\bar{\psi}(x)\gamma^1 \left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Temporal gauge $A_0 = 0$

Gauss's law $-\partial_1 \dot{A}^1 = g\bar{\psi}\gamma^0\psi$

Number of lattice sites

Lattice mass

Single component Dirac spinor

$$H_S = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger U_n \phi_{n+1} - \phi_{n+1}^\dagger U_n^\dagger \phi_n\right) + m_{lat}\sum_{n=0}^{N-1}(-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_{n=0}^{N-2} L_n^2$$

$$L_n - L_{n-1} = Q_n$$

Lattice site number

Link operator on the right of site n

Electric field operator on link to the right of site n

Charge operator

$$Q_n = \phi_n^\dagger \phi_n - \left(1 - (-1)^n\right)/2$$

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

Temporal gauge $A_0 = 0$

Gauss's law $-\partial_1\dot{A}^1 = g\bar{\psi}\gamma^0\psi$

$$H_S = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger U_n\phi_{n+1} - \phi_{n+1}^\dagger U_n^\dagger\phi_n\right) + m_{lat}\sum_{n=0}^{N-1}(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_{n=0}^{N-2}L_n^2$$

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$$L_n - L_{n-1} = Q_n$$

Unitary transformation
on fields and solving Gauss's law

Charge operator

$$Q_n = \phi_n^\dagger\phi_n - \left(1 - (-1)^n\right)/2$$

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

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Unitary transformation
on fields and solving Gauss's law

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$$+m_{lat}\sum_{n=0}^{N-1}(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_{n=0}^{N-2}\left(l_0 + \sum_{k=0}^{n}Q_k\right)^2$$

Charge operator

$$Q_n = \phi_n^\dagger\phi_n - \left(1 - (-1)^n\right)/2$$

Theory

Schwinger model

$$\mathscr{H} = -i\bar{\psi}(x)\gamma^1\left(\partial_x - igA_1\right)\psi(x) + m\bar{\psi}\psi + \frac{1}{2}\left(\dot{A}_1 + \frac{g\theta}{2\pi}\right)$$

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$$H_S = -\frac{i}{2a}\sum_{n=0}^{N-2}\left(\phi_n^\dagger\phi_{n+1} - \phi_{n+1}^\dagger\phi_n\right)$$

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$l_0 = \frac{\theta}{2\pi}$ Constant background electric field

Charge operator

$$Q_n = \phi_n^\dagger\phi_n - \left(1-(-1)^n\right)/2$$

Theory

Staggered Hamiltonian

Jordan-Wigner transformation

$$H_S = x \sum_{n=0}^{N-2} \left(S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+ \right)$$

Kinetic hopping term $x = 1/(ag)^2$

$$+\frac{1}{2} \sum_{n=0}^{N-2} \sum_{k=n+1}^{N-1} (N-k-1) Z_n Z_k$$

Long range Coulomb interaction

$$+\sum_{n=0}^{N-2} \left(\frac{N}{4} - \frac{1}{2} \left\lceil \frac{n}{2} \right\rceil + l_0 (N-n-1) \right) Z_n$$

Electric field term

$$+\frac{m_{lat}}{g} \sqrt{x} \sum_{n=0}^{N-1} (-1)^n Z_n$$

Mass term

$$+l_0^2 (N-1) + \frac{1}{2} l_0 N + \frac{1}{8} N^2$$

Constants

$$Q_n = (Z_n + (-1)^n)/2$$

Theory

Derivation of Lindblad master equation (LME)

$$\frac{d\rho^{(int)}(t)}{dt} = -i\left[H_I^{(int)}, \rho^{(int)}(t)\right]$$

Interaction picture von Neumann equation of motion

Theory

Derivation of Lindblad master equation (LME)

$$\rho^{(int)}(t) = e^{i(H_S+H_E)t}\rho(t)e^{-i(H_S+H_E)t}$$

↑

$$\frac{d\rho^{(int)}(t)}{dt} = -\,i\left[H_I^{(int)}, \rho^{(int)}(t)\right]$$

Interaction picture von Neumann equation of motion

Theory

Derivation of Lindblad master equation (LME)

$$\rho^{(int)}(t) = e^{i(H_S+H_E)t}\rho(t)e^{-i(H_S+H_E)t}$$

$$H_I^{(int)}(t) = e^{i(H_S+H_E)t}H_I e^{-i(H_S+H_E)t} = \int dx O_E^{(int)}(t,x) O_S^{(int)}(t,x)$$

$$\frac{d\rho^{(int)}(t)}{dt} = -i\left[H_I^{(int)}, \rho^{(int)}(t)\right]$$

Interaction picture von Neumann equation of motion

Theory

Derivation of Lindblad master equation (LME)

$$O^{(int)}_{E/S}(t,x) = e^{iH_{E/S}t} O_{E/S}(x) e^{-iH_{E/S}t}$$

$$\rho^{(int)}(t) = e^{i(H_S+H_E)t}\rho(t)e^{-i(H_S+H_E)t}$$

$$H^{(int)}_I(t) = e^{i(H_S+H_E)t} H_I e^{-i(H_S+H_E)t} = \int dx O^{(int)}_E(t,x) O^{(int)}_S(t,x)$$

$$\frac{d\rho^{(int)}(t)}{dt} = -i\left[H^{(int)}_I, \rho^{(int)}(t)\right]$$

Interaction picture von Neumann equation of motion

Theory

Derivation of Lindblad master equation (LME)

$$O_{E/S}^{(int)}(t,x) = e^{iH_{E/S}t} O_{E/S}(x) e^{-iH_{E/S}t}$$

$$\rho^{(int)}(t) = e^{i(H_S+H_E)t}\rho(t)e^{-i(H_S+H_E)t}$$

$$H_I^{(int)}(t) = e^{i(H_S+H_E)t}H_I e^{-i(H_S+H_E)t} = \int dx O_E^{(int)}(t,x) O_S^{(int)}(t,x)$$

$$\frac{d\rho^{(int)}(t)}{dt} = -i\left[H_I^{(int)}, \rho^{(int)}(t)\right]$$

Interaction picture von Neumann equation of motion

Trace out the environment degrees of freedom

Theory

Derivation of Lindblad master equation (LME)

$$O^{(int)}_{E/S}(t,x) = e^{iH_{E/S}t} O_{E/S}(x) e^{-iH_{E/S}t}$$

$$\rho^{(int)}(t) = e^{i(H_S+H_E)t} \rho(t) e^{-i(H_S+H_E)t}$$

$$H_I^{(int)}(t) = e^{i(H_S+H_E)t} H_I e^{-i(H_S+H_E)t} = \int dx O_E^{(int)}(t,x) O_S^{(int)}(t,x)$$

$$\frac{d\rho^{(int)}(t)}{dt} = -i\left[H_I^{(int)}, \rho^{(int)}(t)\right]$$

Interaction picture von Neumann equation of motion

$$\rho_S^{(int)}(t) = \mathrm{Tr}_E\left(\rho^{(int)}(t)\right) = \mathrm{Tr}_E\left(U(t)\rho^{(int)}(0)U^\dagger(t)\right)$$

Trace out the environment degrees of freedom

Theory

Derivation of Lindblad master equation (LME)

$$O_{E/S}^{(int)}(t,x) = e^{iH_{E/S}t} O_{E/S}(x) e^{-iH_{E/S}t}$$

$$\rho^{(int)}(t) = e^{i(H_S+H_E)t} \rho(t) e^{-i(H_S+H_E)t}$$

$$H_I^{(int)}(t) = e^{i(H_S+H_E)t} H_I e^{-i(H_S+H_E)t} = \int dx O_E^{(int)}(t,x) O_S^{(int)}(t,x)$$

$$\frac{d\rho^{(int)}(t)}{dt} = -i \left[H_I^{(int)}, \rho^{(int)}(t) \right]$$

Interaction picture von Neumann equation of motion

$$\rho_S^{(int)}(t) = \mathrm{Tr}_E \left(\rho^{(int)}(t) \right) = \mathrm{Tr}_E \left(U(t) \rho^{(int)}(0) U^\dagger(t) \right)$$

$$U(t) = \mathscr{T} e^{-i \int_0^t H_I^{(int)}(t')dt'}$$

Trace out the environment degrees of freedom

Theory

Derivation of LME

$$\rho_E^{(int)}(t) = \rho_E = \frac{e^{-\beta H_E}}{\text{Tr}_E(e^{-\beta H_E})}$$

Environment is assumed thermal at fixed temperature $T = 1/\beta$

Theory

Derivation of LME

$$\rho_E^{(int)}(t) = \rho_E = \frac{e^{-\beta H_E}}{\mathrm{Tr}_E(e^{-\beta H_E})}$$

Environment is assumed thermal at fixed temperature $T = 1/\beta$

Born approximation: weak coupling between the system and environment

Theory

Derivation of LME

$$\rho_E^{(int)}(t) = \rho_E = \frac{e^{-\beta H_E}}{\mathrm{Tr}_E(e^{-\beta H_E})}$$

Environment is assumed thermal at fixed temperature $T = 1/\beta$

Born approximation: weak coupling between the system and environment

$$\rho_S^{(int)}(t) = \rho_S^{(int)}(0) - \int dx_1 \int dx_2 \frac{\mathrm{sign}(t_1 - t_2)}{2} D(x_1 - x_2)\left[O_S^{(int)}(t_1, x_1)O_S^{(int)}(t_2, x_2), \rho_S^{(int)}\right]$$

$$+ \int dx_1 \int dx_2 D(x_1 - x_2)\left[O_S^{(int)}(t_2, x_2)\rho_S^{(int)}(0)O_S^{(int)}(t_1, x_1) - \frac{1}{2}\left\{O_S^{(int)}(t_1, x_1)O_S^{(int)}(t_2, x_2), \rho_S^{(int)}\right\}\right] + \mathcal{O}\left(\left(tH_I^{(int)}\right)^3\right)$$

Theory

Derivation of LME

$$\rho_E^{(int)}(t) = \rho_E = \frac{e^{-\beta H_E}}{\text{Tr}_E(e^{-\beta H_E})}$$

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Born approximation: weak coupling between the system and environment

$$\rho_S^{(int)}(t) = \rho_S^{(int)}(0) - \int dx_1 \int dx_2 \frac{\text{sign}(t_1 - t_2)}{2} D(x_1 - x_2)\left[O_S^{(int)}(t_1, x_1) O_S^{(int)}(t_2, x_2), \rho_S^{(int)}\right]$$
$$+ \int dx_1 \int dx_2 D(x_1 - x_2)\left[O_S^{(int)}(t_2, x_2)\rho_S^{(int)}(0) O_S^{(int)}(t_1, x_1) - \frac{1}{2}\left\{O_S^{(int)}(t_1, x_1) O_S^{(int)}(t_2, x_2), \rho_S^{(int)}\right\}\right] + \mathcal{O}\left(\left(tH_I^{(int)}\right)^3\right)$$

$$\rho^{(int)}(t=0) = \rho_S^{(int)}(t=0) \otimes \rho_E$$

Assumption on the initial state

Theory

Derivation of LME

$$\rho_E^{(int)}(t) = \rho_E = \frac{e^{-\beta H_E}}{\mathrm{Tr}_E(e^{-\beta H_E})}$$

Environment is assumed thermal at fixed temperature $T = 1/\beta$

Born approximation: weak coupling between the system and environment

$$\rho_S^{(int)}(t) = \rho_S^{(int)}(0) - \int dx_1 \int dx_2 \frac{\mathrm{sign}(t_1 - t_2)}{2} D(x_1 - x_2) \left[O_S^{(int)}(t_1, x_1) O_S^{(int)}(t_2, x_2), \rho_S^{(int)} \right]$$

$$+ \int dx_1 \int dx_2 D(x_1 - x_2) \left[O_S^{(int)}(t_2, x_2) \rho_S^{(int)}(0) O_S^{(int)}(t_1, x_1) - \frac{1}{2} \left\{ O_S^{(int)}(t_1, x_1) O_S^{(int)}(t_2, x_2), \rho_S^{(int)} \right\} \right] + \mathcal{O}\left(\left(t H_I^{(int)} \right)^3 \right)$$

$$\rho^{(int)}(t=0) = \rho_S^{(int)}(t=0) \otimes \rho_E$$

Assumption on the initial state

$$D(x_1 - x_2) = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \mathrm{Tr}_E \left(O_E^{(int)}(t_1, x_1) O_E^{(int)}(t_2, x_2) \rho_E \right)$$

