Nucleon electromagnetic form factors using N_f=2+1+1 twisted-mass fermions at physical point

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Lattice gauge theories: Introduction

- Continuum field theories: Infinite DOF.

- Perturbative regularization: Fails at low energies e.g. QCD.

- Discretize the space-time: Replace continuum with a grid of lattice sites with 1/a as UV cutoff.

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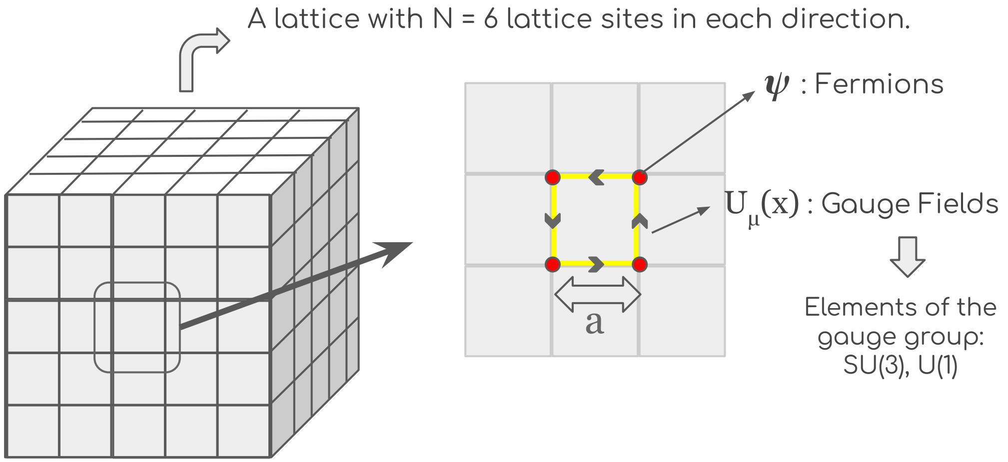

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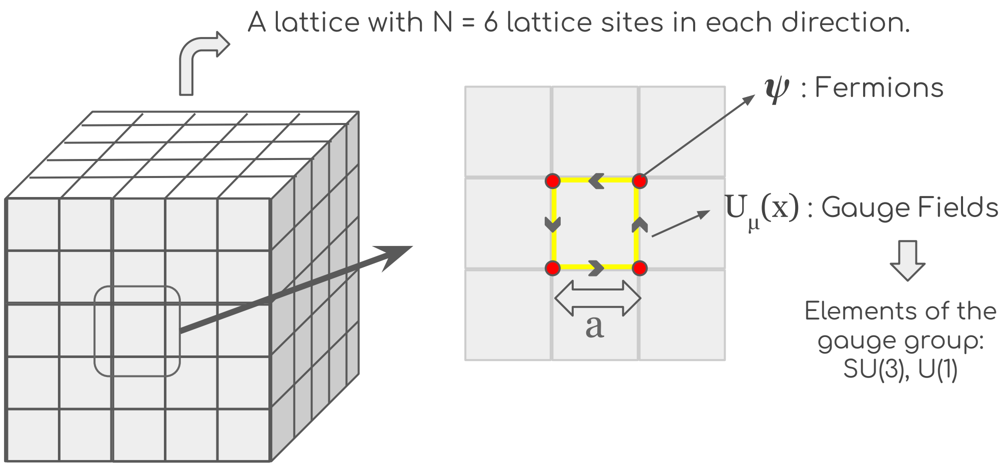

In this formulation the expectation value of an observable (without fermions) is given by:

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int D[U] O(U) e^{-S(U)} \qquad \mathcal{Z} = \int D[U] e^{-S(U)}$$

Standard approach: Hybrid Monte Carlo

- Generate conjugate momentum field.

$$\langle O \rangle = \frac{1}{\mathcal{Z}'} \int D[U] D[P] O(U) e^{-(P^2/2 + S(U))}$$

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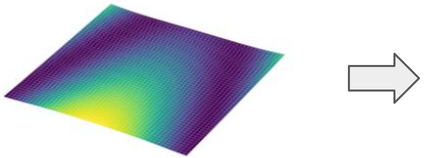
Initial field configuration



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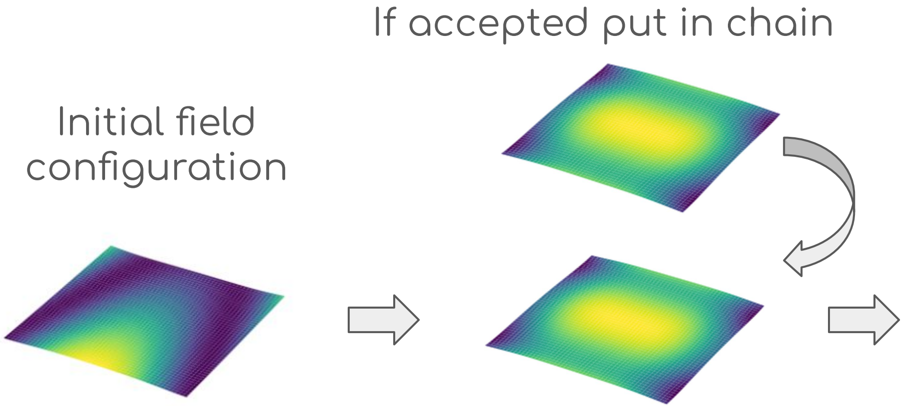

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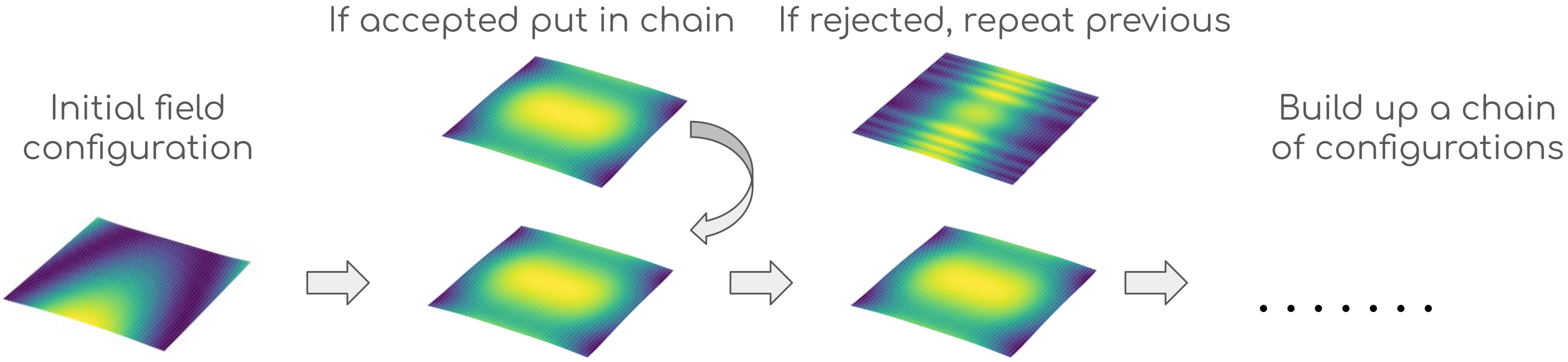
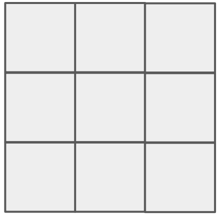


Continuum limit

- Continuum limit needed to extract physical values.



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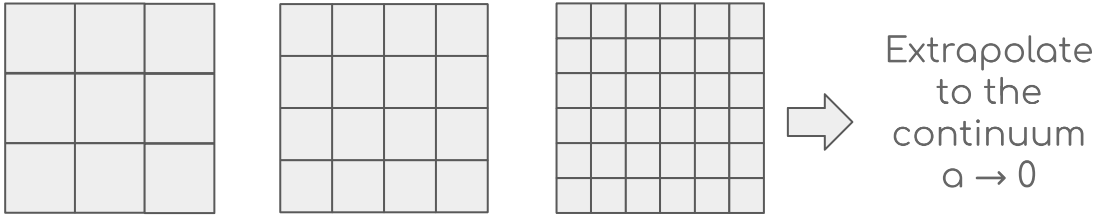

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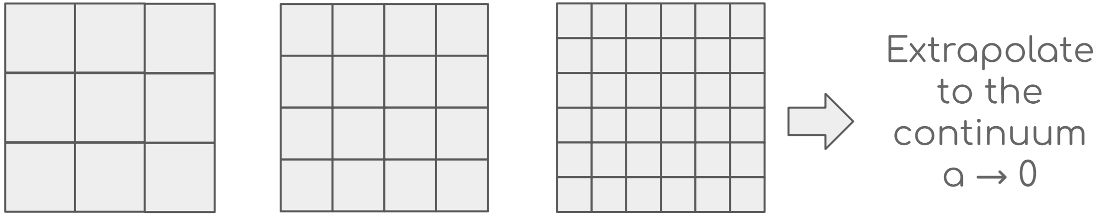

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Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64	$64^3 \times 128$	0.07957(13)	140.2(2)	3.62
cC211.060.80	$80^3 \times 160$	0.06821(13)	136.7(2)	3.78
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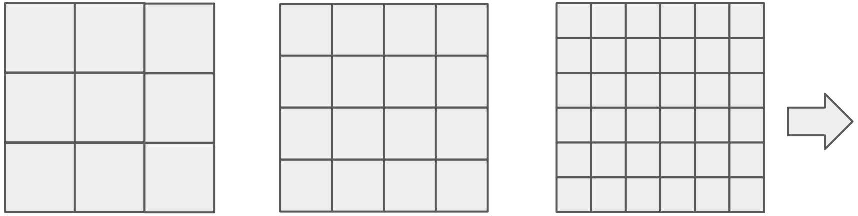

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- We use clover improved, twisted-mass fermions (O(a) improved).

Correlators in lattice QCD

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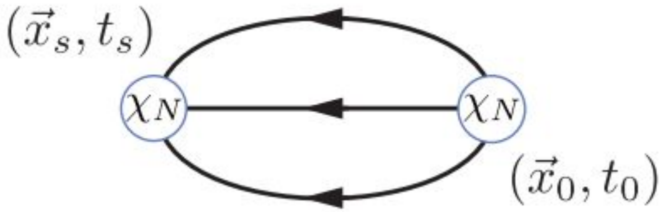

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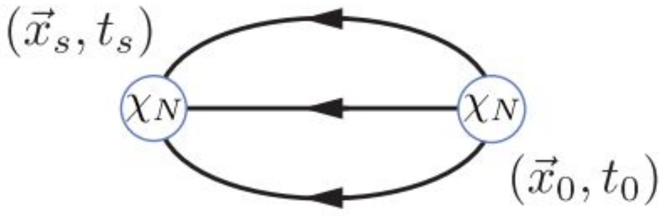

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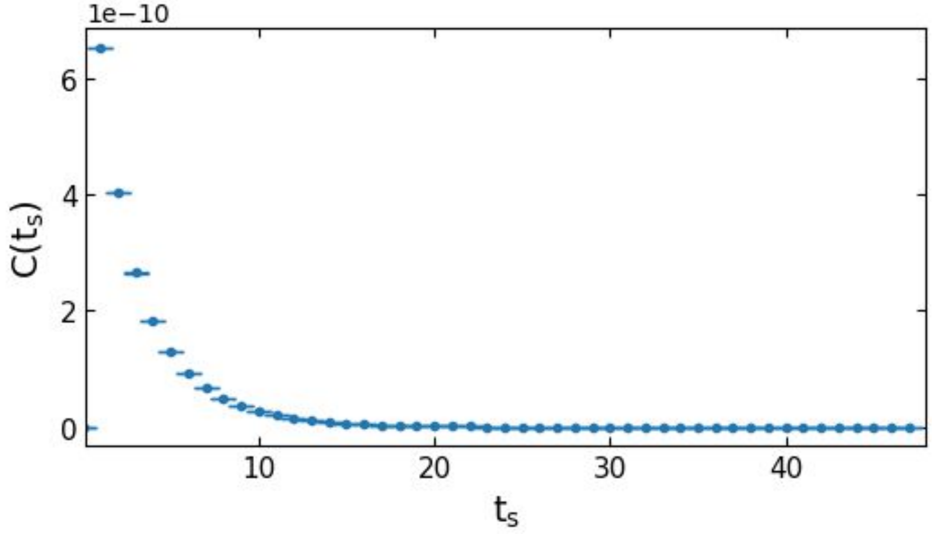

