Predictions of slow longitudinal mode 1 instability in storage rings with harmonic cavities

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Outline

1. Introduction
2. Prediction with macroparticle tracking
3. Prediction based on instability theory
4. Conclusions

Mode 1 Instability

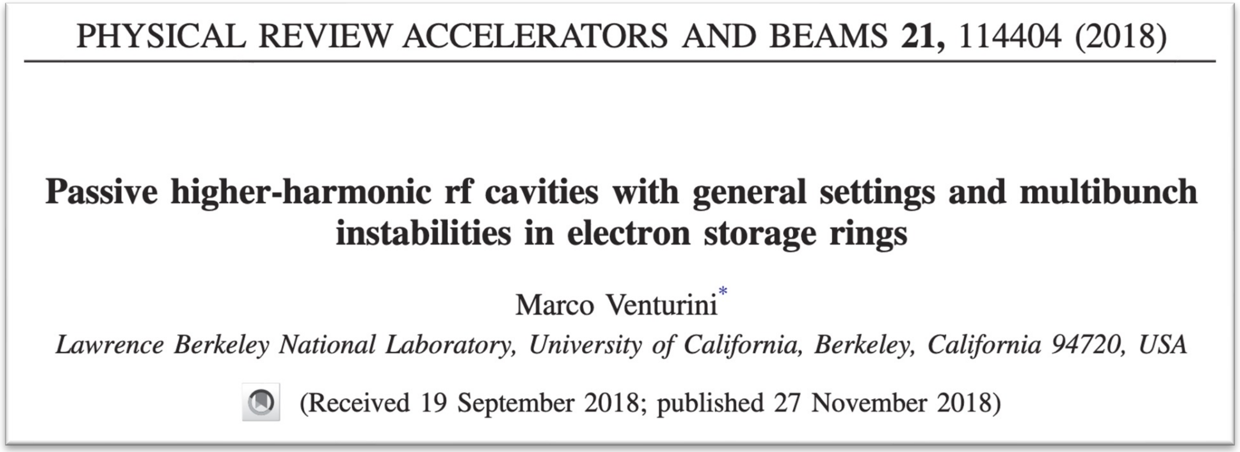
- 4th generation storage rings with passive harmonic cavities (HCs) for bunch lengthening
- Longitudinal plane. Uniform filling pattern. NC and SC passive HCs
- Coherent oscillation of coupled-bunch mode 1 with low frequency (few Hz)
- Driven above a critical harmonic voltage, may limit performance with HC
- Predicted in simulations during HC studies (ALS-U[1], HALF[2], SOLEIL-II[3], DIAMOND-II[4]). Observed experimentally at MAX-IV[5]

1) M. Venturini, PRAB **21** 114404 (2018)
2) T. He, et al. PRAB **25** 024401 (2022)
3) A. Gamelin, Harmonic cavity studies for the SOLEIL Upgrade, I.FAST Workshop 2022
4) T. Olsson, Collective Effects in the Diamond-II Storage Ring. LEL Workshop 2022
5) F. Cullinan, et al, Longitudinal Beam Dynamics in Ultra-low Emittance Rings, I.FAST Workshop 2022

Recent developments to predict the mode 1 instability

M. Venturini, 2018

- Ring: ALS-U. Normal-conducting cavities
- Perturbation theory (mode analysis) in **arbitrary longitudinal potential for any coupled-bunch mode**. Elements of the theory in S. Krinsky, 1985; A. Mosnier, 1999; Y. Cai 2011
- Focused on dipolar motion of mode 0 (Robinson) and mode 1. Simplified formulas for quadratic (single-rf) and quartic potential (flat-potential)
- Predicted **low coherent oscillation frequency** for mode 1
- Highlighted mode 1 instability **driven by the imaginary part of HCs impedance**
- Mode 1 ruled out the option of reusing ALS HCs



PHYSICAL REVIEW ACCELERATORS AND BEAMS **21,** 114404 (2018)

Passive higher-harmonic rf cavities with general settings and multibunch instabilities in electron storage rings