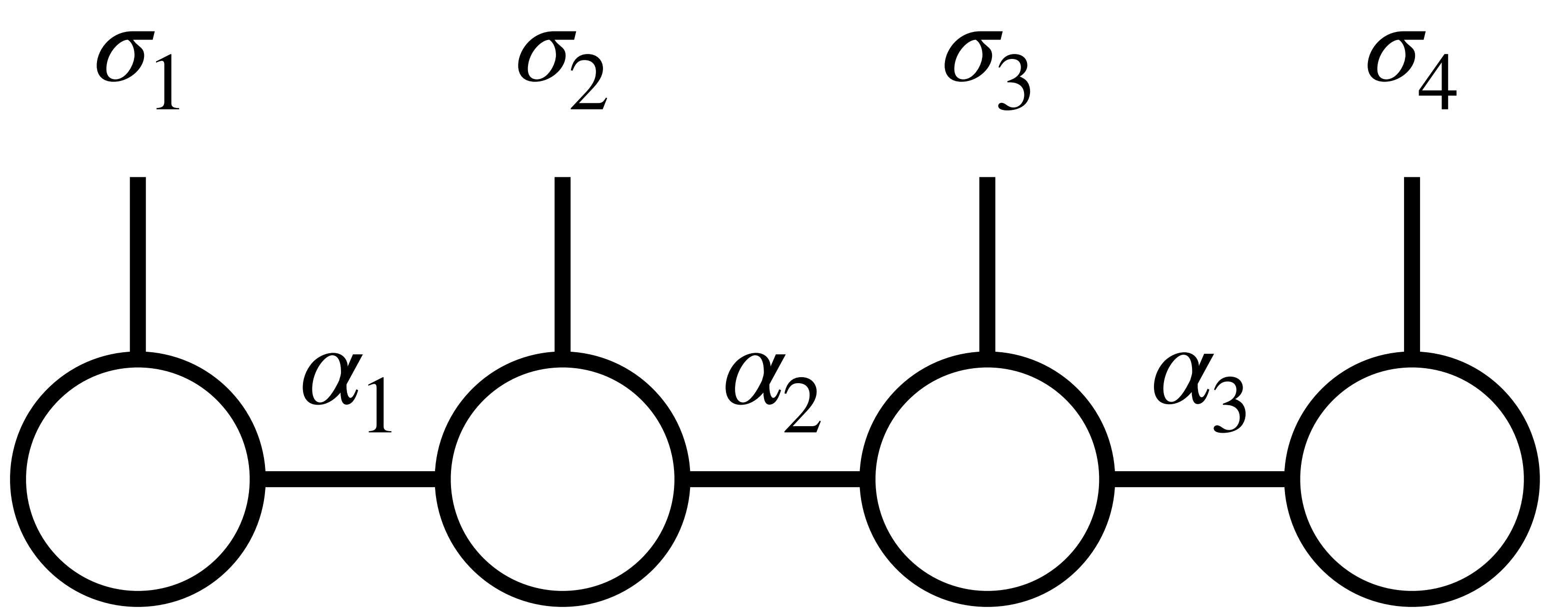
Environment correlator

Methods

MPS representation

$$\psi_{\sigma_1 \ldots \sigma_N} \approx A^{\sigma_1}_{\alpha_1} A^{\sigma_2}_{\alpha_1,\alpha_2} \ldots A^{\sigma_{N-1}}_{\alpha_{N-2},\alpha_{N-1}} A^{\sigma_N}_{\alpha_{N-1}}$$

MPS



Methods

MPS representation

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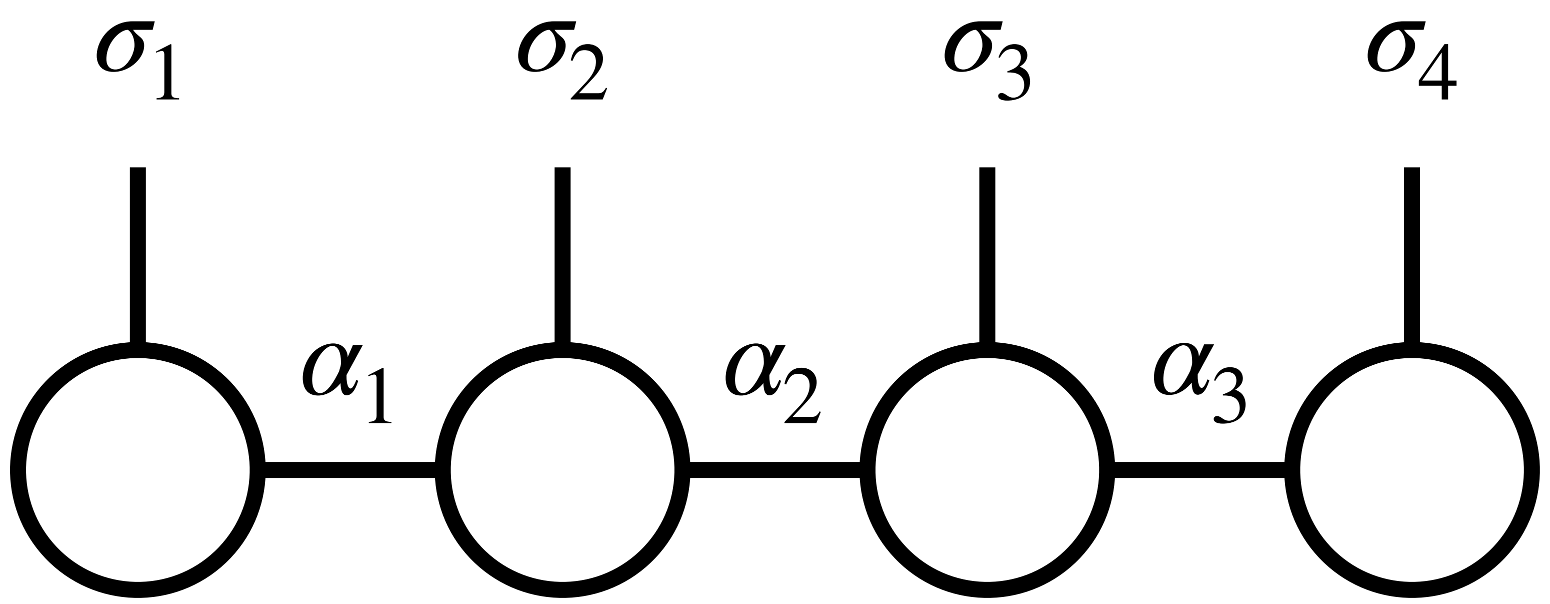

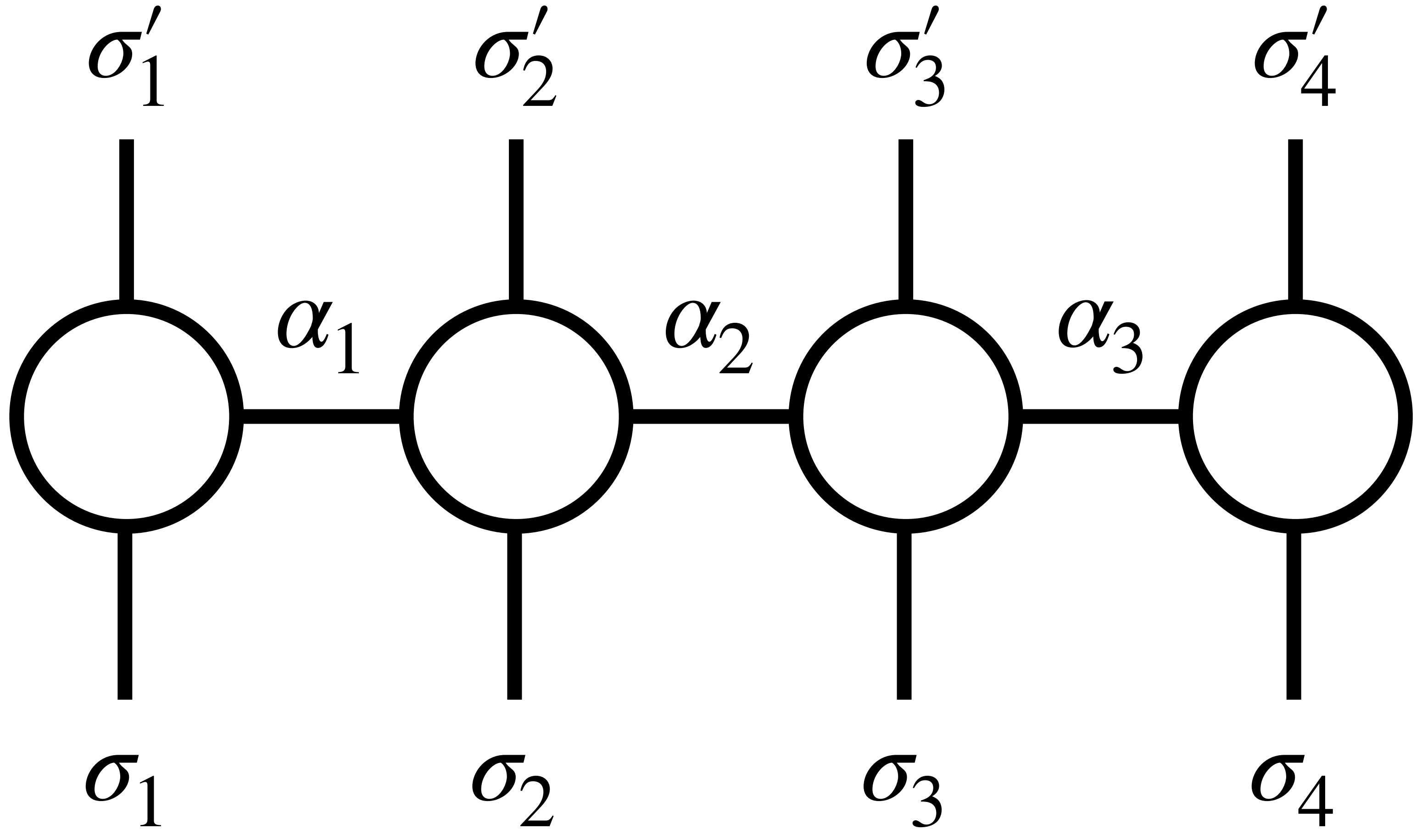

MPO

Methods

MPS representation

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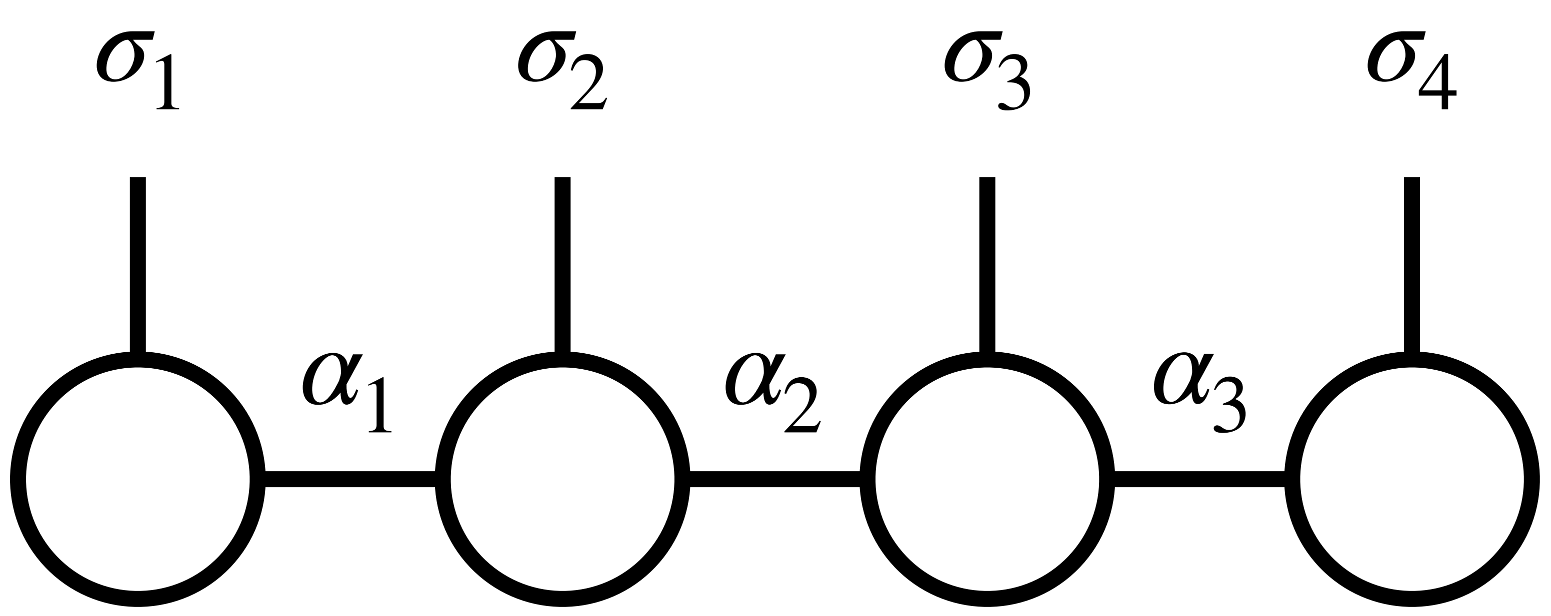
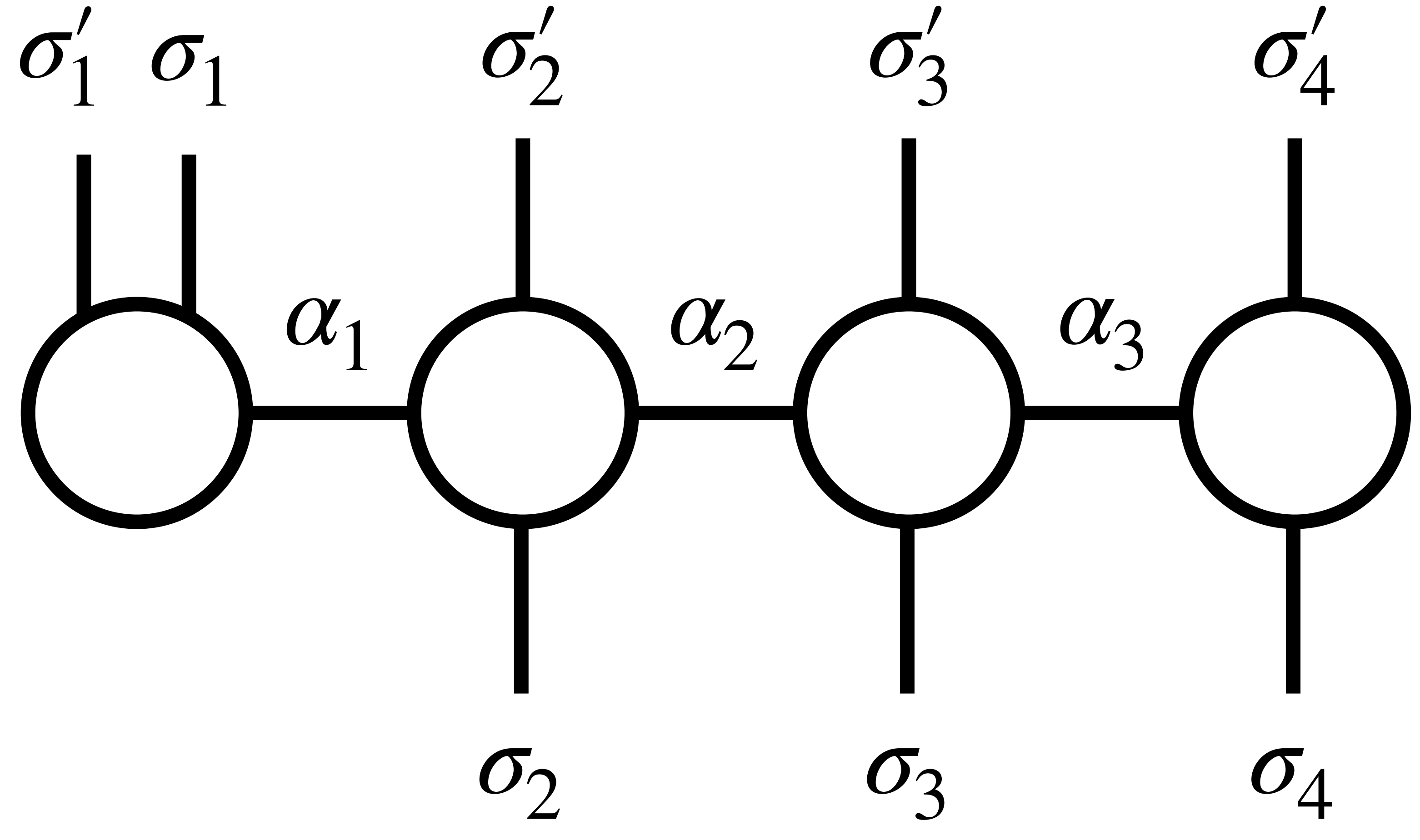
MPS