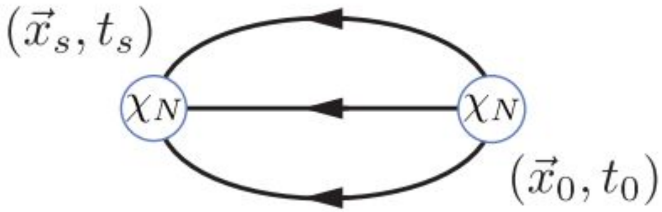

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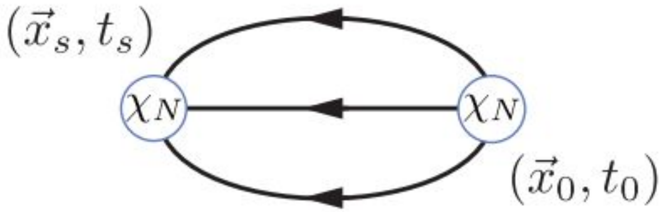

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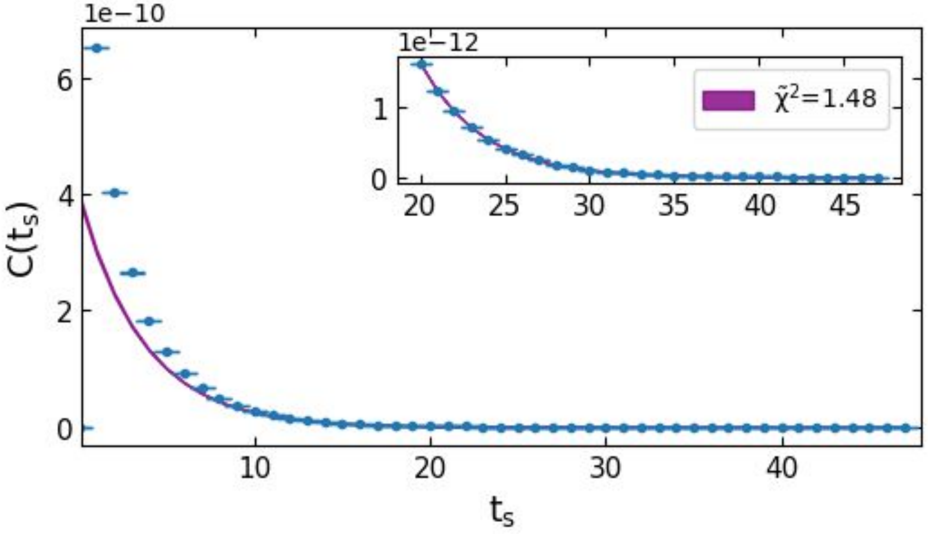

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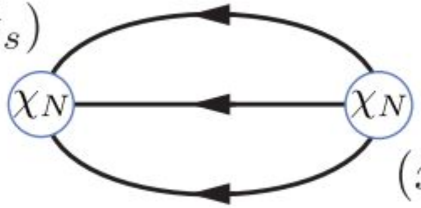

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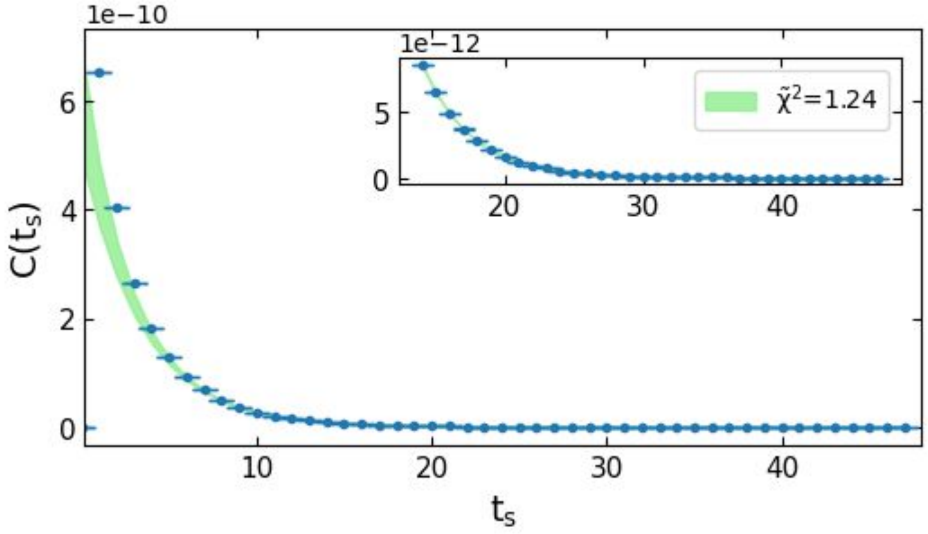

$$C(\Gamma_0, \vec{0}, t) = c_0(\vec{0}) e^{-E_0(\vec{0}) t_s} + c_1(\vec{0}) e^{-E_1(\vec{0}) t_s} + \ldots$$

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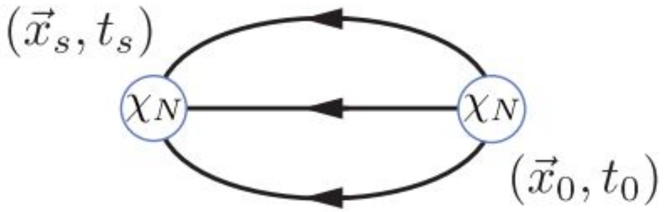

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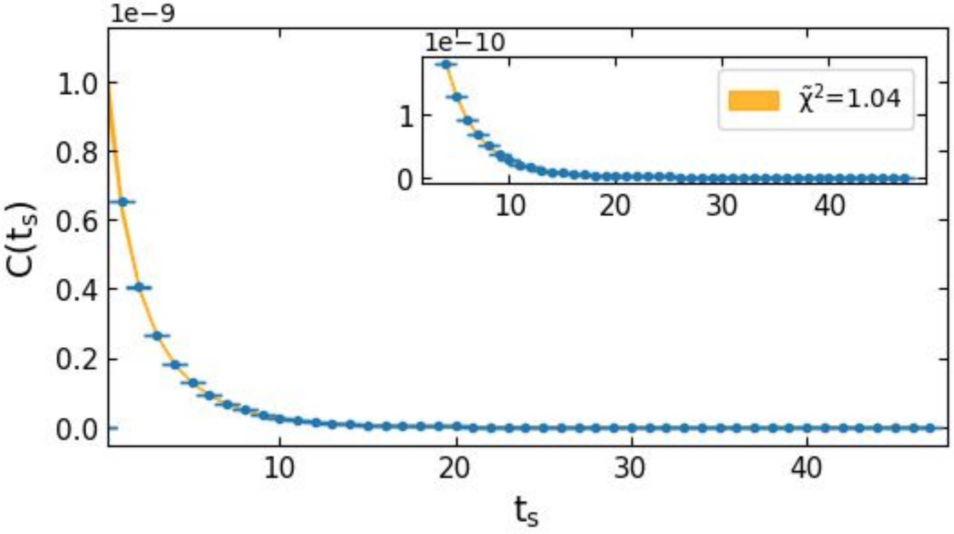

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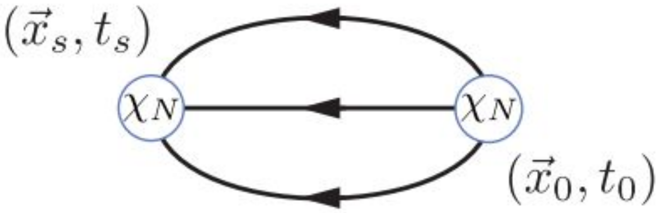

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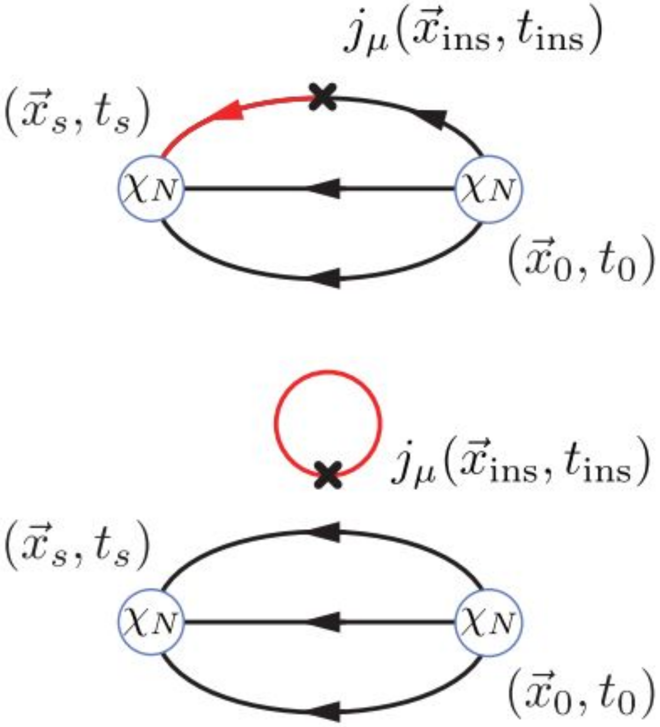
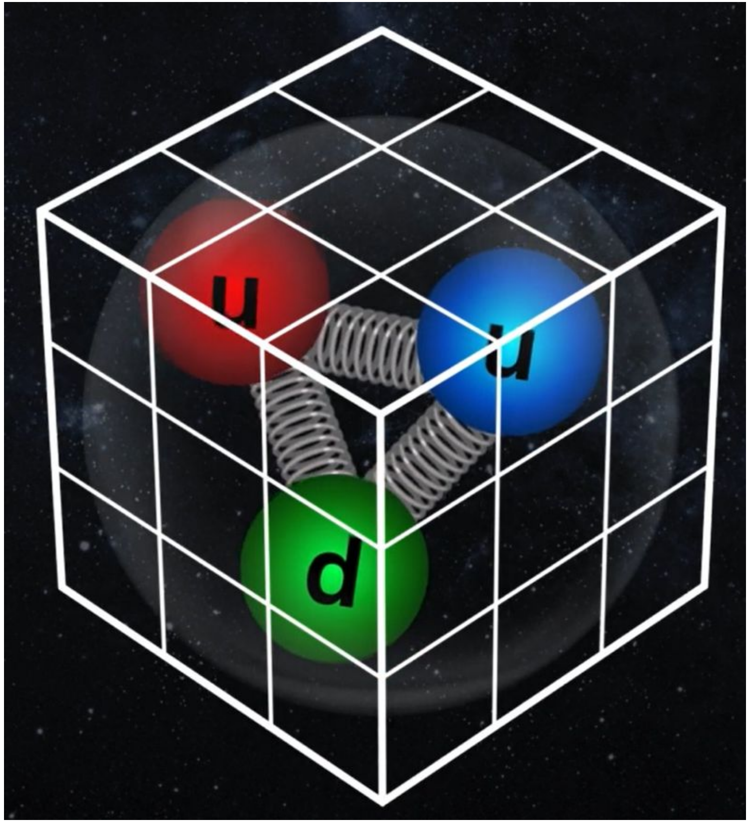
- The three point function is given by:

$$\mathcal{C}_\mu(\Gamma_\nu, \vec{q}, \vec{p}'; t_s, t_{\rm ins}, t_0) = \sum_{\vec{x}_{\rm ins}, \vec{x}_s} e^{i(\vec{x}_{\rm ins} - \vec{x}_0)\cdot\vec{q}} e^{-i(\vec{x}_s - \vec{x}_0)\cdot\vec{p}'} \times \mathrm{Tr}\left[\Gamma_\nu \langle \chi_N(t_s, \vec{x}_s) j_\mu(t_{\rm ins}, \vec{x}_{\rm ins}) \bar{\chi}_N(t_0, \vec{x}_0) \rangle\right].$$



Electromagnetic form factors

- ➢ Interested in theoretically probing the structure of nucleons using lattice QCD.