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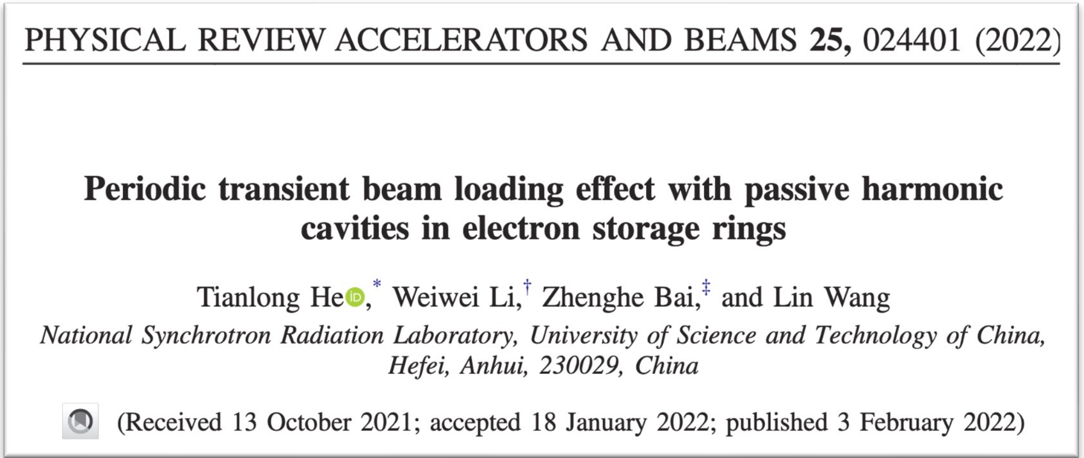
(Received 19 September 2018; published 27 November 2018)

T. He, et al., 2022

- Ring: HALF. Superconducting cavities
- **Periodic transient beam loading (PTBL)** predicted with **macroparticle tracking** and **semi-analytical equilibrium solver.** Explored the characteristics of PTBL
- GPU accelerated tracking with thousands of macroparticles for millions of turns
- Thresholds from non-convergence of semi-analytical equilibrium agrees with tracking. Possible due to the property of **low coherent frequency of PTBL**
- Steady-state time-domain perturbation approach
- Applied a **mode 1 phase perturbation** to the system and studied if it is amplified
- **Analytical formula** with results that agree well with macroparticle tracking



PHYSICAL REVIEW ACCELERATORS AND BEAMS **25,** 024401 (2022)

Periodic transient beam loading effect with passive harmonic cavities in electron storage rings

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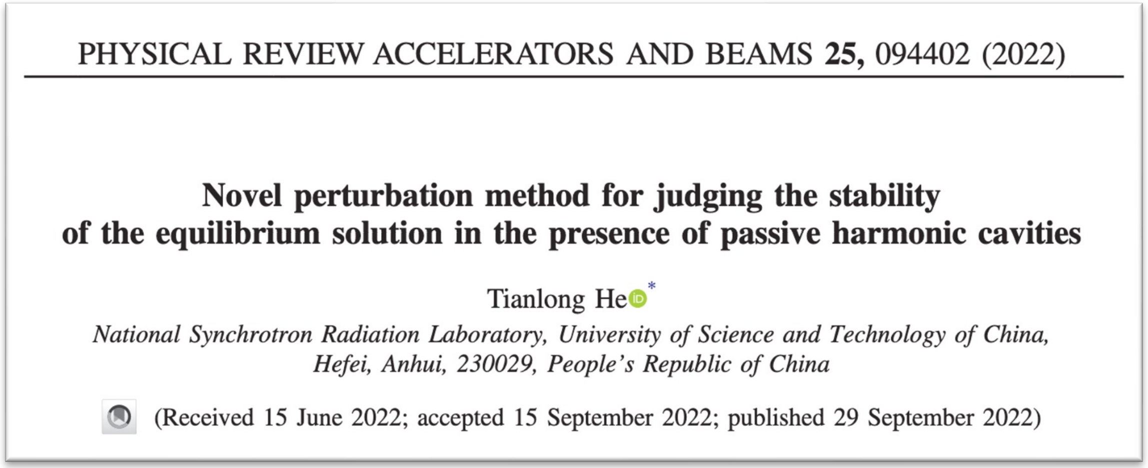
(Received 13 October 2021; accepted 18 January 2022; published 3 February 2022)



PHYSICAL REVIEW ACCELERATORS AND BEAMS **25,** 094402 (2022)

Novel perturbation method for judging the stability of the equilibrium solution in the presence of passive harmonic cavities