Methods

MPS representation

$$\psi_{\sigma_1 \dots \sigma_N} \approx A^{\sigma_1}_{\alpha_1} A^{\sigma_2}_{\alpha_1,\alpha_2} \dots A^{\sigma_{N-1}}_{\alpha_{N-2},\alpha_{N-1}} A^{\sigma_N}_{\alpha_{N-1}}$$

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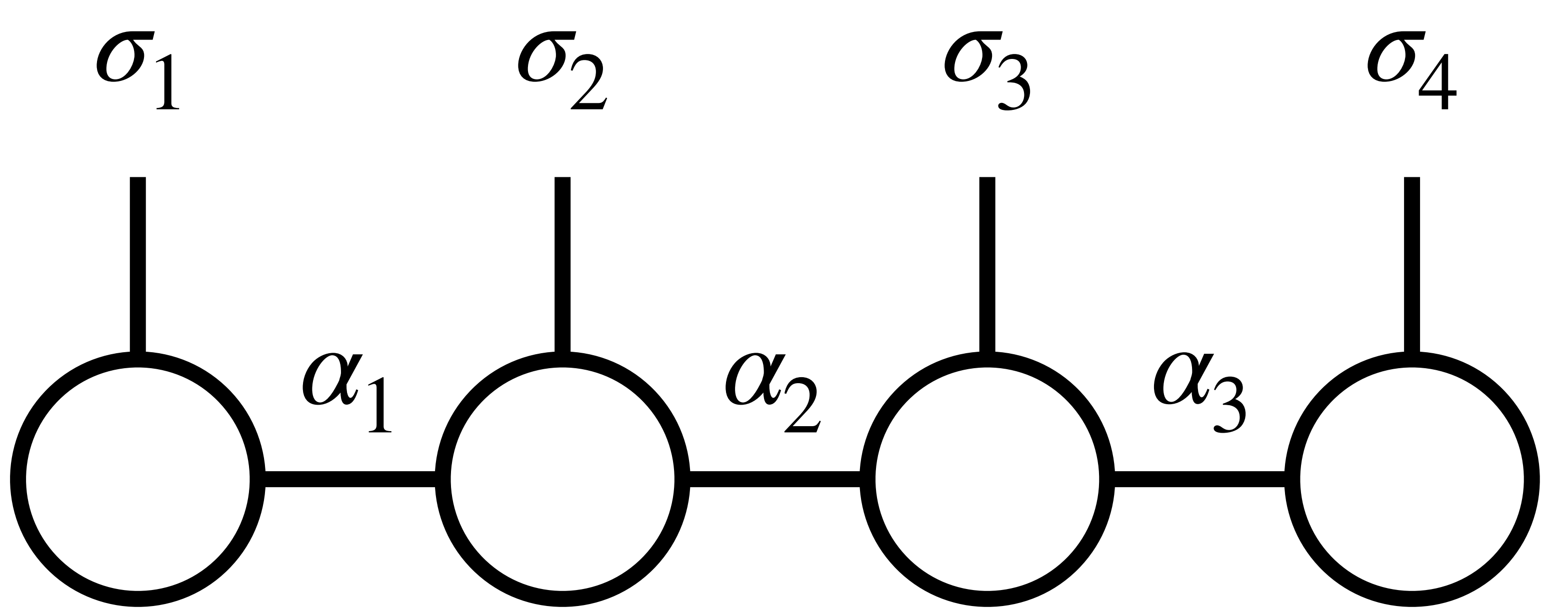

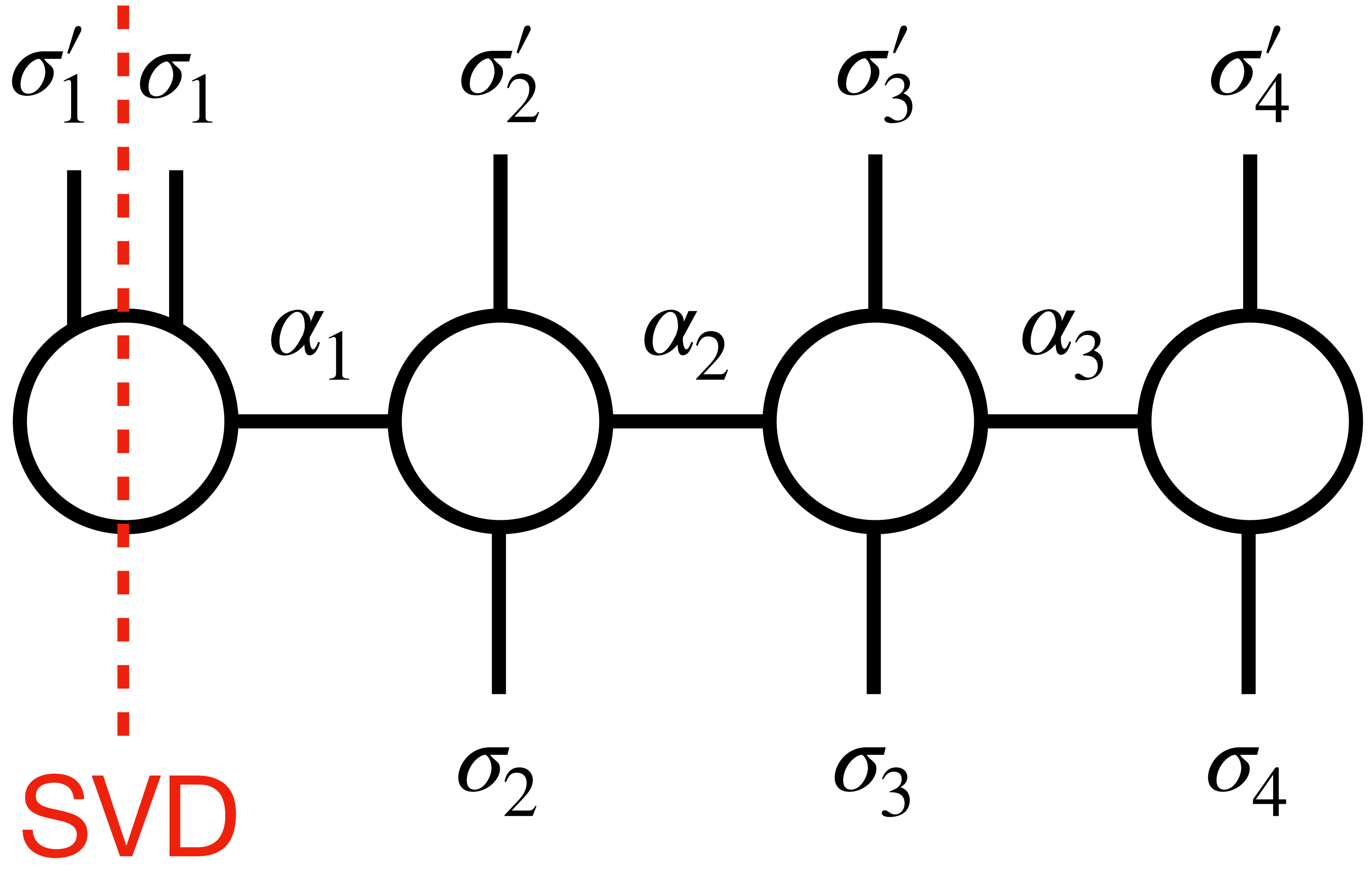

Methods

MPS representation

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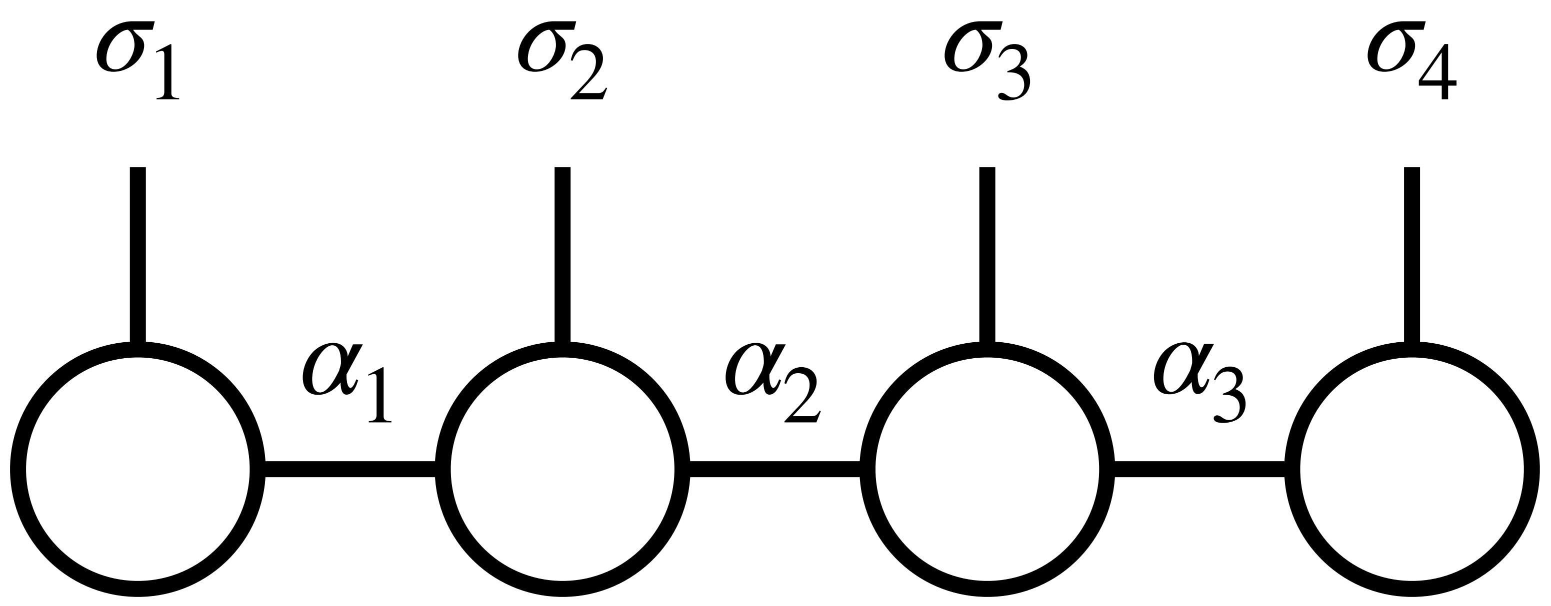
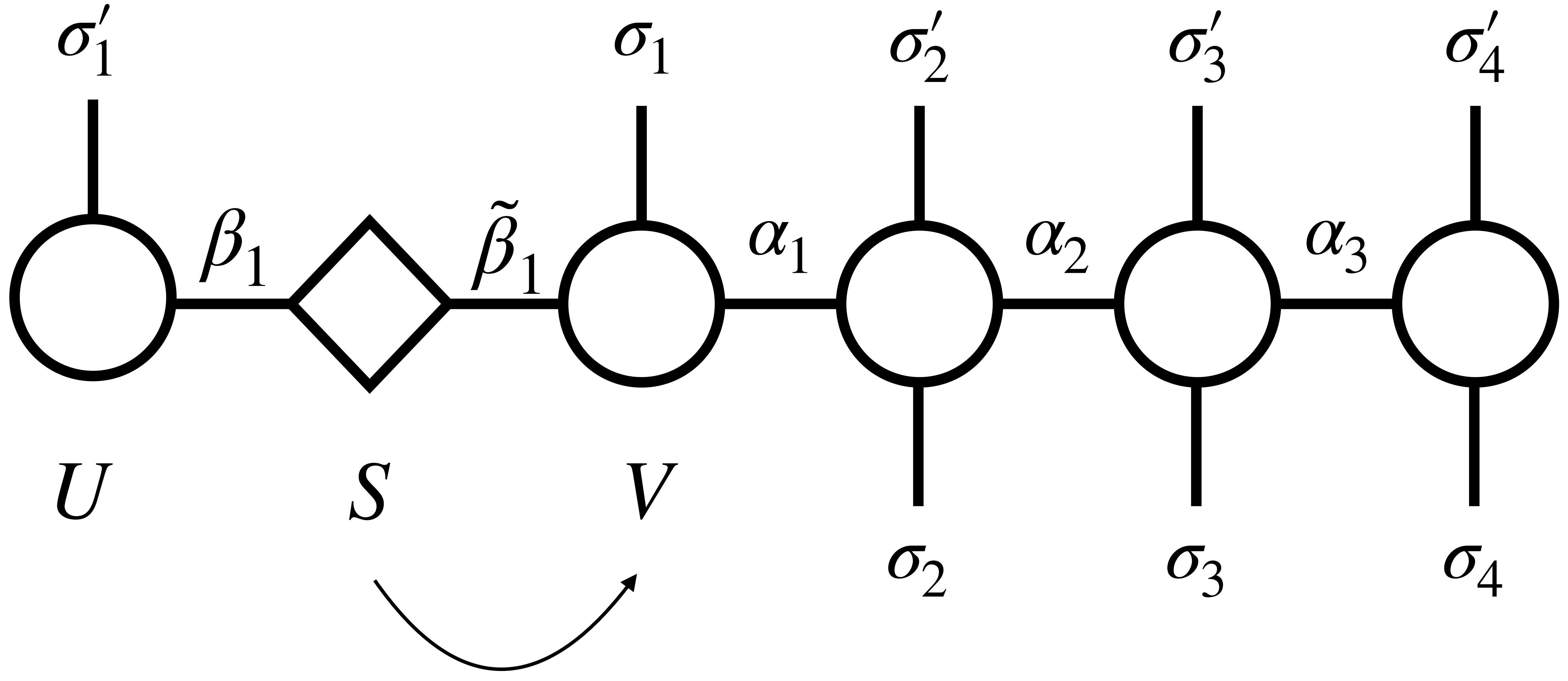