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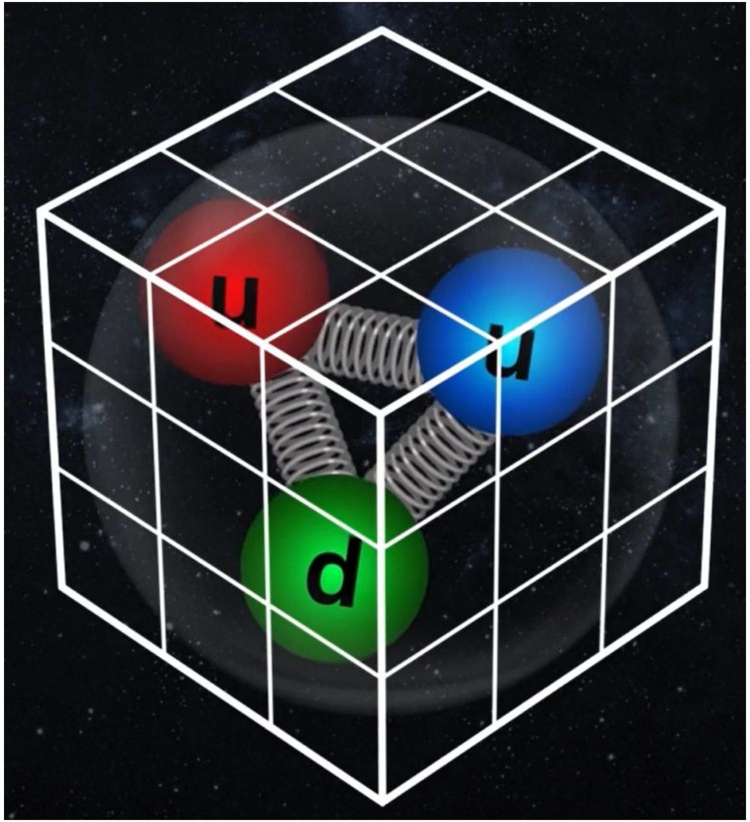

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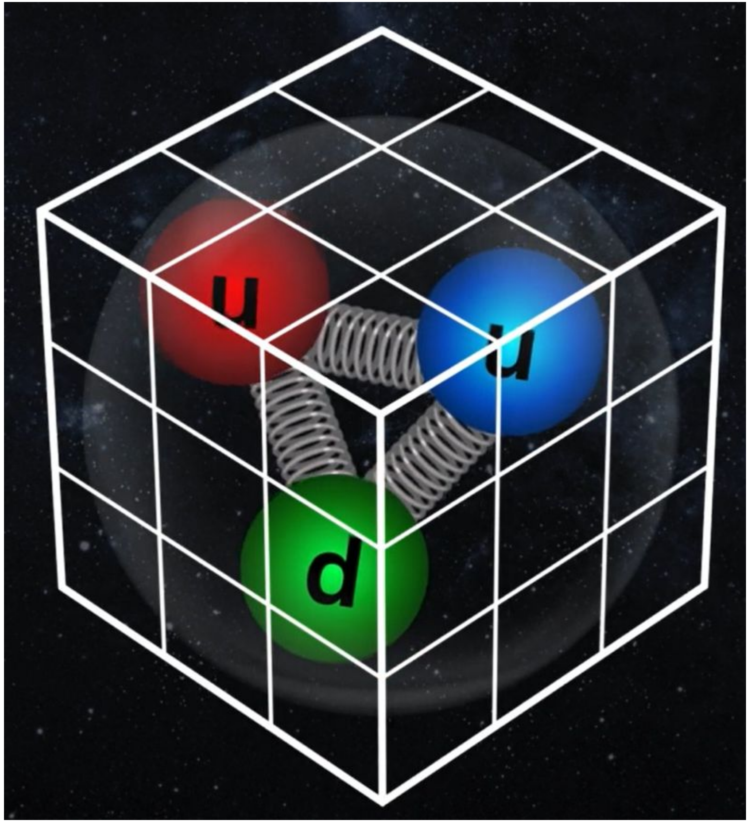
- The nucleon matrix element of for the electromagnetic current is given by:

$$\langle N(p',s')|j_\mu|N(p,s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}_N(p',s') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right] u_N(p,s)$$



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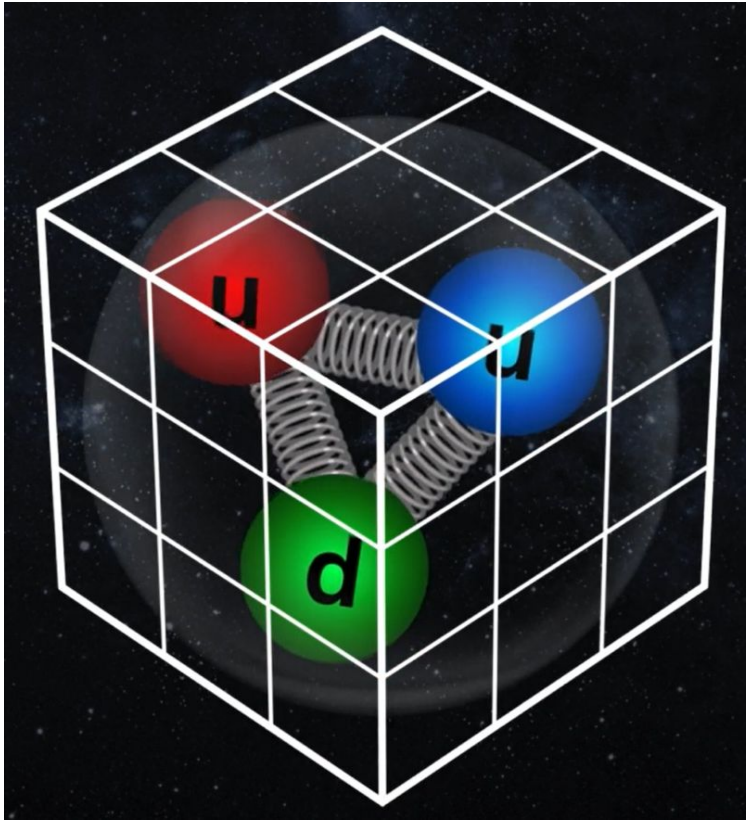
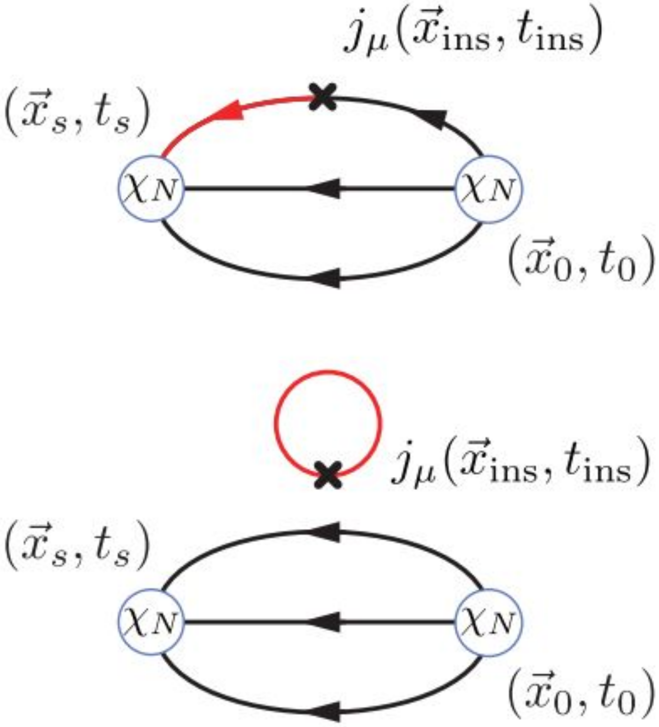
$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



Nucleon matrix element on lattice

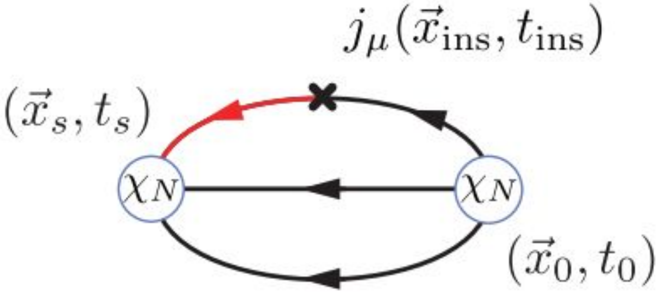
- We take the two-point and three-point functions to momentum space.

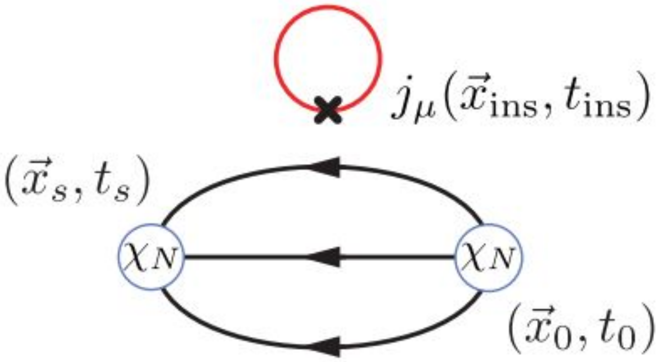


Nucleon matrix element on lattice

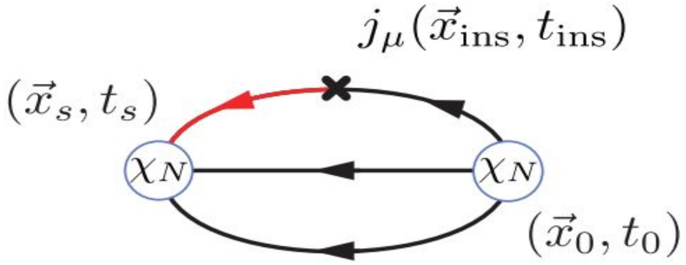
- We take the two-point and three-point functions to momentum space.
- We construct the following ratio to get rid of exponentials and overlaps.

$$\Pi_\mu(\Gamma_\nu,\vec{p}\,',\vec{p};t_s,t_{ins}) = \frac{C_\mu(\Gamma_\nu,\vec{p}\,',\vec{p};t_s,t_{ins})}{C(\Gamma_0,\vec{p}\,';t_s)} \times \sqrt{\frac{C(\Gamma_0,\vec{p};t_s-t_{ins})C(\Gamma_0,\vec{p}\,';t_{ins})C(\Gamma_0,\vec{p}\,';t_s)}{C(\Gamma_0,\vec{p}\,';t_s-t_{ins})C(\Gamma_0,\vec{p};t_{ins})C(\Gamma_0,\vec{p};t_s)}}$$





Connected and disconnected contributions



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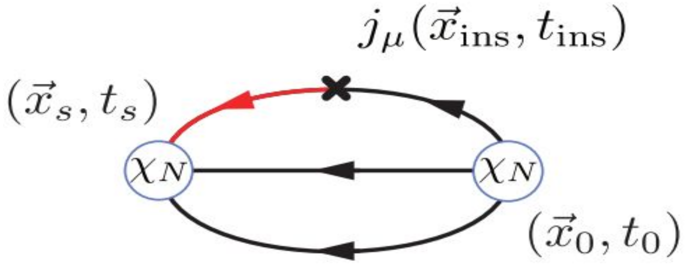