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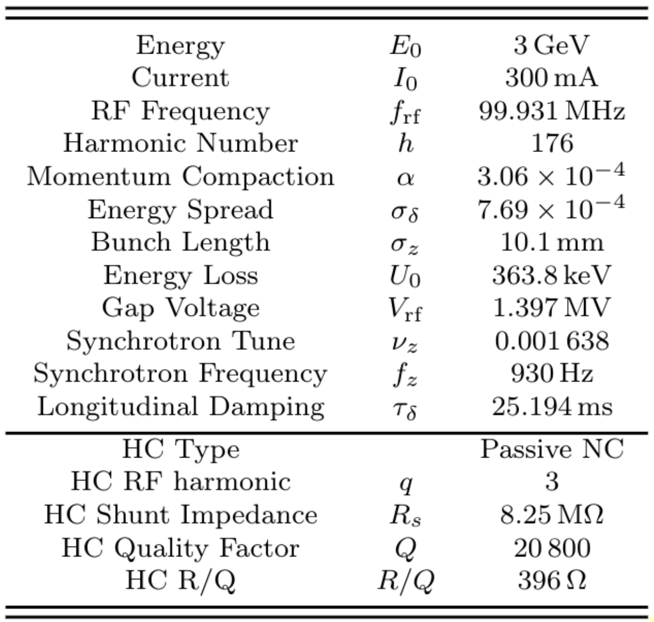
(Received 15 June 2022; accepted 15 September 2022; published 29 September 2022)

Prediction with macroparticle tracking

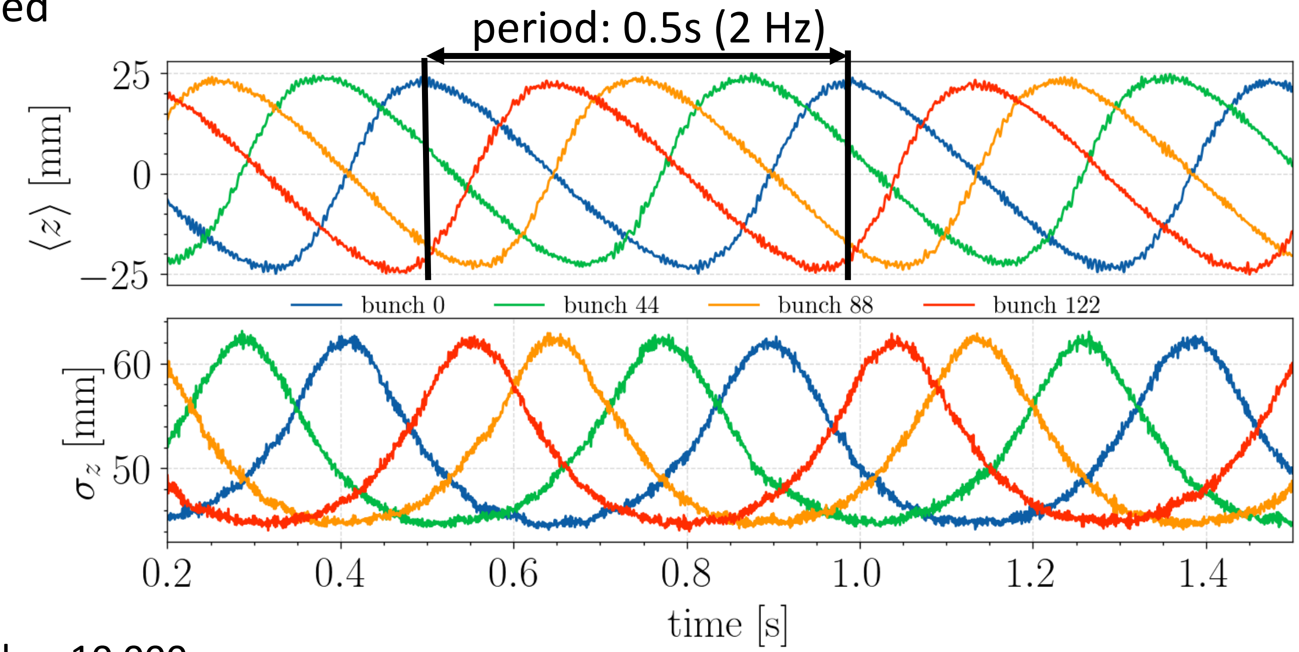
Validation of tracking code

- Longitudinal tracking code in python3 developed at LNLS*
- Test a case where mode 1 instability is already expected