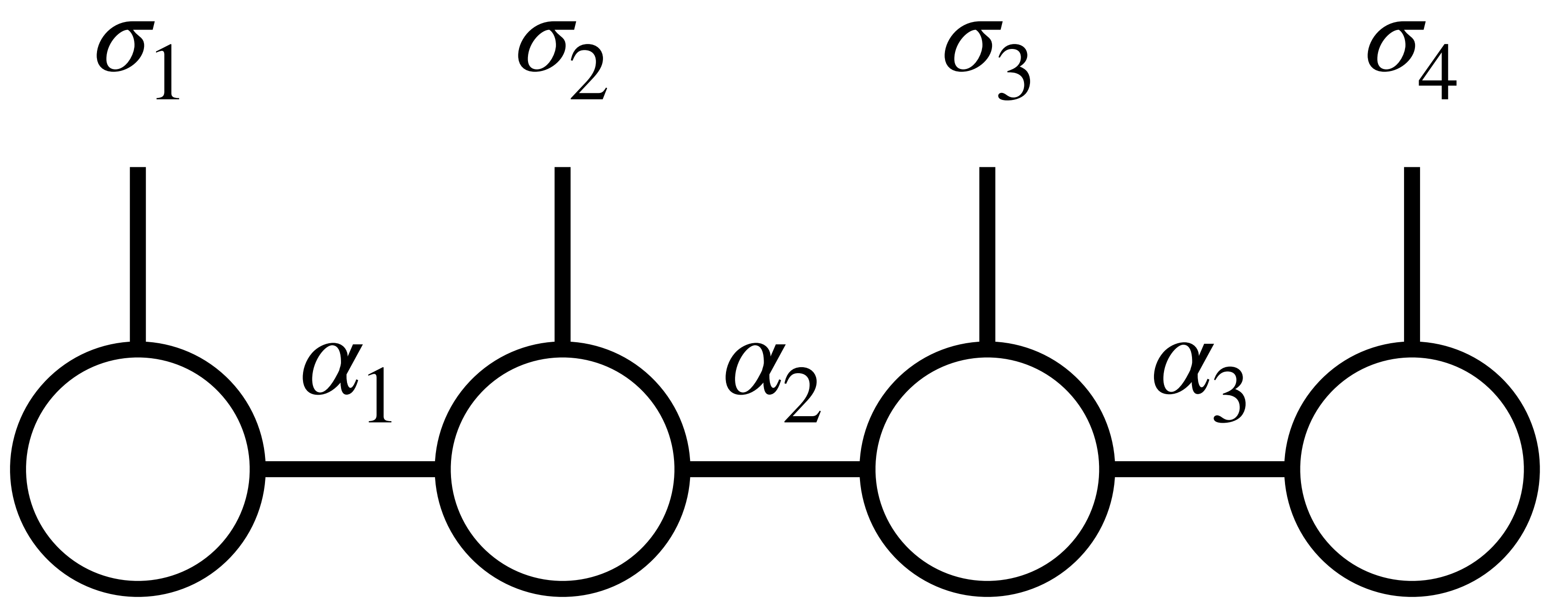


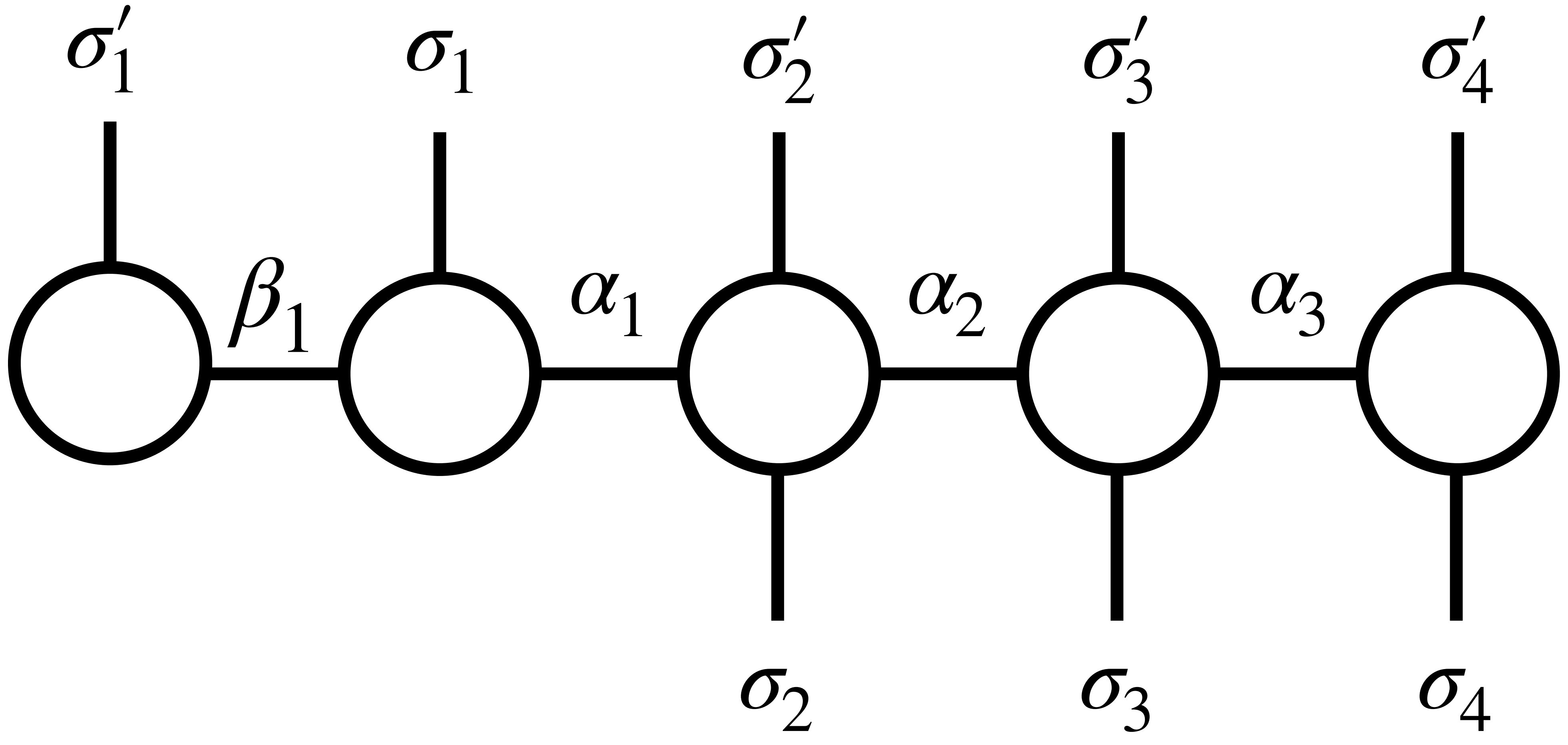
Methods

MPS representation

$$\psi_{\sigma_1 \ldots \sigma_N} \approx A^{\sigma_1}_{\alpha_1} A^{\sigma_2}_{\alpha_1,\alpha_2} \ldots A^{\sigma_{N-1}}_{\alpha_{N-2},\alpha_{N-1}} A^{\sigma_N}_{\alpha_{N-1}}$$

MPS





Methods

MPS representation

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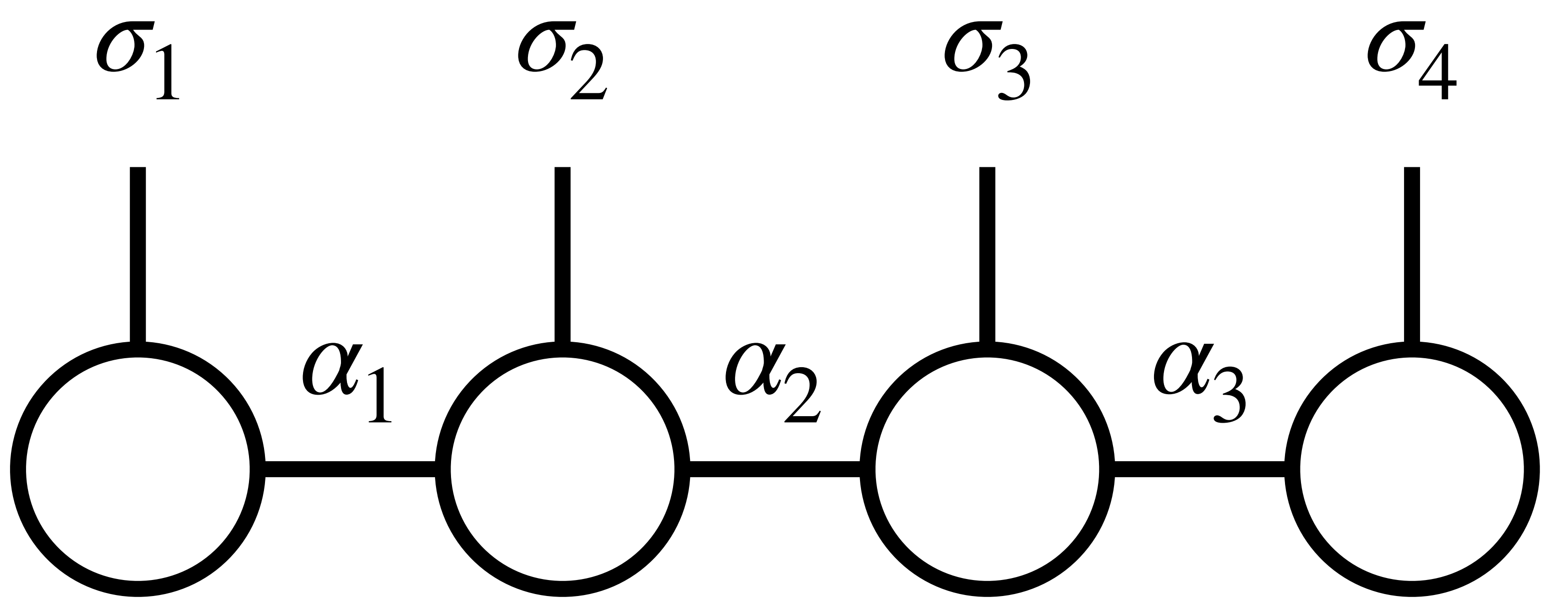
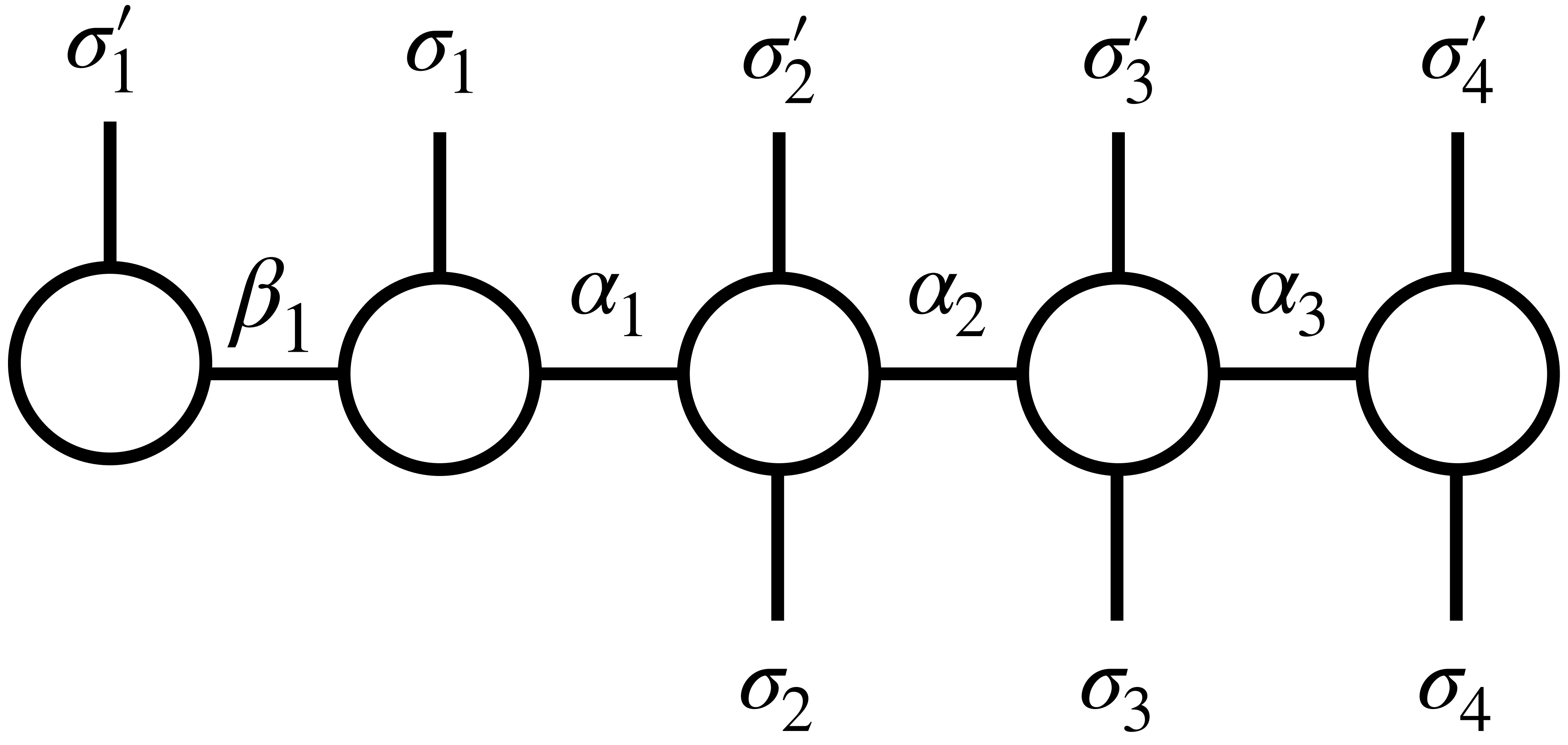
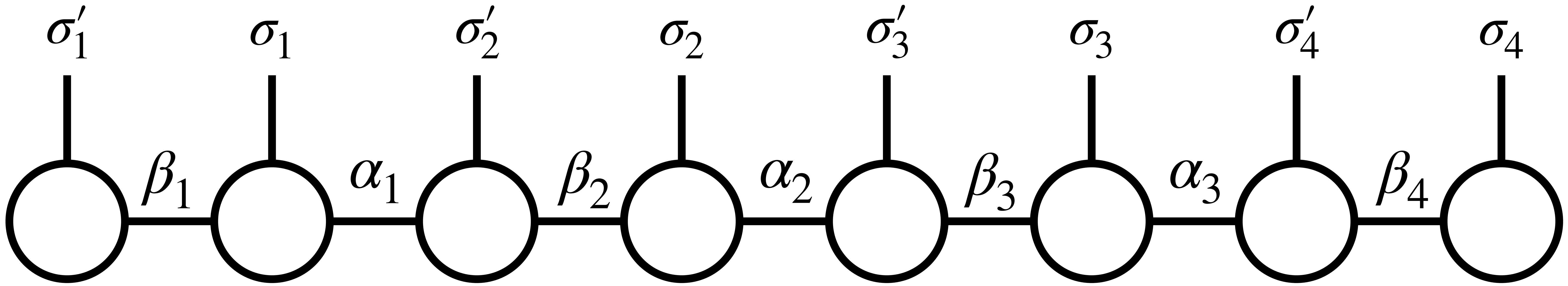
MPS