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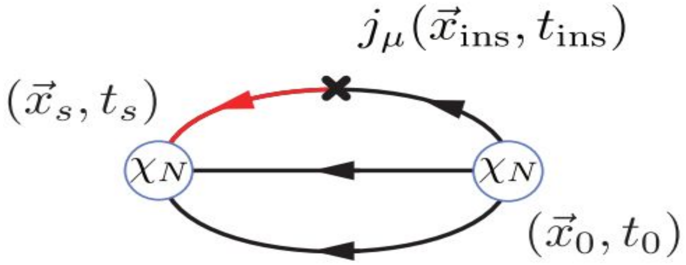

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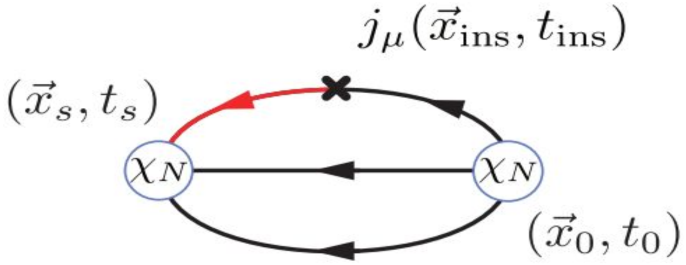

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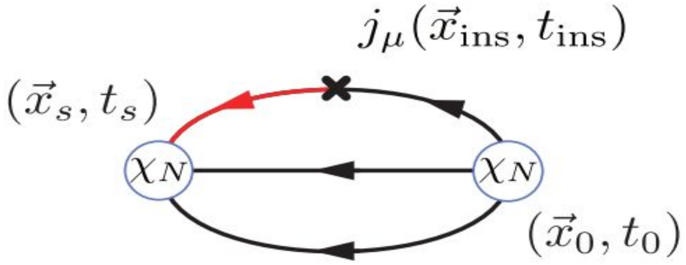

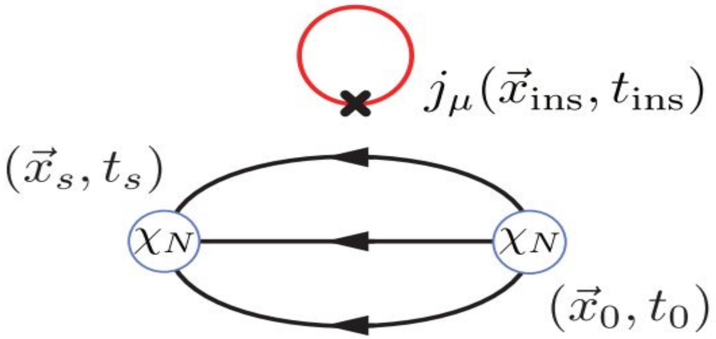

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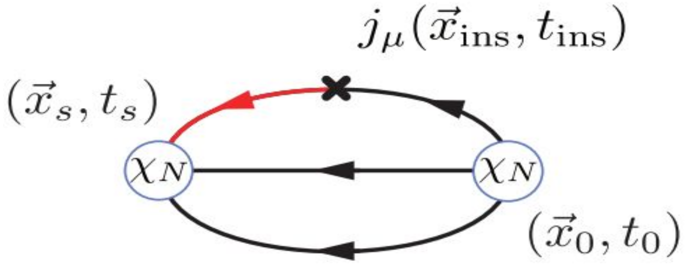

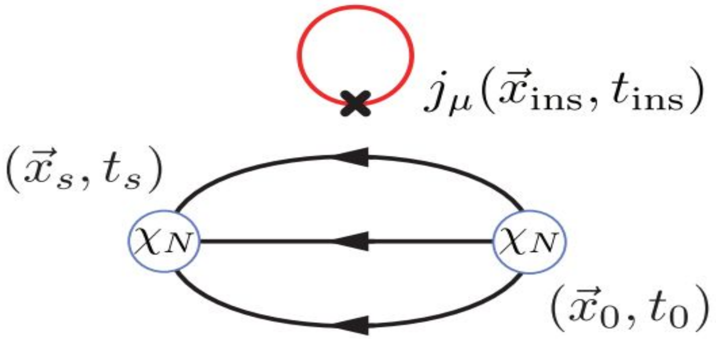

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- ➢ Disconnected loop computed using deflation, hierarchical probing, dilution.

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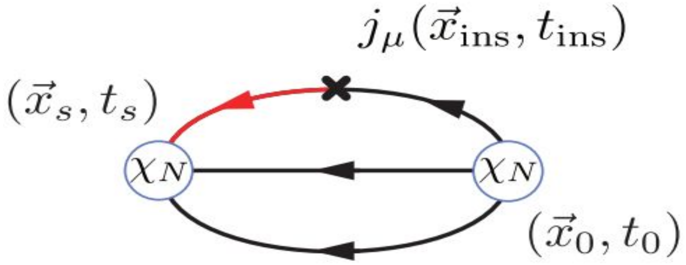

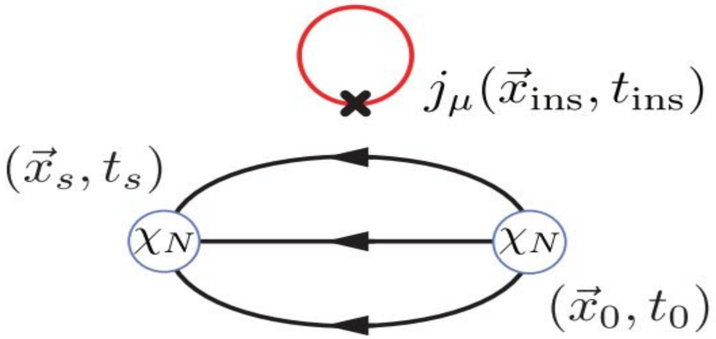

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- ➢ The number of source positions are increased for increasing t_s, to counter increase in noise.
- ➢ Lattice conserved current used, no renormalization needed.

- ➢ Disconnected contribution is obtained from correlating high statistics two-point function with disconnected quark loop. Alexandrou et. al [1812.10311]
- ➢ Disconnected loop computed using deflation, hierarchical probing, dilution.
- ➢ Local current used, renormalization required.

Statistics

➢ Statistics for connected three point functions.

cB211.072.64		
n_{conf}=750		
t_s/a	t_s[fm]	n_{src}
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128

cC211.060.80		
n_{conf}=400		
t_s/a	t_s[fm]	n_{src}
6	0.41	1
8	0.55	2
10	0.69	4
12	0.82	10
14	0.96	22
16	1.10	48
18	1.24	45
20	1.37	116
22	1.51	246

cD211.054.96		
n_{conf}=500		
t_s/a	t_s[fm]	n_{src}
8	0.46	1
10	0.57	2
12	0.68	4
14	0.80	8
16	0.91	16
18	1.03	32
20	1.14	64
22	1.25	16
24	1.37	32
26	1.48	64

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22	1.25	16
24	1.37	32
26	1.48	64

Ensemble	$n_{\rm conf}$	n_{ev}	$n_{\rm src}$
cB211.072.64	750	200	477
cC211.060.80	400	450	650
cD211.054.96	500	-	480

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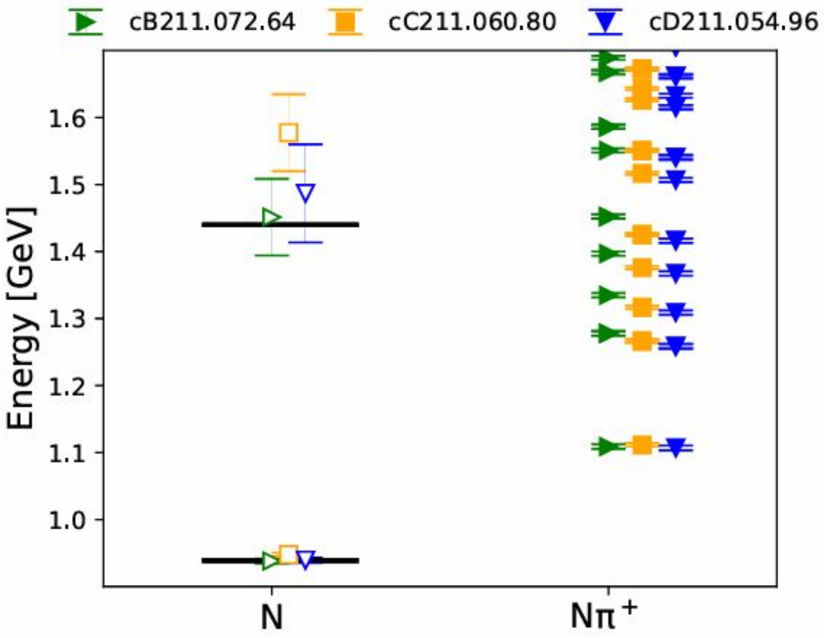

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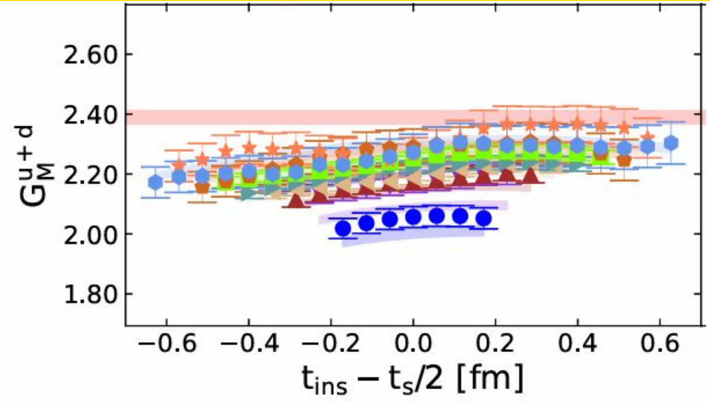

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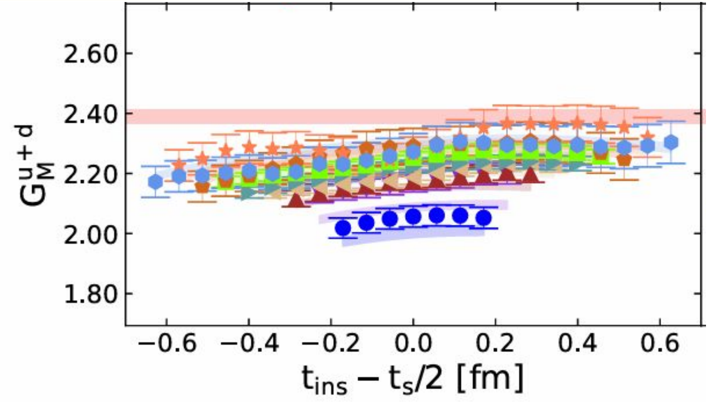

Ensemble	$t_s^{low,3pt}/a$	$t_s^{low,2pt}/a$	t_{ins}^{sink}/a
cB211.072.64	8, 10, 12, 14	1, 2, 3	2, 3, 4
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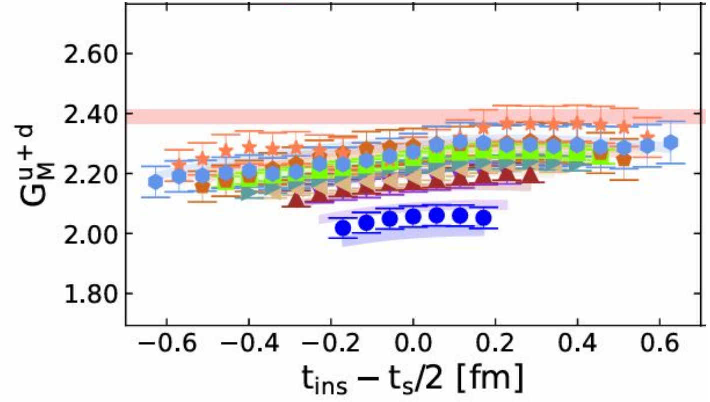