MAX-IV Parameters

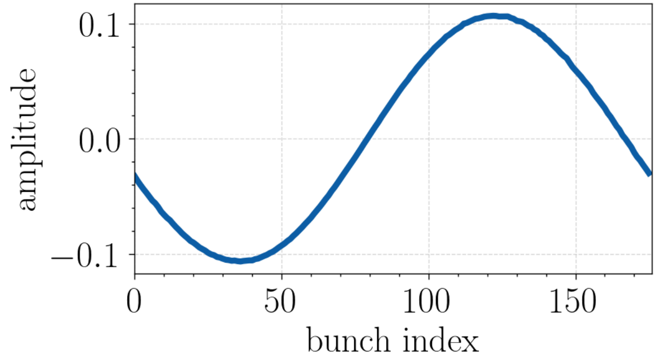


Energy	E_0	3 GeV
Current	I_0	300 mA
RF Frequency	f_{rf}	99.931 MHz
Harmonic Number	h	176
Momentum Compaction	α	3.06×10^{-4}
Energy Spread	σ_δ	7.69×10^{-4}
Bunch Length	σ_z	10.1 mm
Energy Loss	U_0	363.8 keV
Gap Voltage	V_{rf}	1.397 MV
Synchrotron Tune	ν_z	0.001 638
Synchrotron Frequency	f_z	930 Hz
Longitudinal Damping	τ_δ	25.194 ms
HC Type		Passive NC
HC RF harmonic	q	3
HC Shunt Impedance	R_s	8.25 MΩ
HC Quality Factor	Q	20 800
HC R/Q	R/Q	396 Ω



Nr. of Macroparticles: 10 000
Nr. of Turns: 1 500 000

Most unstable coupled-bunch mode



No beam loading from main cavity
Passive HC at flat potential (300mA, RF voltage 1.397 MV)
Detune: 75 kHz
Harmonic voltage: 448 kV

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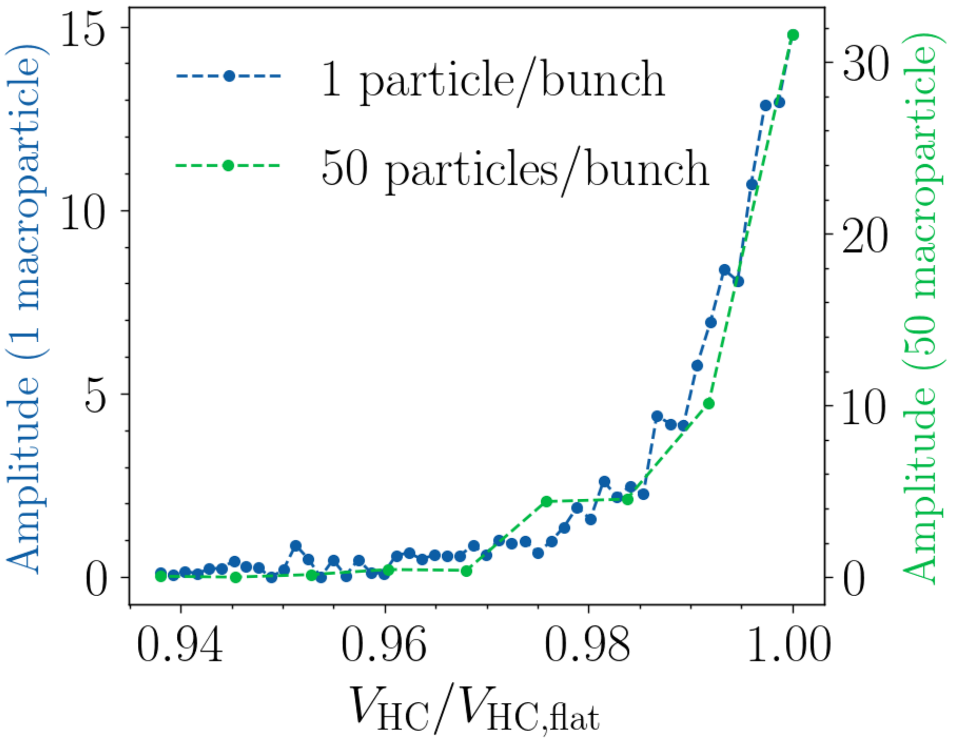
* Open access code
https://github.com/lnls-fac/collective_effects/blob/master/pycolleff/pycolleff/longitudinal_tracking.py