Purified density matrix MPO represented with an MPS

Methods

Time evolution

$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -iH_S \otimes I + iI \otimes H_S + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2) \otimes L^\dagger(x_1) - \frac{1}{2}\left(L^\dagger(x_1)L(x_2) \otimes I + I \otimes L^\dagger(x_1)L(x_2)\right)\right)$$

Methods

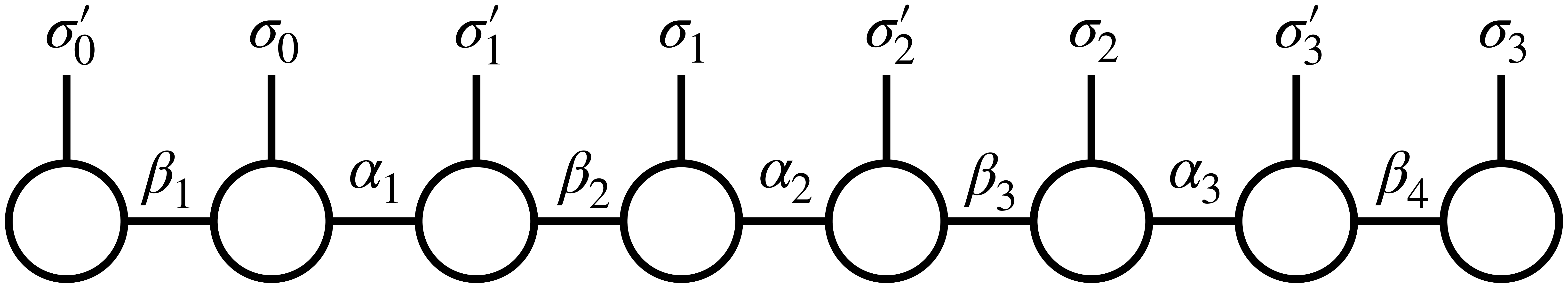
Time evolution

$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -\underline{iH_S} \otimes I + \underline{iI} \otimes H_S + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(\underline{L(x_2)} \otimes L^\dagger(x_1) - \frac{1}{2}\left(\underline{L^\dagger(x_1)L(x_2)} \otimes I + \underline{I} \otimes L^\dagger(x_1)L(x_2)\right)\right)$$

Methods

Time evolution



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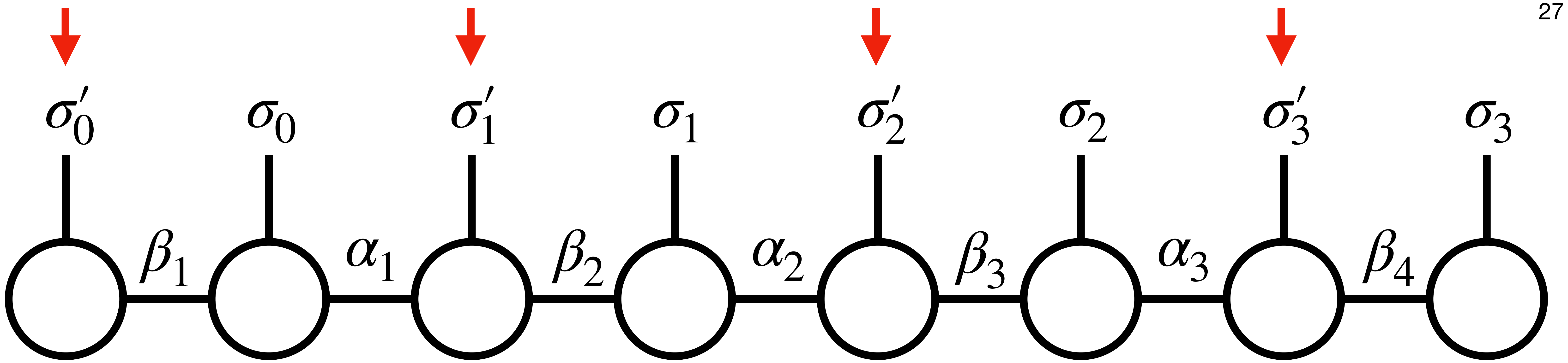
Methods

Time evolution

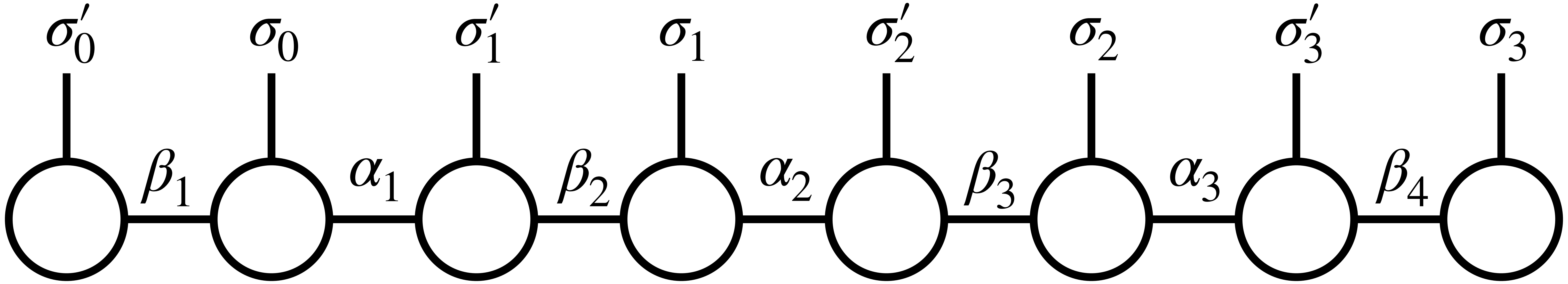


$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -\underline{iH_S} \otimes I + \underline{iI} \otimes H_S + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(\underline{L(x_2)} \otimes L^\dagger(x_1) - \frac{1}{2}\left(\underline{L^\dagger(x_1)L(x_2)} \otimes I + \underline{I} \otimes L^\dagger(x_1)L(x_2)\right)\right)$$

Methods

Time evolution

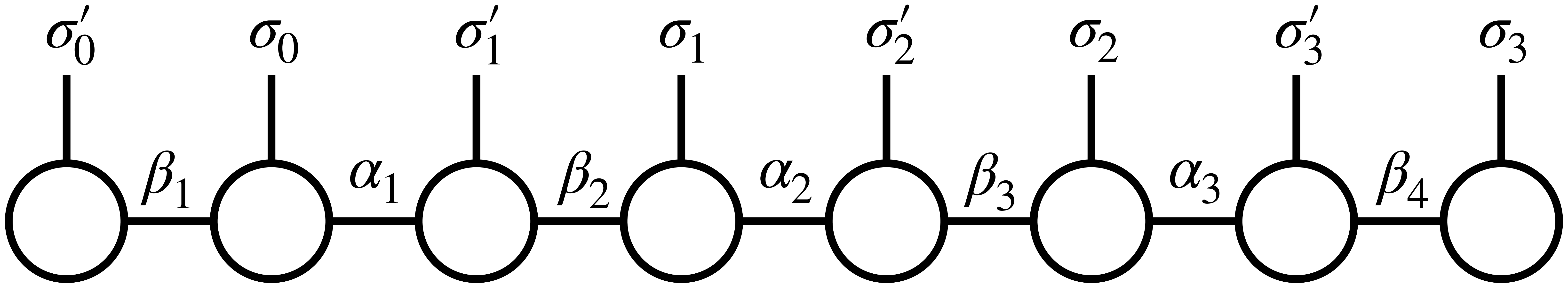


$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -iH_S \otimes I + iI \otimes H_S + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2) \otimes L^\dagger(x_1) - \frac{1}{2}\left(L^\dagger(x_1)L(x_2) \otimes I + I \otimes L^\dagger(x_1)L(x_2)\right)\right)$$

Methods

Time evolution

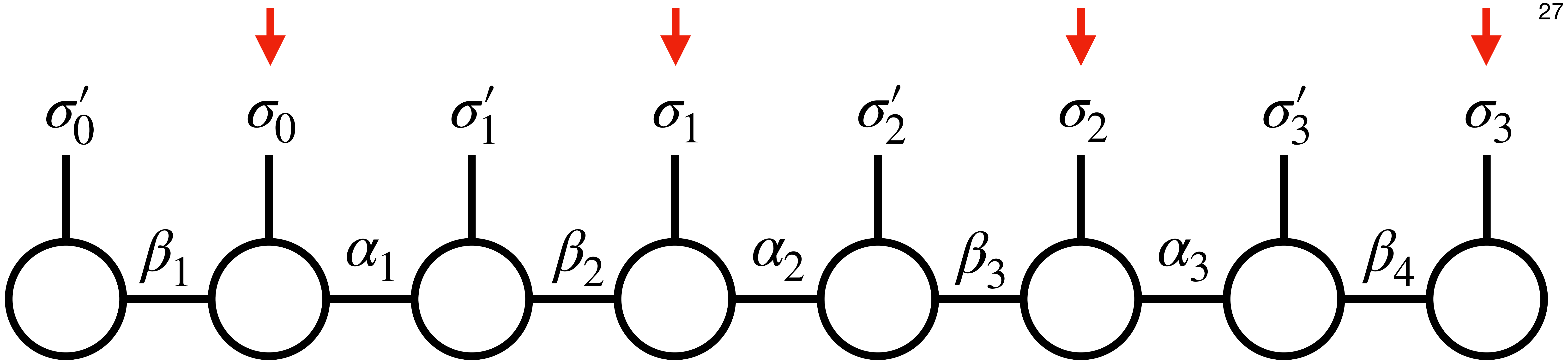


$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -iH_S \otimes \underline{I} + iI \otimes \underline{H_S} + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2) \otimes \underline{L^\dagger(x_1)} - \frac{1}{2}\left(L^\dagger(x_1)L(x_2) \otimes \underline{I} + I \otimes \underline{L^\dagger(x_1)L(x_2)}\right)\right)$$