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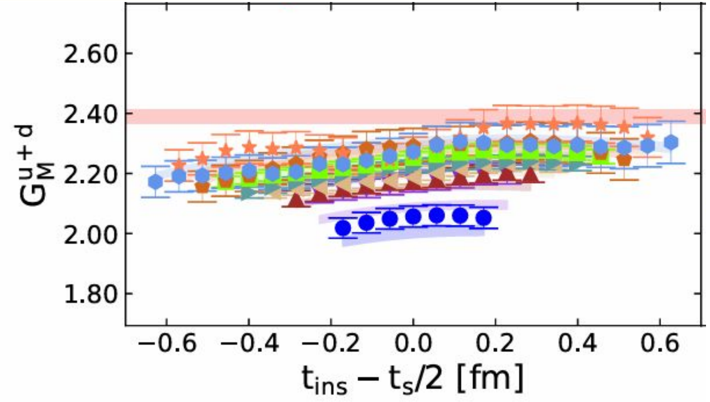

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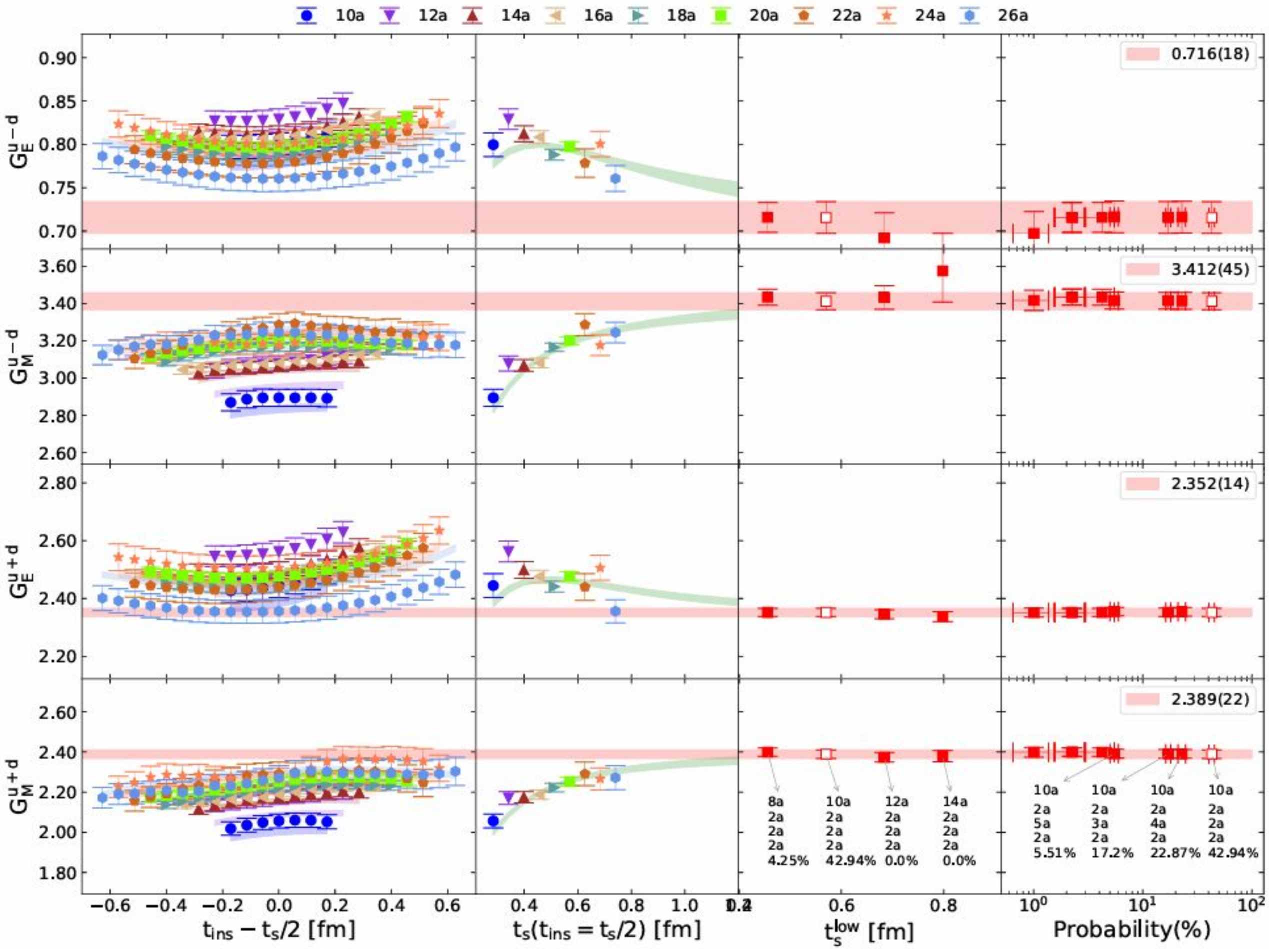

Isovector form factors

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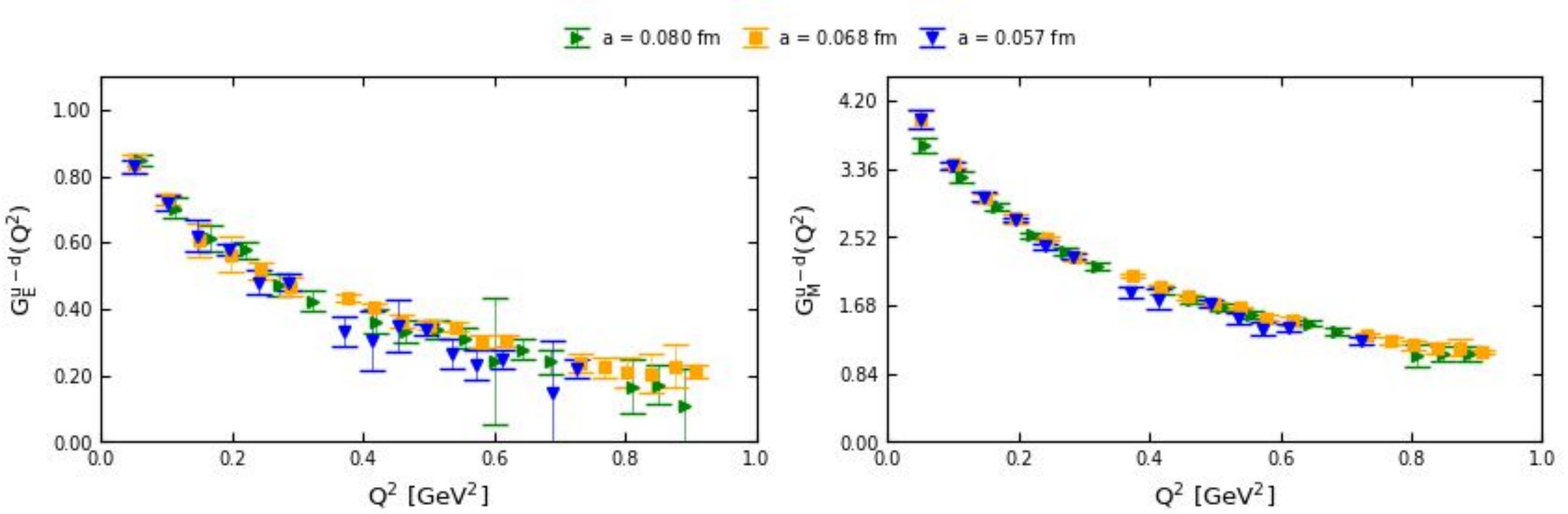

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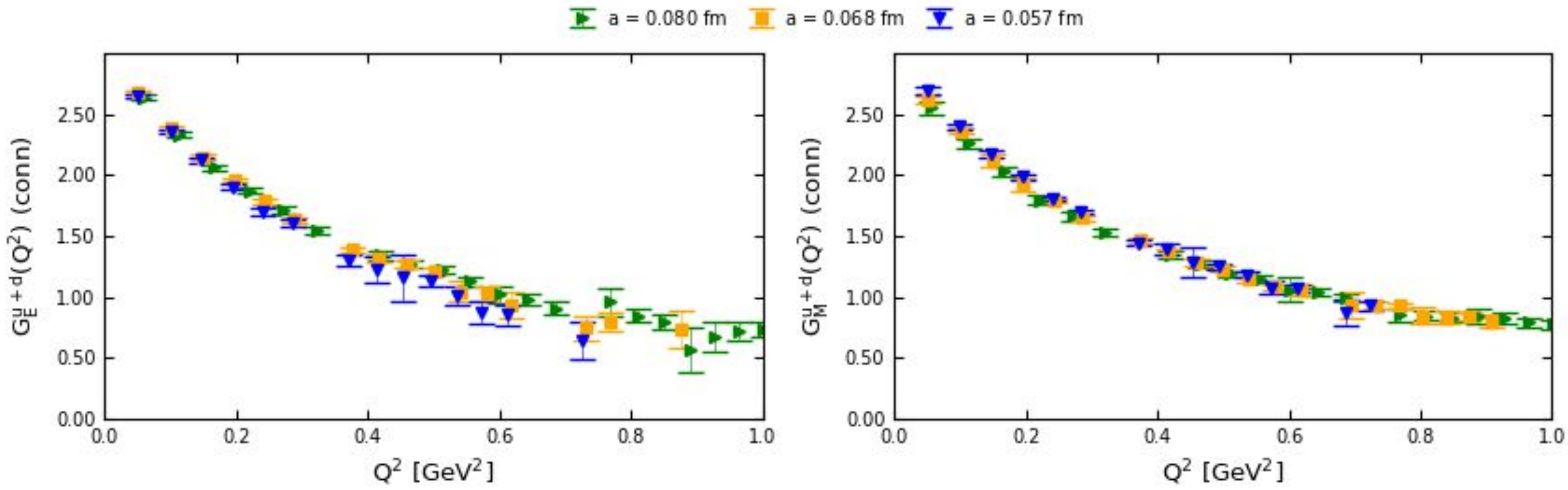

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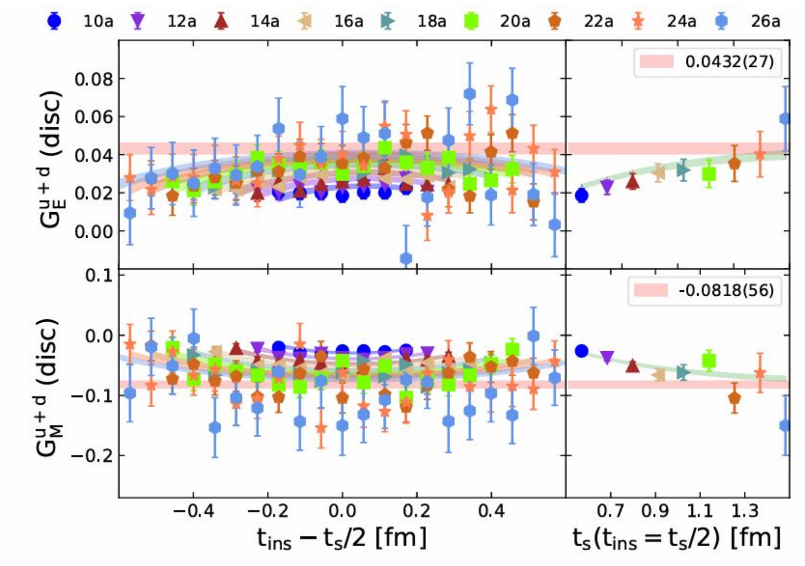