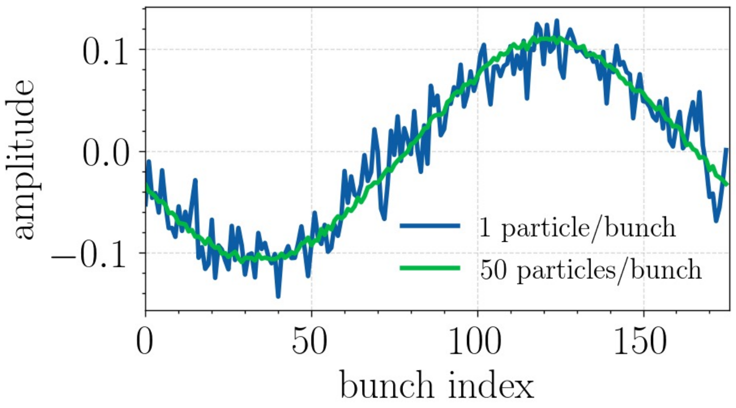
Simplifying the simulation

- Does the instability depend on bunch details? → Tests with reduced number of macroparticles
- Mode 1 instability happens even in the limiting case of 1 macroparticle per bunch!
  - Bunch centroids (point bunch approximation) should capture the underlying mechanism for instability
  - Non-linearities inside bunch should be negligible