Methods

Time evolution

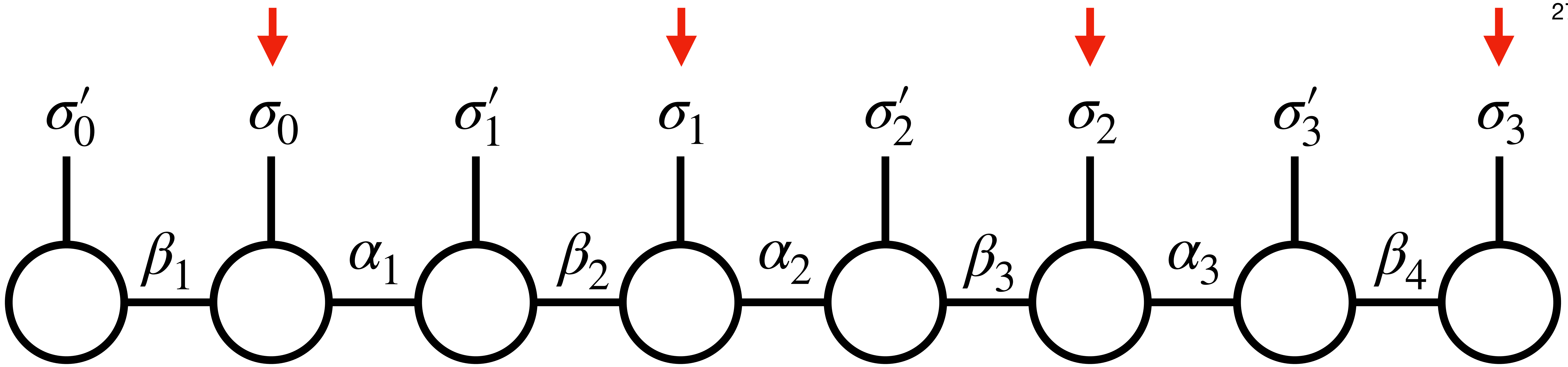


$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -iH_S \otimes \underline{I} + iI \otimes \underline{H_S} + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2) \otimes \underline{L^\dagger(x_1)} - \frac{1}{2}\left(L^\dagger(x_1)L(x_2) \otimes \underline{I} + I \otimes \underline{L^\dagger(x_1)L(x_2)}\right)\right)$$

Methods

Time evolution



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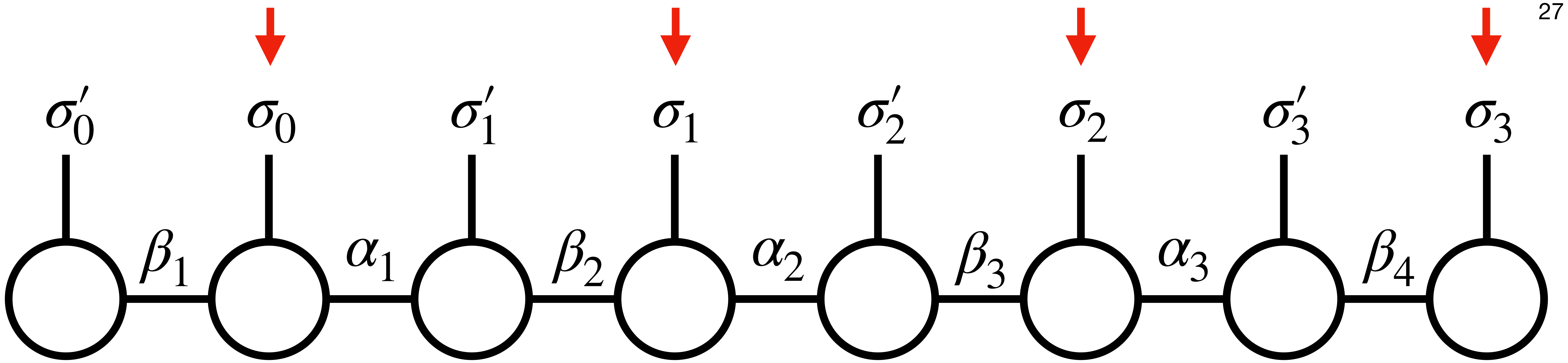
$$\rho_S(t) = e^{\mathscr{L}t}\rho_S(t=0) = e^{\mathscr{L}_{odd}+\mathscr{L}_{even}+\mathscr{L}_{Taylor}}\rho_S(t=0)$$

Methods

Time evolution



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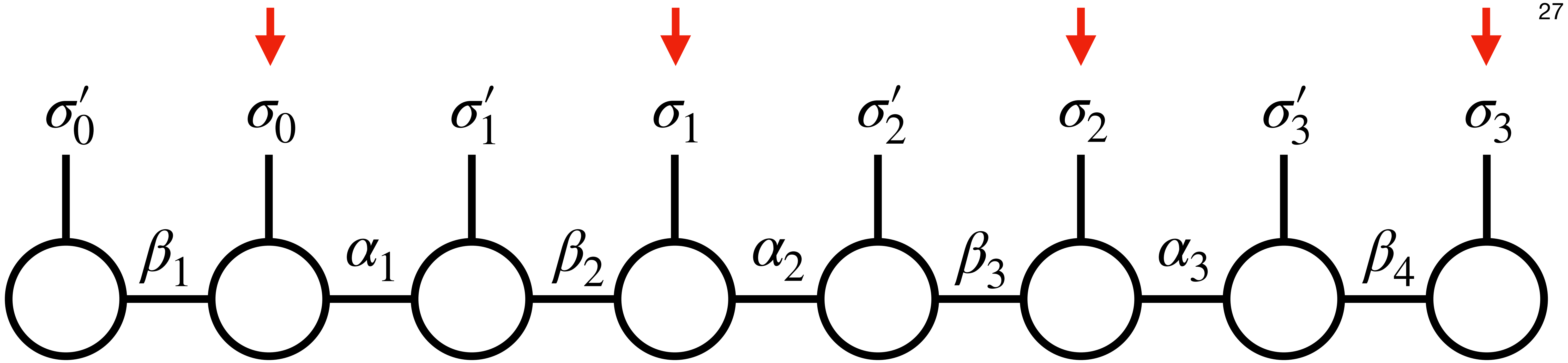
$$e^{\tau\mathscr{L}_{Taylor}} \approx 1 + \sum_{j=1}^{\kappa} \frac{\left(\tau\mathscr{L}_{Taylor}\right)^j}{j!}, \qquad \kappa = 2$$

Methods

Time evolution



$$\dot{\rho}_S(t) = \mathscr{L}\rho_S(t)$$

$$\mathscr{L} = -iH_S \otimes I + iI \otimes H_S + a^2 \sum_{x_1,x_2} D(x_1 - x_2)\left(L(x_2) \otimes L^\dagger(x_1) - \frac{1}{2}\left(L^\dagger(x_1)L(x_2) \otimes I + I \otimes L^\dagger(x_1)L(x_2)\right)\right)$$

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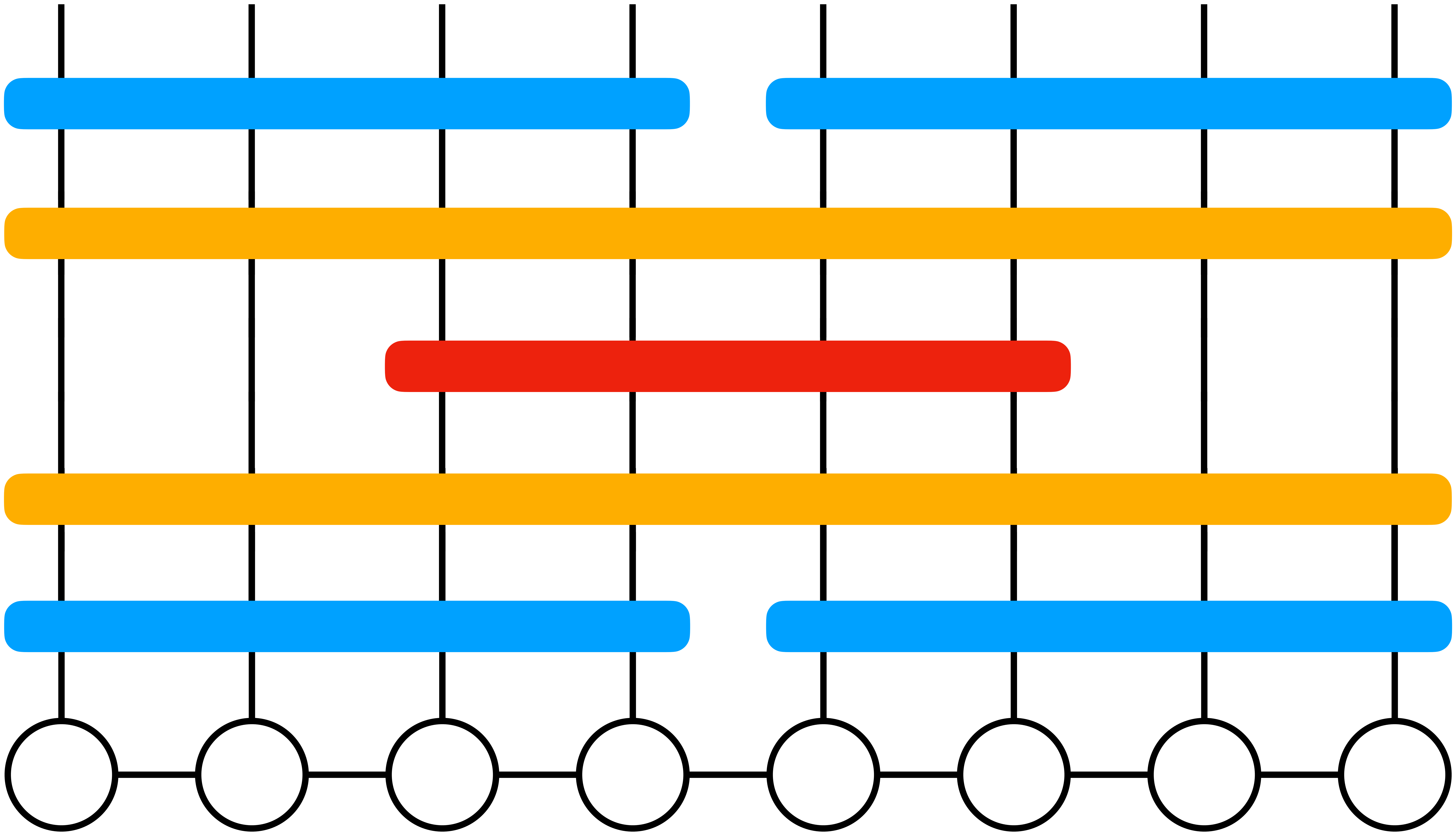
$$e^{\tau\mathscr{L}_{Taylor}} \approx 1 + \sum_{j=1}^{\kappa} \frac{\left(\tau\mathscr{L}_{Taylor}\right)^j}{j!}, \qquad \kappa = 2$$

$$D(x_1 - x_2) = D\delta_{x_1,x_2}$$

Integration scheme

$$e^{\mathscr{L}\tau} \approx e^{\mathscr{L}_{even}\frac{\tau}{2}} e^{\mathscr{L}_{Taylor}\frac{\tau}{2}} e^{\mathscr{L}_{odd}\tau} e^{\mathscr{L}_{Taylor}\frac{\tau}{2}} e^{\mathscr{L}_{even}\frac{\tau}{2}}$$



$e^{\mathscr{L}_{even}\frac{\tau}{2}}$

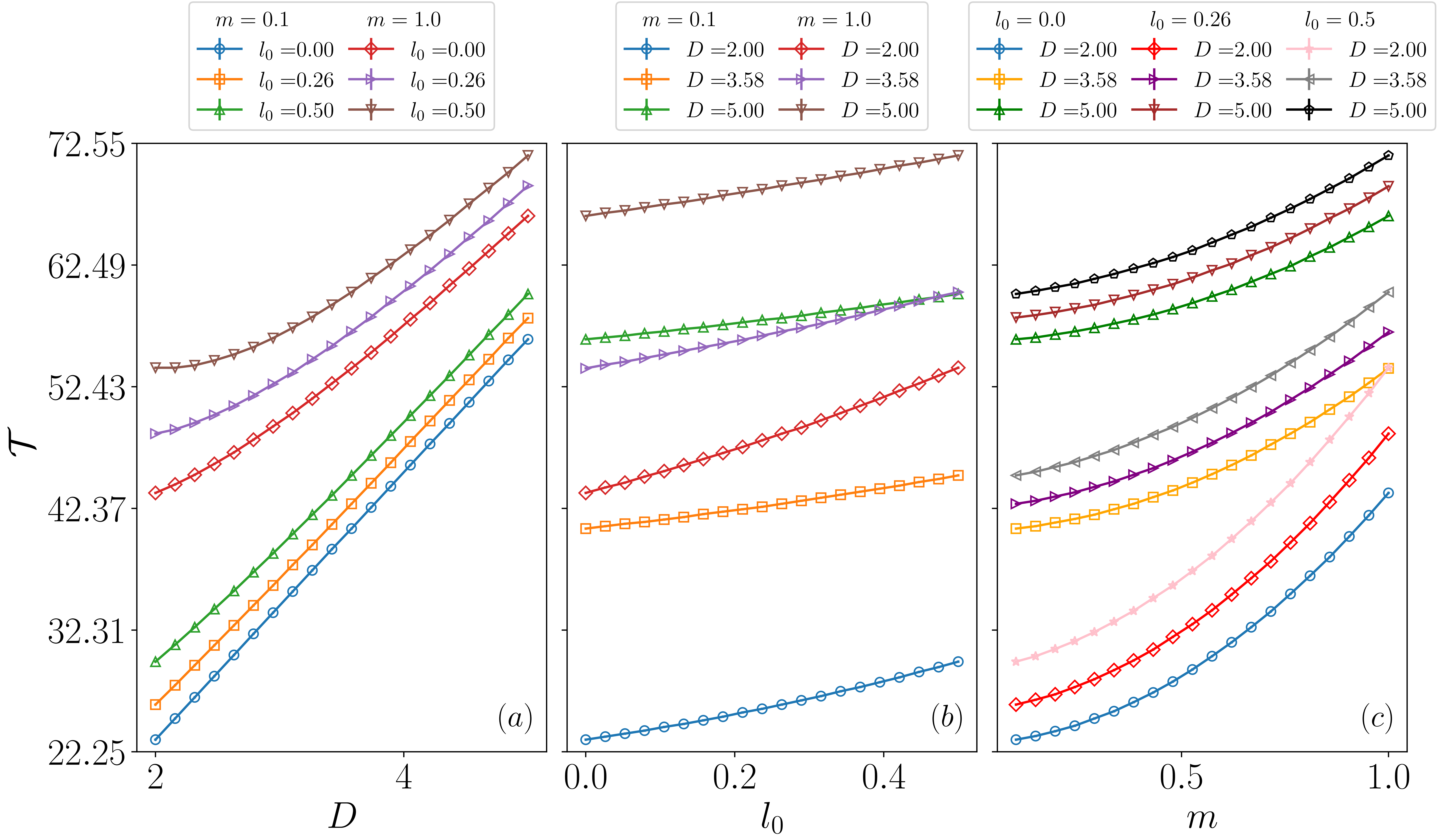
$e^{\mathscr{L}_{Taylor}\frac{\tau}{2}}$