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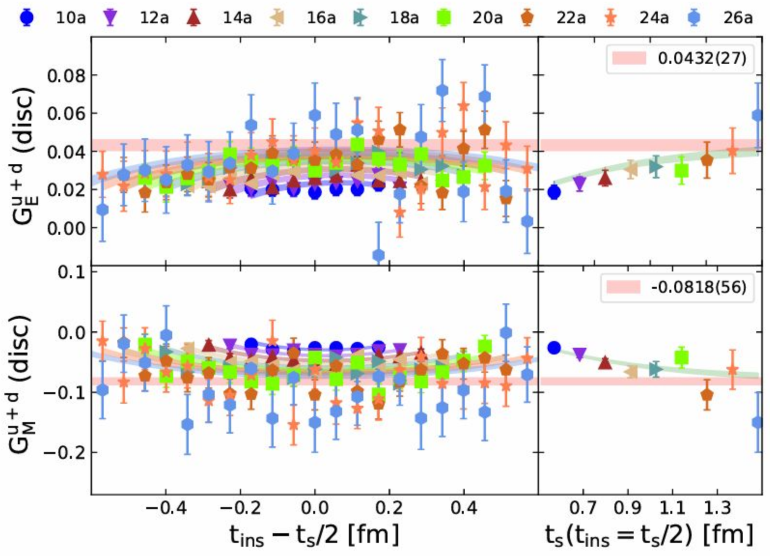
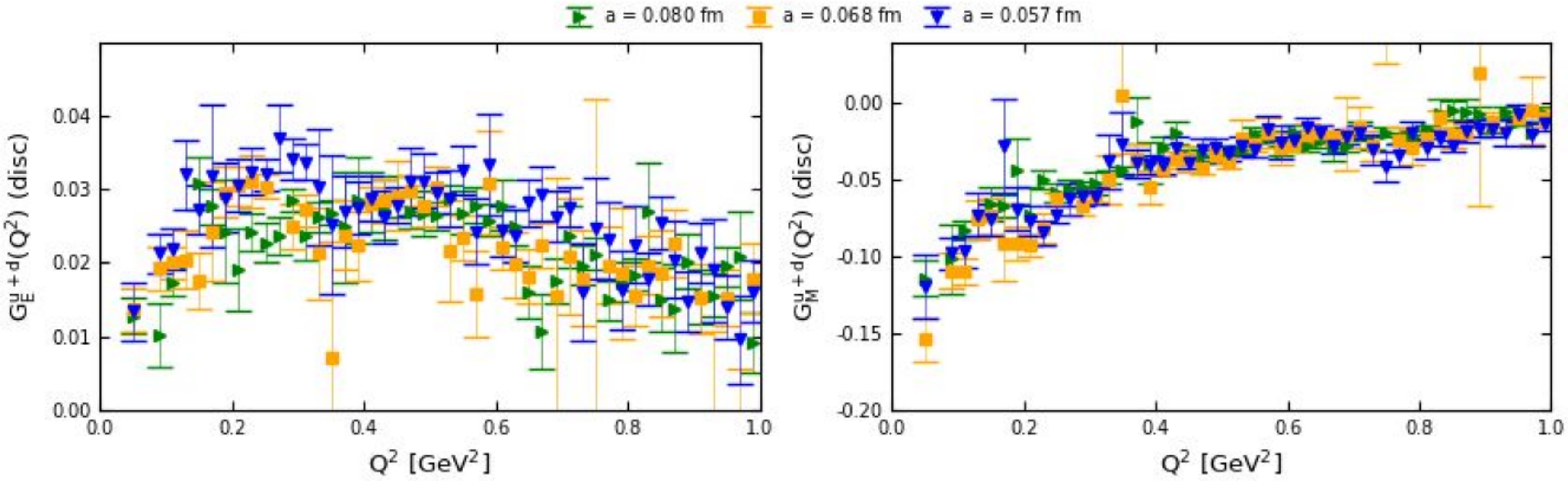


Ensemble	$t_s^{\text{low,3pt}}/a$	$t_{\text{ins}}^{\text{source}}/a$	$t_s^{\text{low,2pt}}/a$
cB211.72.64	8	2	2
cC211.60.80	10	2	3
cD211.54.96	10	2	4

Disconnected contributions to isoscalar form factor

- With momenta in the sink the total number of Q^2 increases to O(100).
- We show binned results weighted by the errors.



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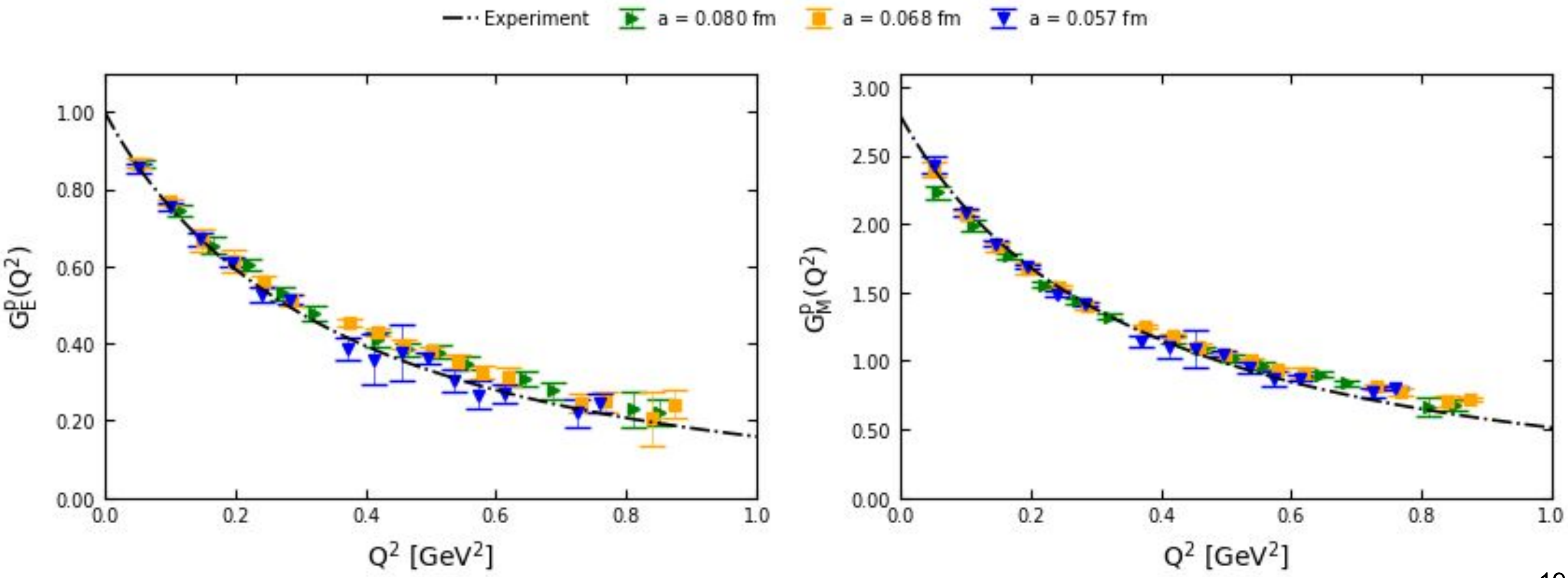

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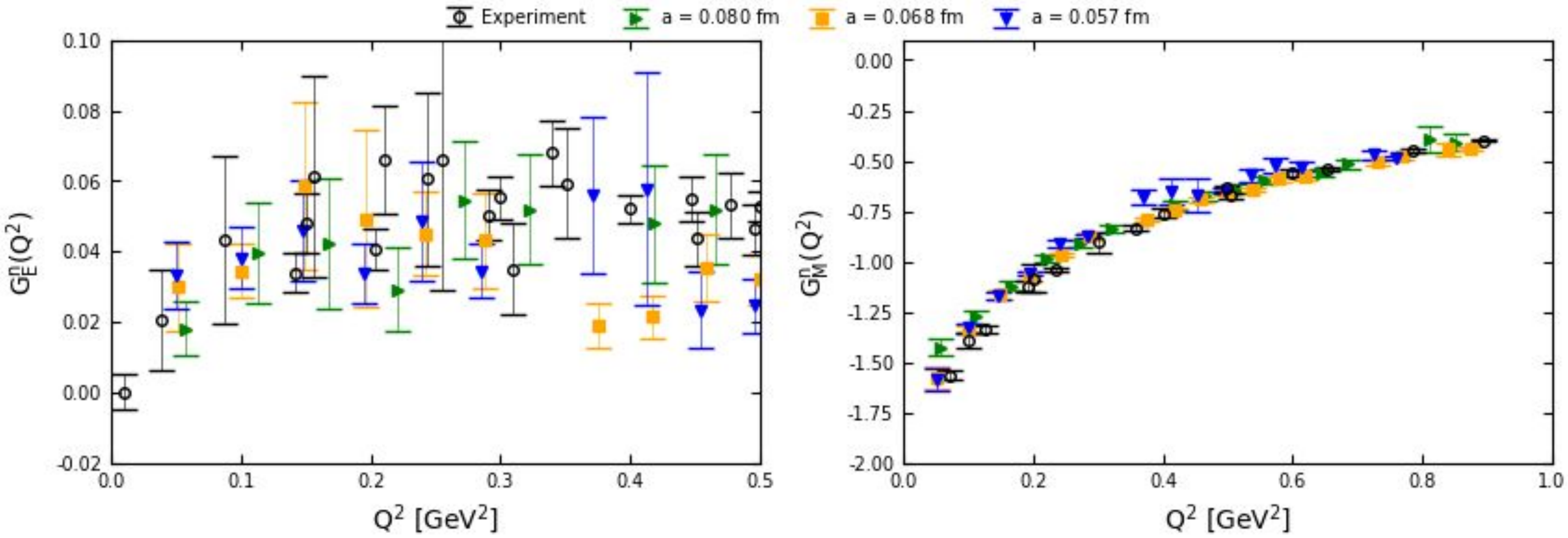

Determination of radius and magnetic moment

- Once we have the parameterization of Q^2 and a^2, the radius can be obtained by:

$$\langle r_X^2 \rangle^q = \frac{-6}{G_X^q(0)} \left. \frac{\partial G_X^q(q^2)}{\partial q^2} \right|_{q^2=0}$$

- The moments are obtained simply by taking the value at $Q^2 = 0$:

$$G_M^p(0) = \mu_p, \quad G_M^n(0) = \mu_n$$

Parameterization of Q^2 Dependance and continuum limit

Dipole

$$G(Q^2) = \frac{g}{\left(1 + \frac{Q^2}{12} r^2\right)^2}$$

$$G(Q^2, a^2) = \frac{g(a^2)}{\left(1 + \frac{Q^2}{12} r^2(a^2)\right)^2}$$

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z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k(Q^2)$$

$$z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}$$

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Galster-like

$$G(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B} \frac{1}{\left(1 + \frac{Q^2}{0.71\,\text{GeV}^2}\right)^2}$$

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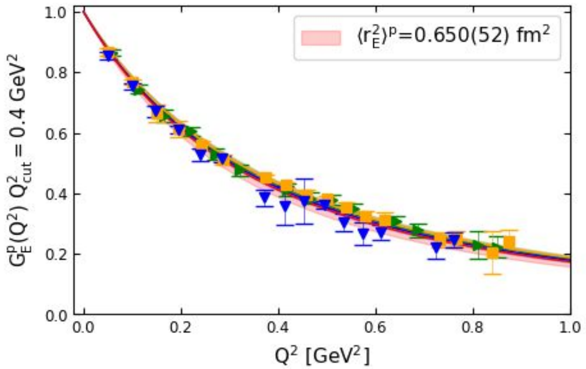
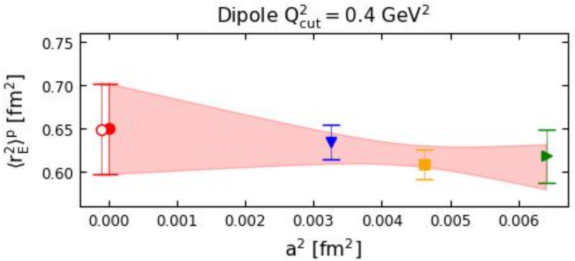
	Dipole	Z-expansion	Galster-like
Proton G_E	1 step + 2 step	1 step	-
Proton G_M	1 step + 2 step	1 step	-
Neutron G_E	-	-	1 step
Neutron G_M	1 step + 2 step	1 step	-