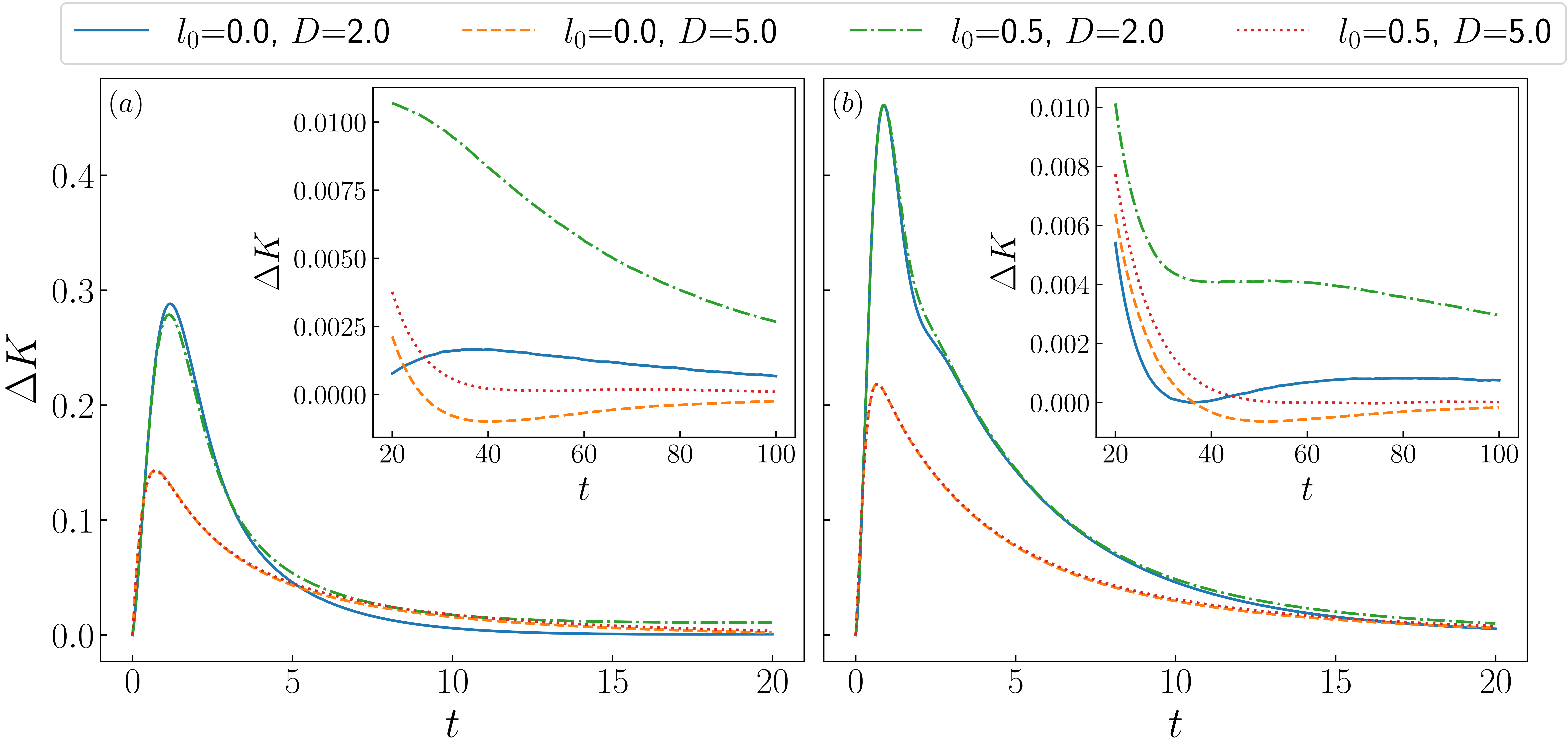
$e^{\mathscr{L}_{odd}\tau}$

$e^{\mathscr{L}_{Taylor}\frac{\tau}{2}}$

$e^{\mathscr{L}_{even}\frac{\tau}{2}}$

Thermalization time vs parameters

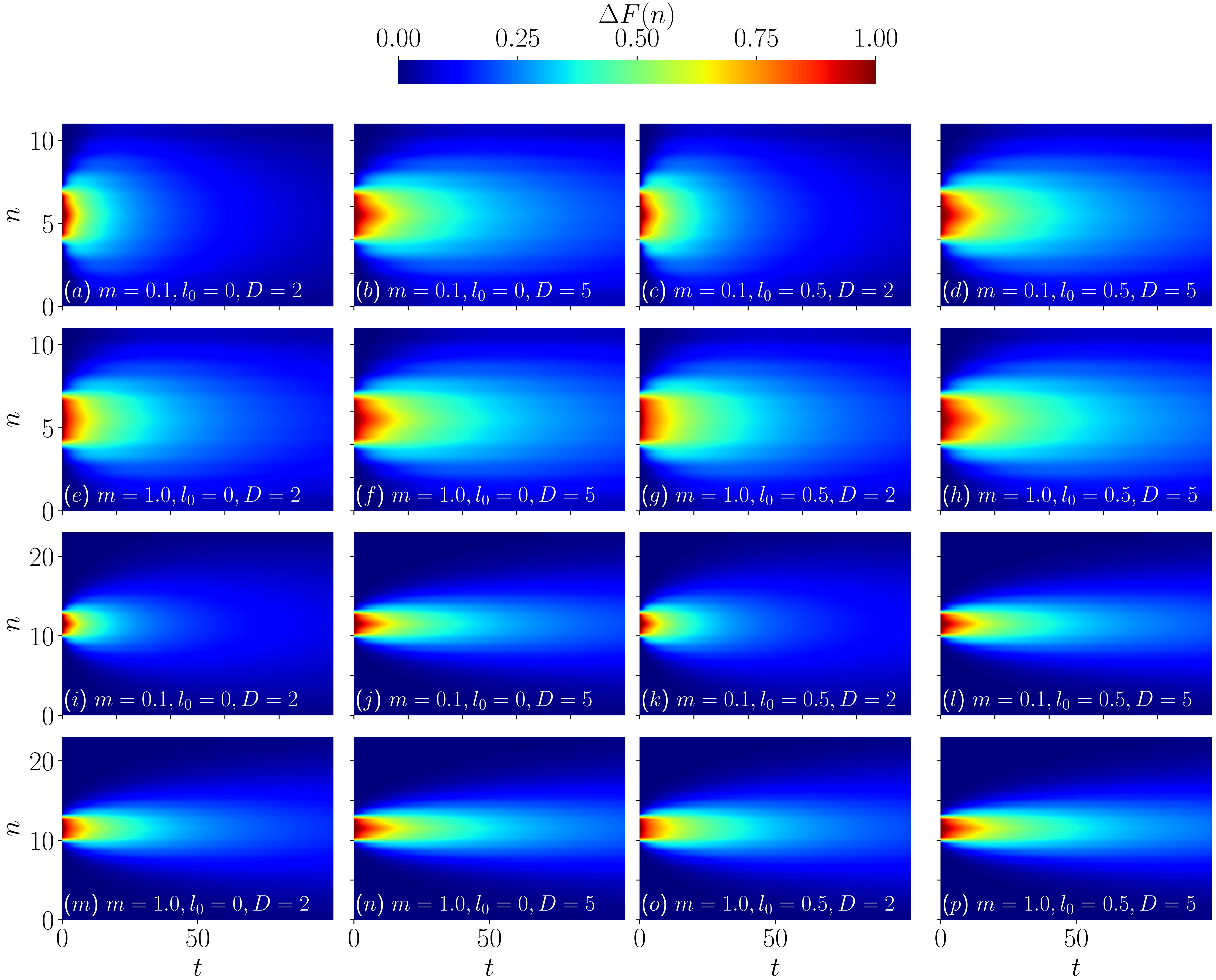


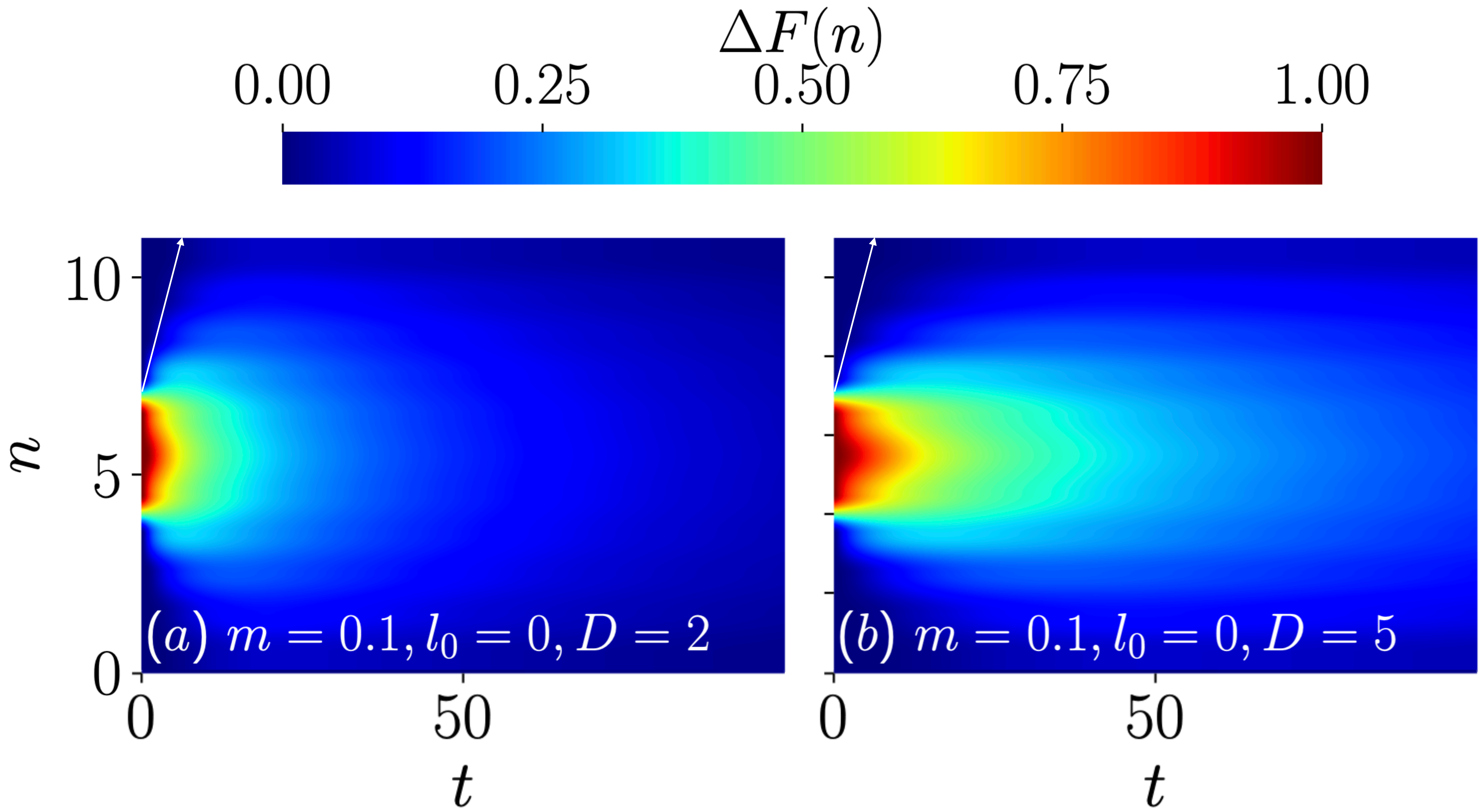


l_0=0.0, D=2.0
l_0=0.0, D=5.0
l_0=0.5, D=2.0
l_0=0.5, D=5.0
(a)
(b)
ΔK
t
0.0
0.1
0.2
0.3
0.4
0
5
10
15
20
0.0100
0.0075
0.0050
0.0025
0.0000
0.010
0.008
0.006
0.004
0.002
0.000
40
60
80
100