Proton electric form factors with an example fit





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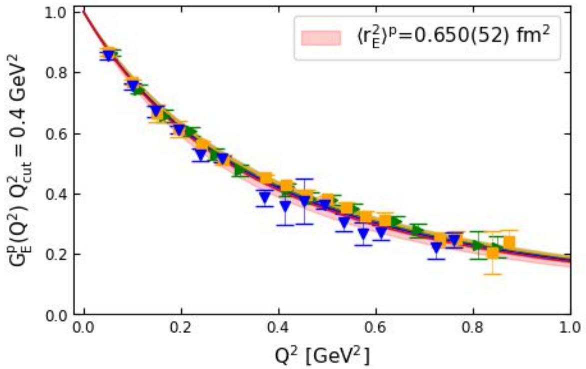

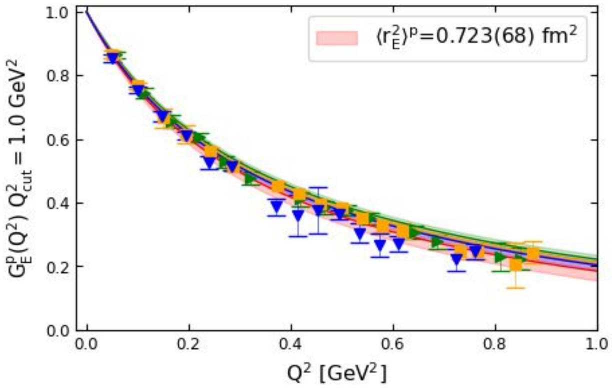

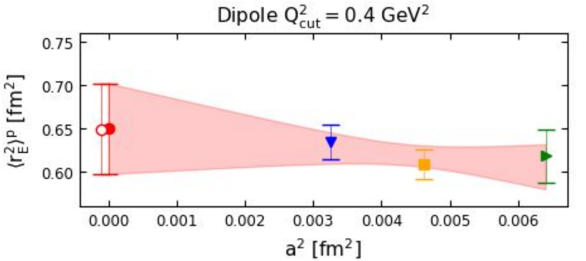

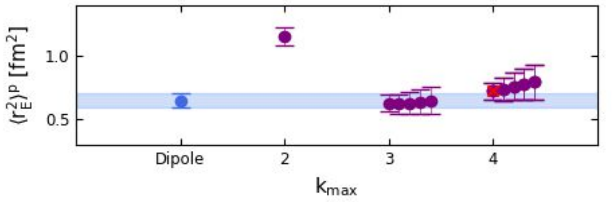

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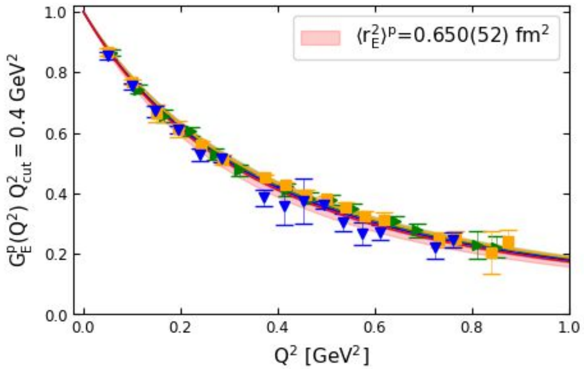
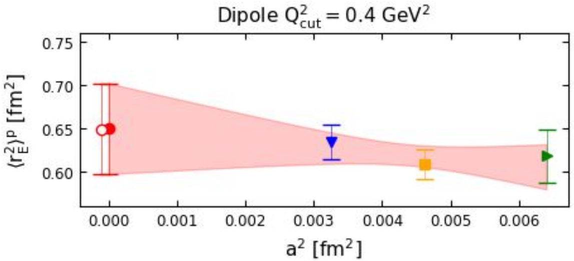
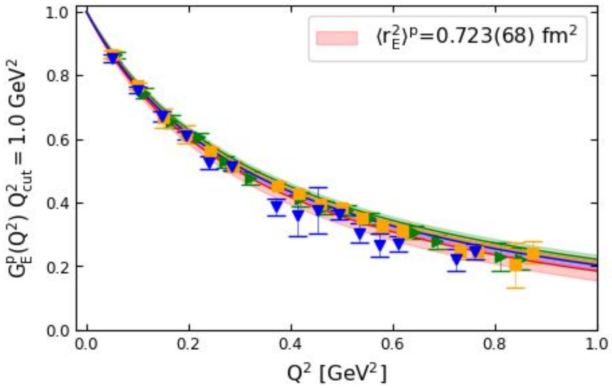
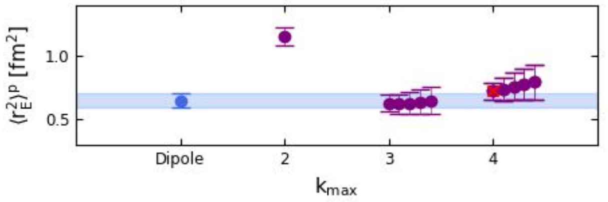




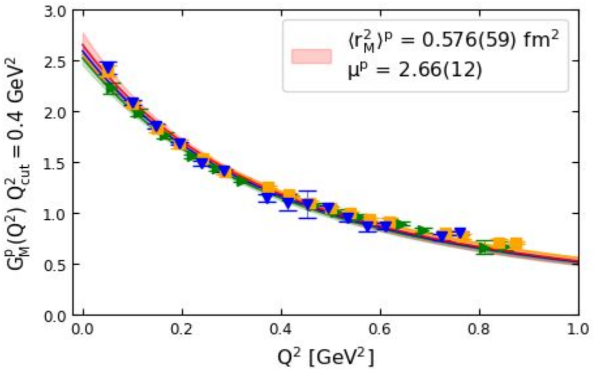


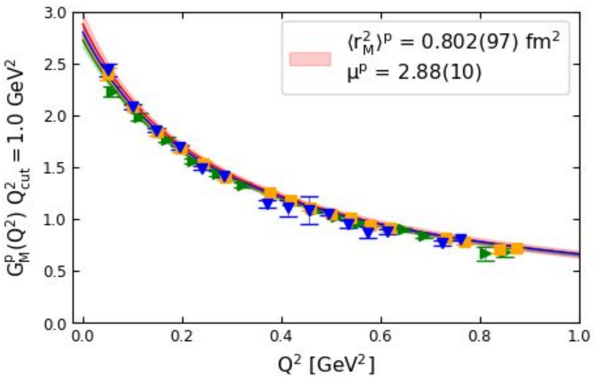


Ensemble	$\langle r_E^2\rangle^p$ [fm^2]	$\tilde{\chi}^2$
cB211.72.64	0.619(31)	0.518
cC211.60.80	0.609(17)	0.635
cD211.54.96	0.635(20)	1.969
$a = 0$, 1-step	0.650(52)	1.042
$a = 0$, 2-step	0.650(52)	-

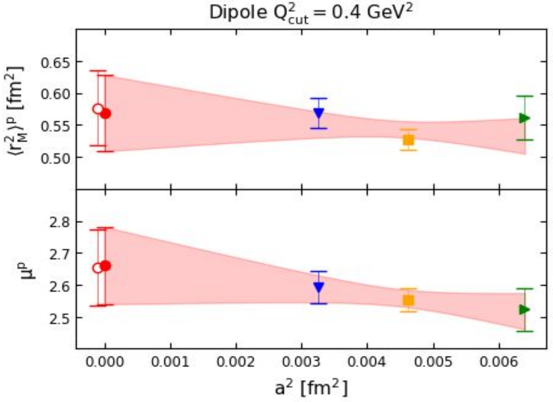
Q^2_{cut} [GeV^2]	$\langle r_E^2\rangle^p$ [fm^2]	$\tilde{\chi}^2$
0.40	0.700(76)	0.770
0.50	0.713(72)	0.638
0.70	0.720(69)	0.595
0.85	0.722(68)	0.583
1.00	0.723(68)	0.585

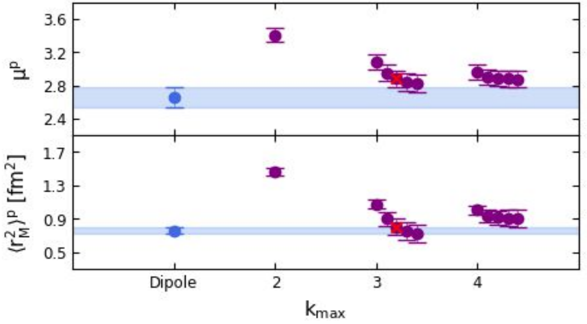
Proton magnetic form factors with an example fit





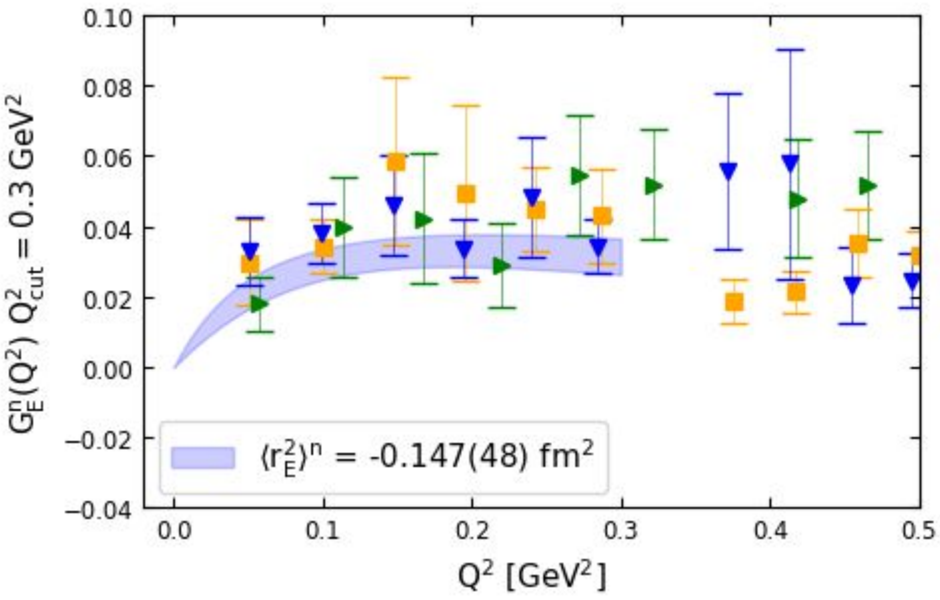
Ensemble	μ^p	$\langle r_M^2\rangle^p$ [fm^2]	$\tilde{\chi}^2$
cB211.72.64	2.524(67)	0.562(34)	1.016
cC211.60.80	2.553(37)	0.527(17)	2.230
cD211.54.96	2.592(49)	0.569(24)	2.732
$a = 0$, 1-step	2.66(12)	0.576(59)	2.326
$a = 0$, 2-step	2.66(12)	0.569(60)	-



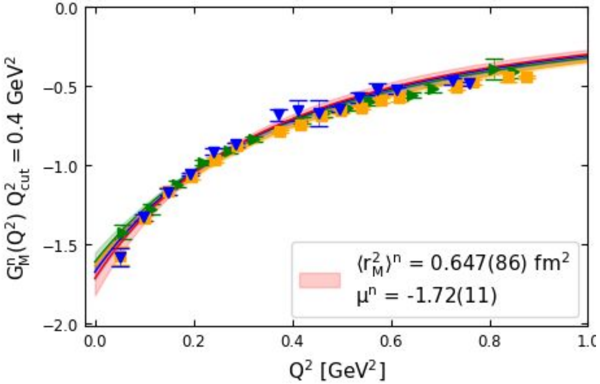


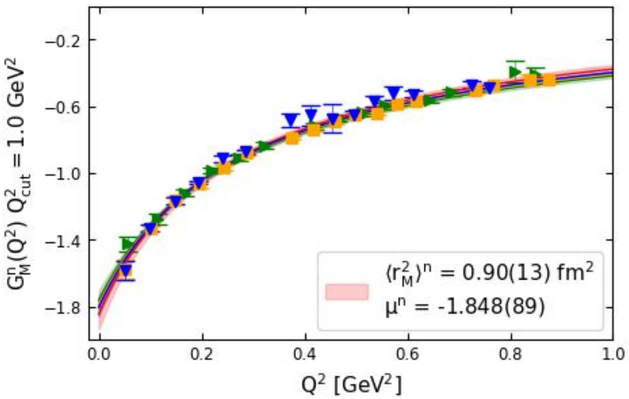
Q^2_{cut}[GeV^2]	μ^p	$\langle r_M^2\rangle^p$ [fm^2]	$\tilde{\chi}^2$
0.40	2.97(12)	0.98(12)	1.172
0.50	2.92(11)	0.91(11)	1.007
0.70	2.89(10)	0.82(10)	1.159
0.85	2.89(10)	0.826(99)	1.099
1.00	2.88(10)	0.802(97)	1.095

Neutron electric form factor with an example fit

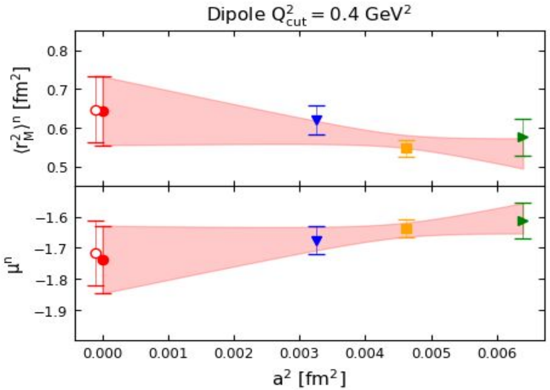


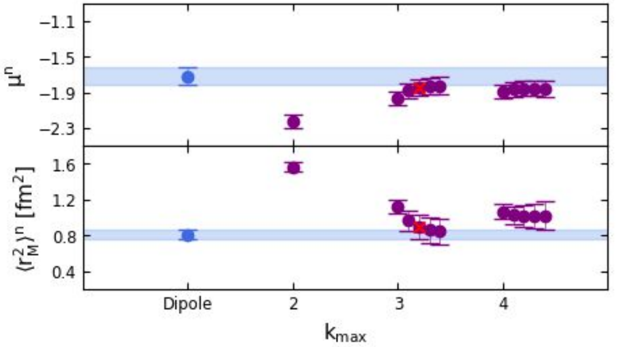
Neutron magnetic form factor with an example fit





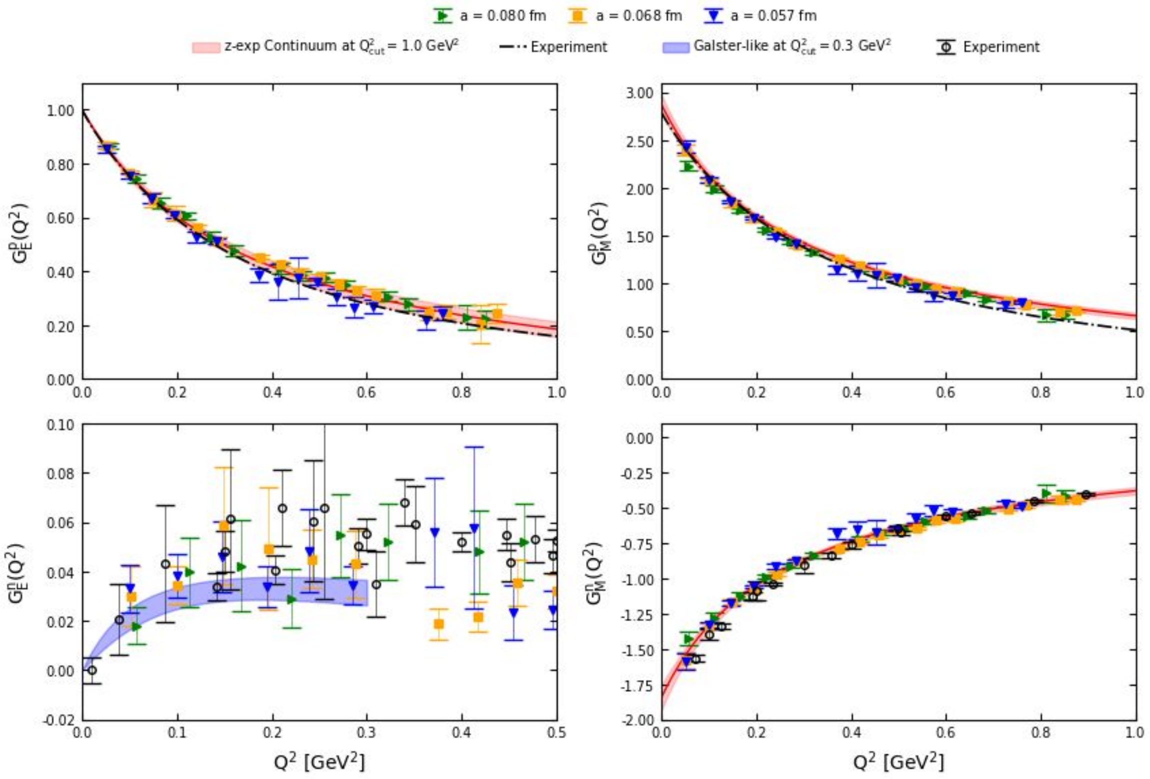
Ensemble	μ^n	$\langle r_M^2 \rangle^n$ [fm^2]	χ^2/N_{dof}
cB211.72.64	-1.612(59)	0.575(48)	0.770
cC211.60.80	-1.637(30)	0.547(21)	1.883
cD211.54.96	-1.676(45)	0.619(37)	2.182
$a = 0$, 1-step	-1.72(11)	0.647(86)	2.072
$a = 0$, 2-step	-1.74(11)	0.644(89)	-



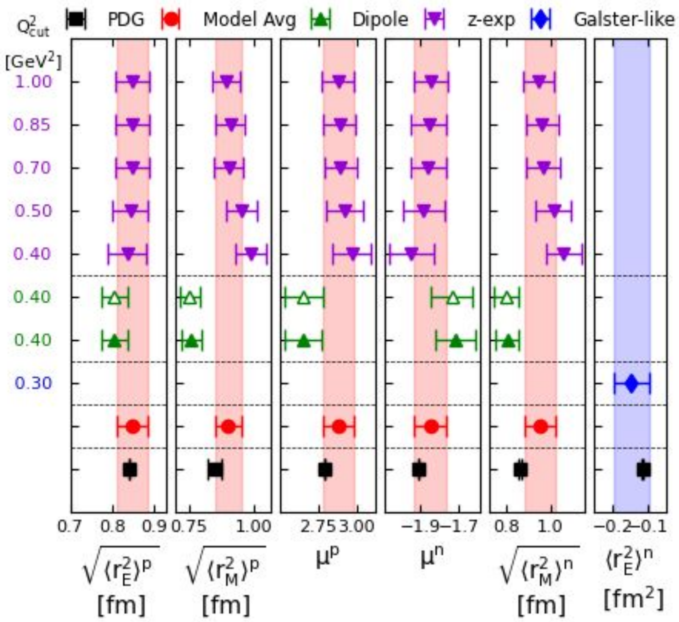


Q^2_{cut}[GeV^2]	μ^n	$\langle r_M^2 \rangle^n$ [fm^2]	χ^2/N_{dof}
0.40	-1.95(12)	1.11(17)	0.987
0.50	-1.88(10)	1.02(16)	0.890
0.70	-1.861(92)	0.93(14)	0.910
0.85	-1.859(91)	0.92(14)	0.821
1.00	-1.848(89)	0.90(13)	0.811

Results

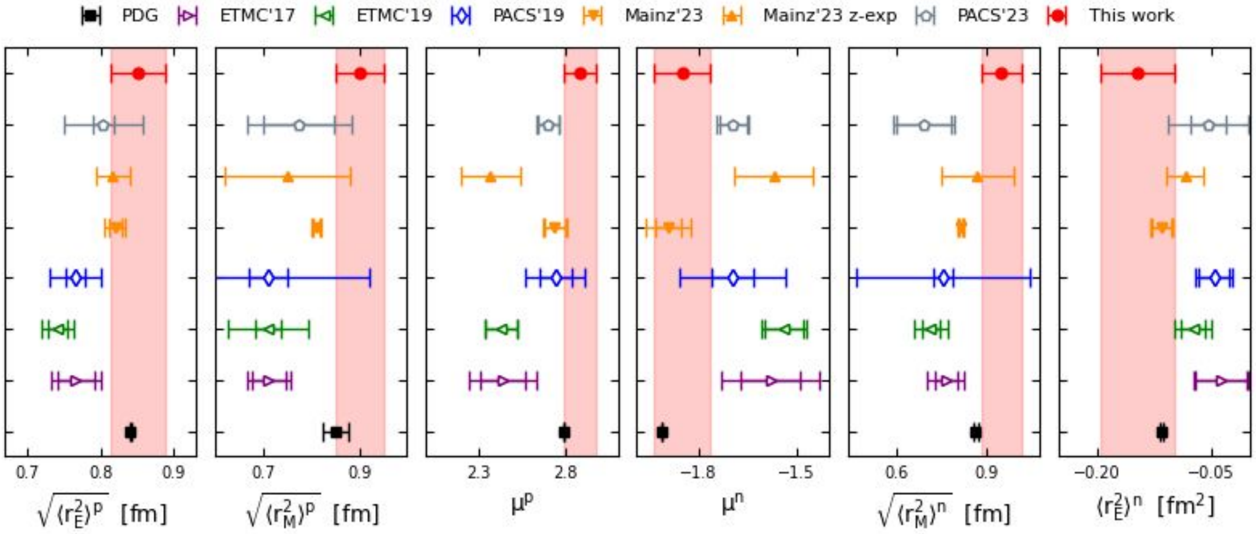


a = 0.080 fm
a = 0.068 fm
a = 0.057 fm
z-exp Continuum at Q^2_{cut} = 1.0 GeV^2
Experiment
Galster-like at Q^2_{cut} = 0.3 GeV^2
Experiment
$G_E^p(Q^2)$
$G_M^p(Q^2)$
$G_E^n(Q^2)$
$G_M^n(Q^2)$
Q^2 [GeV^2]
Q^2 [GeV^2]



Q^2_{cut} [GeV^2]
PDG
Model Avg
Dipole
z-exp
Galster-like
1.00
0.85
0.70
0.50
0.40
0.40
0.40
0.30
$\sqrt{\langle r_E^2\rangle^p}$ [fm]
$\sqrt{\langle r_M^2\rangle^p}$ [fm]
μ^p
μ^n
$\sqrt{\langle r_M^2\rangle^n}$ [fm]
$\langle r_E^2\rangle^n$ [fm^2]

Results



Final results:
[2502.11301]

$\sqrt{\langle r_E^2\rangle^p}$ [fm]	μ^p	$\sqrt{\langle r_M^2\rangle^p}$ [fm]	μ^n	$\sqrt{\langle r_M^2\rangle^n}$ [fm]	$\langle r_E^2\rangle^n$ [fm^2]
0.850(37)	2.883(96)	0.901(51)	-1.851(85)	0.949(69)	-0.147(48)

Summary and Conclusion

- We have results for electromagnetic form factors at continuum limit, at physical point.
- Results include light disconnected contributions with additional sink momenta.
- Multistate fit ensuring ground state convergence.
- Possible improvement: Add analysis results from another lattice volume.

Thank you!

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