$m = 0.1$ $m = 1$

Subtracted electric field vs D, m, l_0



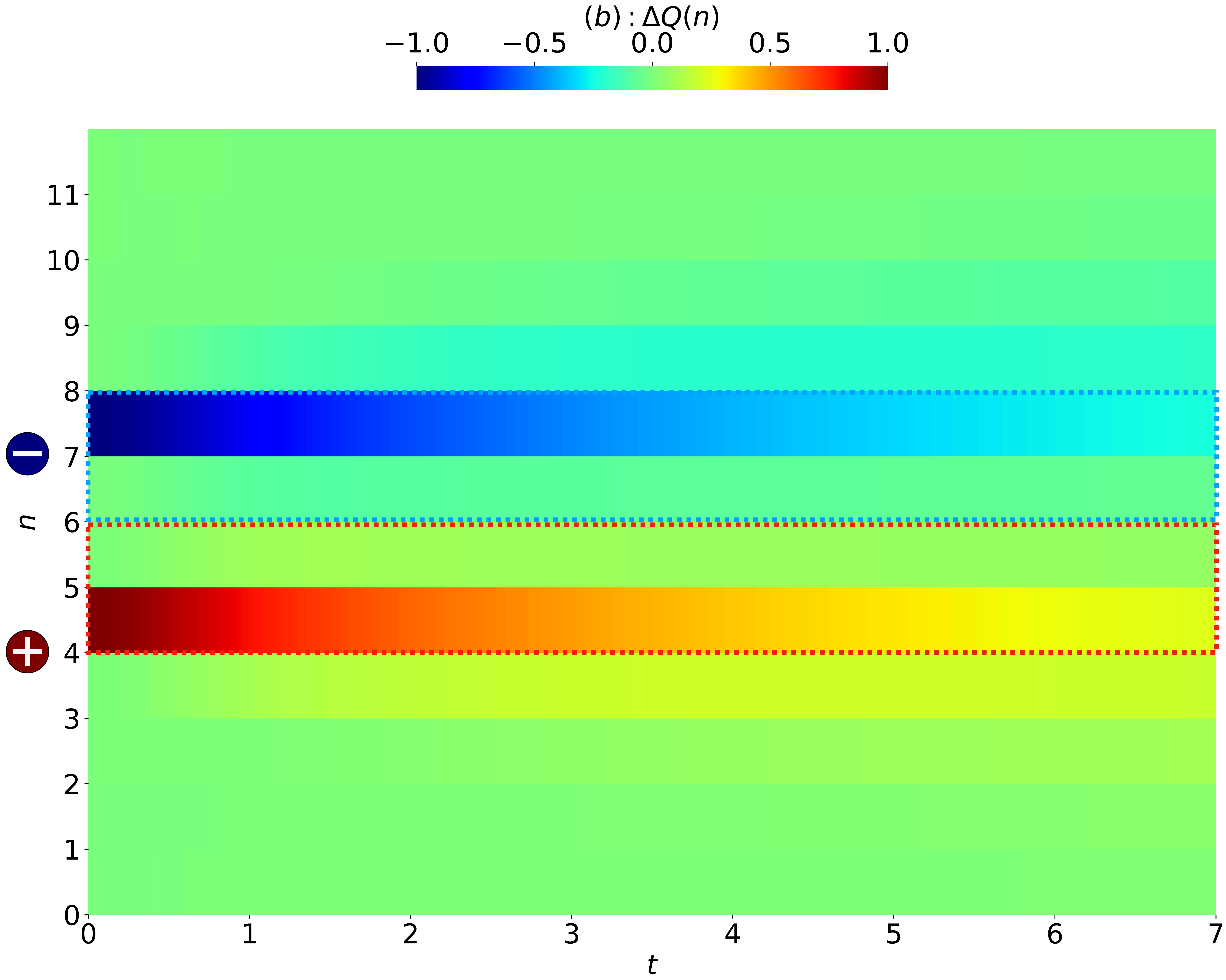


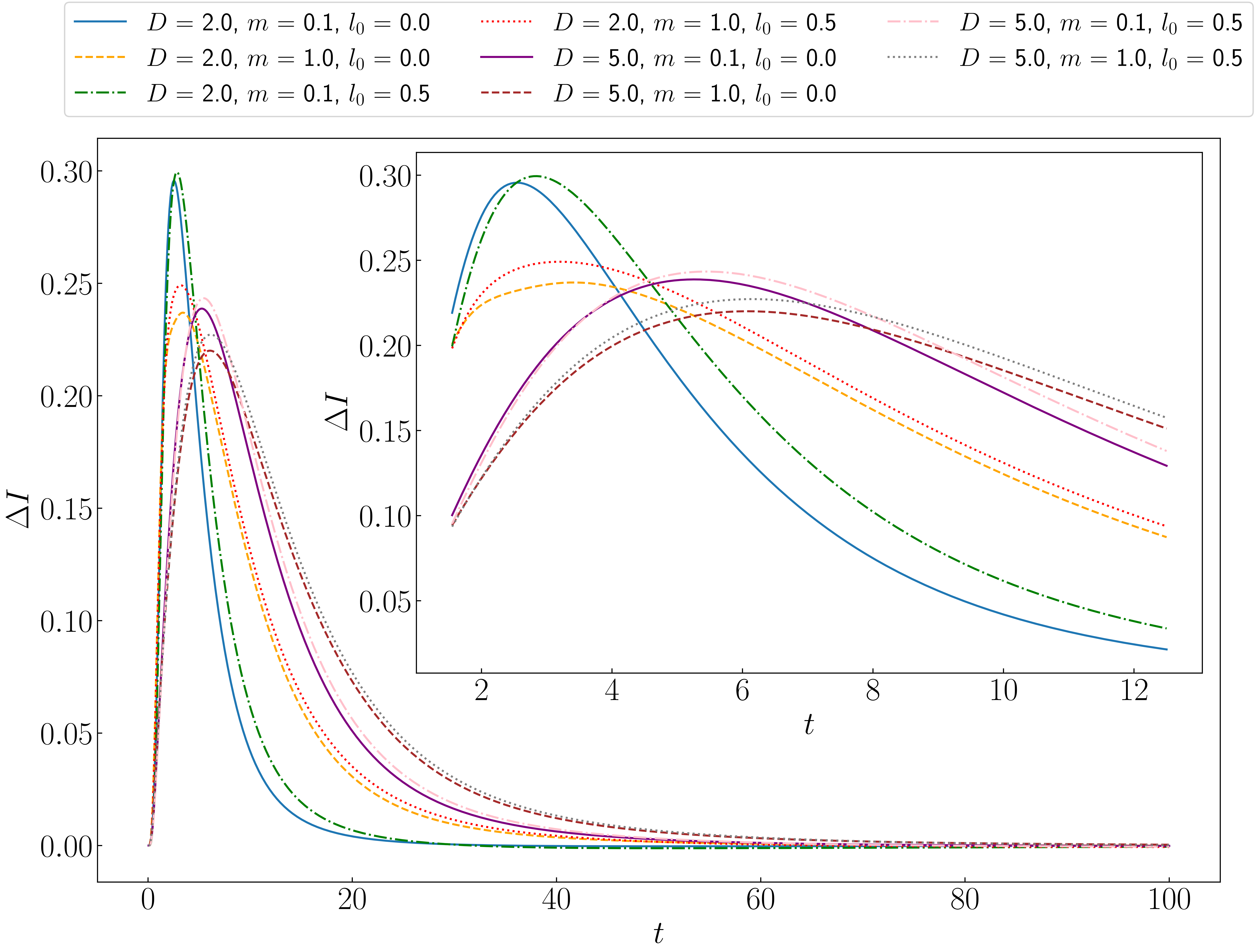
$\Delta F(n)$
0.00
0.25
0.50
0.75
1.00
n
10
5
0
t
0
50
$(a)\ m = 0.1, l_0 = 0, D = 2$
$(b)\ m = 0.1, l_0 = 0, D = 5$

Results

Mutual information

- von Neumann entropy: $S(\rho_S) = -\,\mathrm{Tr}(\rho_S ln(\rho_S))$
- Mutual information: $I(A, B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$


$(b) : \Delta Q(n)$
−1.0
−0.5
0.0
0.5
1.0
11
10
9
8
7
6
5
4
3
2
1
0
n
0
1
2
3
4
5
6
7
t

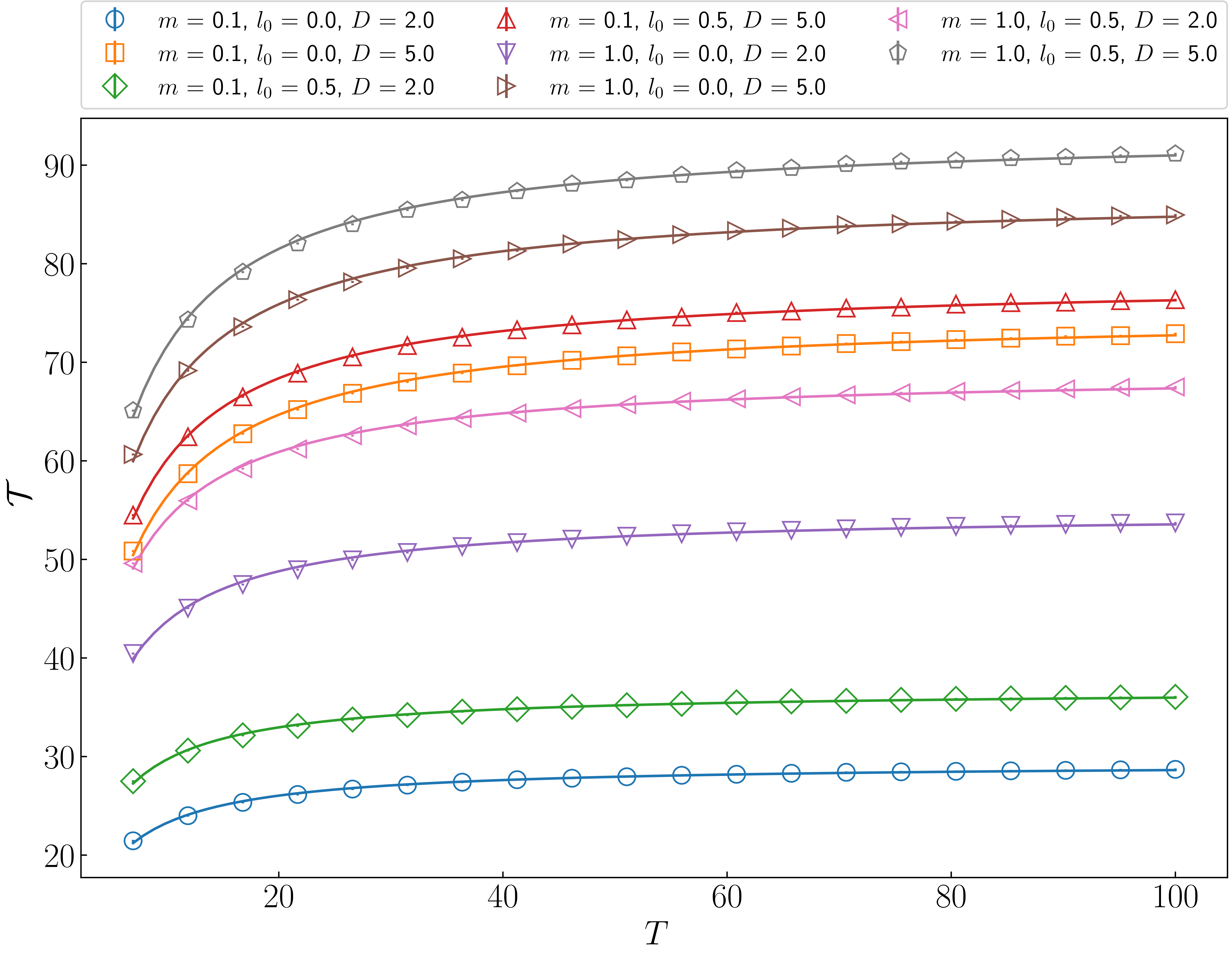

$D = 2.0$, $m = 0.1$, $l_0 = 0.0$
$D = 2.0$, $m = 1.0$, $l_0 = 0.0$
$D = 2.0$, $m = 0.1$, $l_0 = 0.5$
$D = 2.0$, $m = 1.0$, $l_0 = 0.5$
$D = 5.0$, $m = 0.1$, $l_0 = 0.0$
$D = 5.0$, $m = 1.0$, $l_0 = 0.0$
$D = 5.0$, $m = 0.1$, $l_0 = 0.5$
$D = 5.0$, $m = 1.0$, $l_0 = 0.5$
ΔI
t
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
20
40
60
80
100
2
4
6
8
10
12

Results

Thermalization time vs temperature

- Thermalization time $\sim 1/J^\dagger J$
- Jump operator: $J(n) = O(n) - \frac{1}{4T}\left[H_S, O(n)\right] \qquad O(n) = (-1)^n \frac{Z_n + 1}{2a}$
- TT inversely proportional to relaxation rate so we fit: $f(T) = \frac{a}{(b + c/T)^2}$

PRD 108.094518



$m = 0.1,\ l_0 = 0.0,\ D = 2.0$
$m = 0.1,\ l_0 = 0.0,\ D = 5.0$
$m = 0.1,\ l_0 = 0.5,\ D = 2.0$
$m = 0.1,\ l_0 = 0.5,\ D = 5.0$
$m = 1.0,\ l_0 = 0.0,\ D = 2.0$
$m = 1.0,\ l_0 = 0.0,\ D = 5.0$
$m = 1.0,\ l_0 = 0.5,\ D = 2.0$
$m = 1.0,\ l_0 = 0.5,\ D = 5.0$
$\mathcal{T}$
20
30
40
50
60
70
80
90
20
40
60
80
100
T

Thermalization time vs temperature in QCD

- Relaxation rate = 1/thermalization time = $\Gamma \sim \frac{a^2}{2N} \sum_{k=0}^{N-1} D(k) J_k^\dagger J_k$
- Choosing delta function environment correlator $\implies D(k) =$ constant
- In QCD: $D(k) \sim T^3$

PRL 100.052301
PRD 108.094518

Schwinger boson thermalization dynamics