Searching for voltage threshold



Most unstable coupled-bunch mode

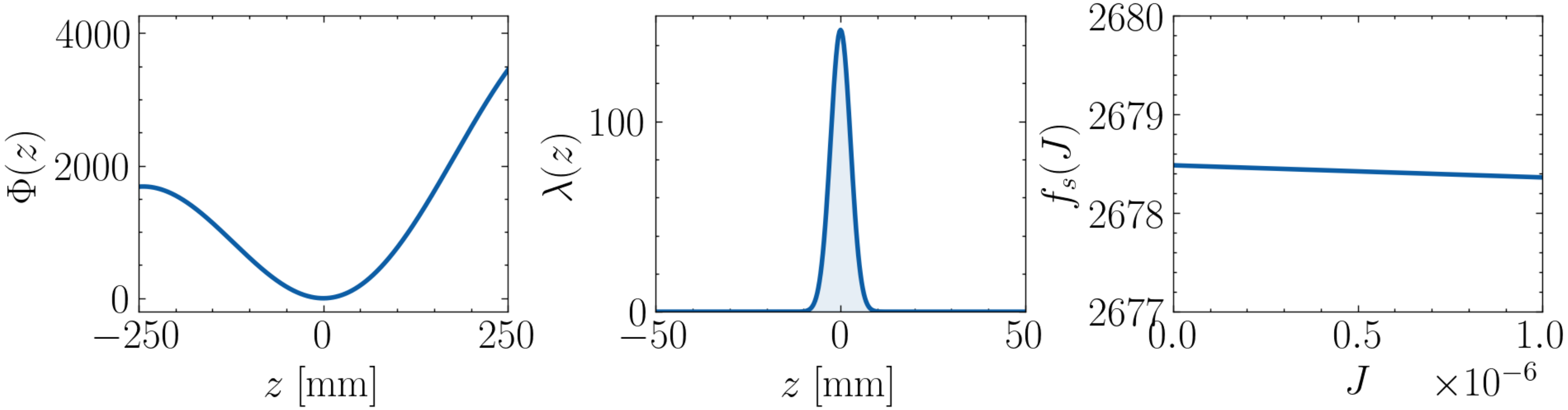


Prediction based on instability theory

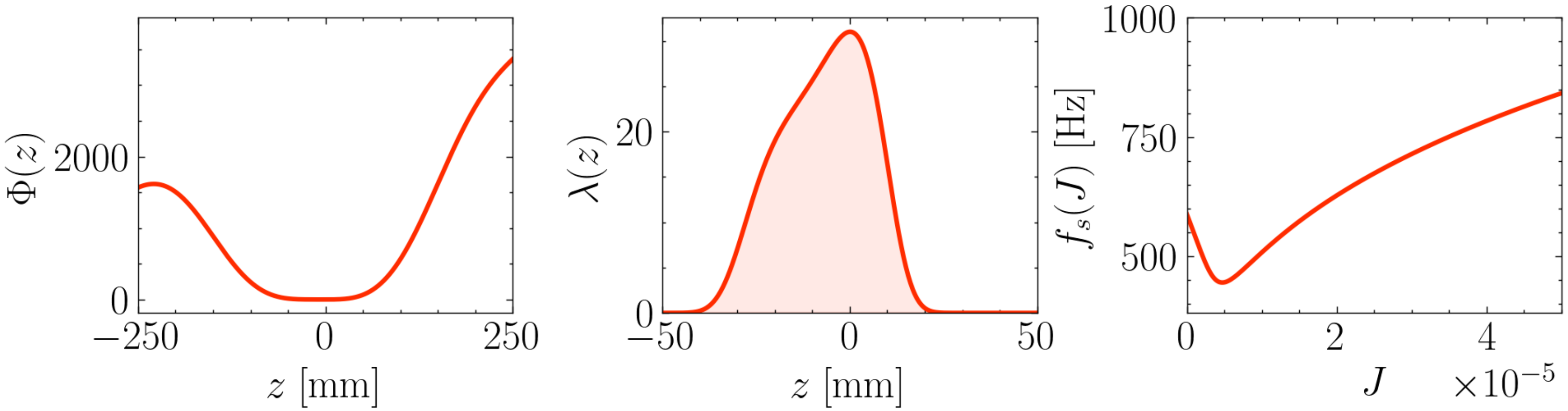
Step 1: solve the equilibrium with HCs

- Self-consistent solution of Haïssinski equation
- Equilibrium longitudinal potential and bunch profiles

Single-rf: linear dynamics, quadratic potential, Gaussian bunch, constant synchrotron frequency



Double-rf: non-linear dynamics, arbitrary potential, non-Gaussian bunch, amplitude-dependent synchrotron frequency



Open access code
https://github.com/lnls-fac/collective_effects/blob/master/pycolleff/pycolleff/longitudinal_equilibrium.py

Step 2: evaluate the stability of mode 1

- HC fields introduce non-linearity and modify the bunch profile. Out of scope of linear instability theory for Gaussian bunches. Linear theory can still be useful though
- Stability diagram analysis:
  - Calculate the complex coherent frequencies for the linear system
  - Evaluate the location of unstable modes on the stability boundary (effect of non-linearities)
  - Applied for instabilities with HCs: M. Venturini, PRAB **21** 114404 (2018) and R. Lindberg, PRAB **21** 124402 (2018)
- Hypothesis: the coherent frequency of mode 1 is too low to have enough overlap with spread of incoherent frequencies → Non-linearities/Landau damping effects can be neglected

Goal: apply a linear theory for Gaussian bunches to predict the mode 1 instability

Longitudinal Mode-Coupling Instability (LMCI)

Suzuki, T., Chin, Y., & Satoh, K. (1983). Mode-coupling theory and bunch lengthening in SPEAR II. Particle Accelerators, 13(3-4), 179-198.
Suzuki, T. (1983). Theory of longitudinal bunched-beam instabilities based on the Fokker-Planck equation. Particle Accelerators, 14, 91-108

- Linear single-particle dynamics in polar coordinates (r, ϕ) **Quadratic potential, Gaussian bunch**
- Small perturbation from equilibrium drives dynamic wakefields with coherent frequency

$$\psi(r,\phi,t) = \psi_0(r) + \psi_1(r,\phi)e^{-i\Omega t}$$

- Expansion in **azimuthal modes m** $\psi_1(r,\phi) = \sum_{m=-\infty}^{+\infty} R_m(r)e^{im\phi}$
- Expansion in orthogonal polynomials (Laguerre) → **radial modes k** associated with each m
- Insert this perturbation to linearize Fokker-Planck equation and solve for coherent frequencies
- Coherent frequencies are eigenvalues of interaction matrix including **mode coupling of azimuthal and radial modes.** Instability if $\mathrm{Im}\Omega > 0$
- Uniform filling. For analysis of coupled-bunch mode ℓ sample impedance at $(p+\ell)\omega_0$. For mode 1, $\ell = 1$

Open access code
https://github.com/lnls-fac/collective_effects/blob/master/pycolleff/pycolleff/colleff.py

Adaptations for the analysis

- Calculation in a fictitious quadratic potential with longer bunch

$$\boxed{\sigma_{z,0} \rightarrow \sigma_{z,\mathrm{HC}}}$$

- Incoherent average synchrotron frequency is also an input

Compatible with bunch length $\omega_s\sigma_z = \alpha c\sigma_\delta$

Local frequency averaged by z-distribution

$$\langle\omega_s\rangle_z = \int_{-\infty}^{\infty}\lambda(z)\sqrt{-\frac{\alpha hc}{2\pi E_0\omega_{\mathrm{rf}}}V'(z)}dz$$

Similar values

Global frequency averaged by action-distribution

$$\langle\omega_s\rangle_J = 2\pi\int_0^{\infty}\Psi(J)\omega_s(J)dJ$$

Typically lower values

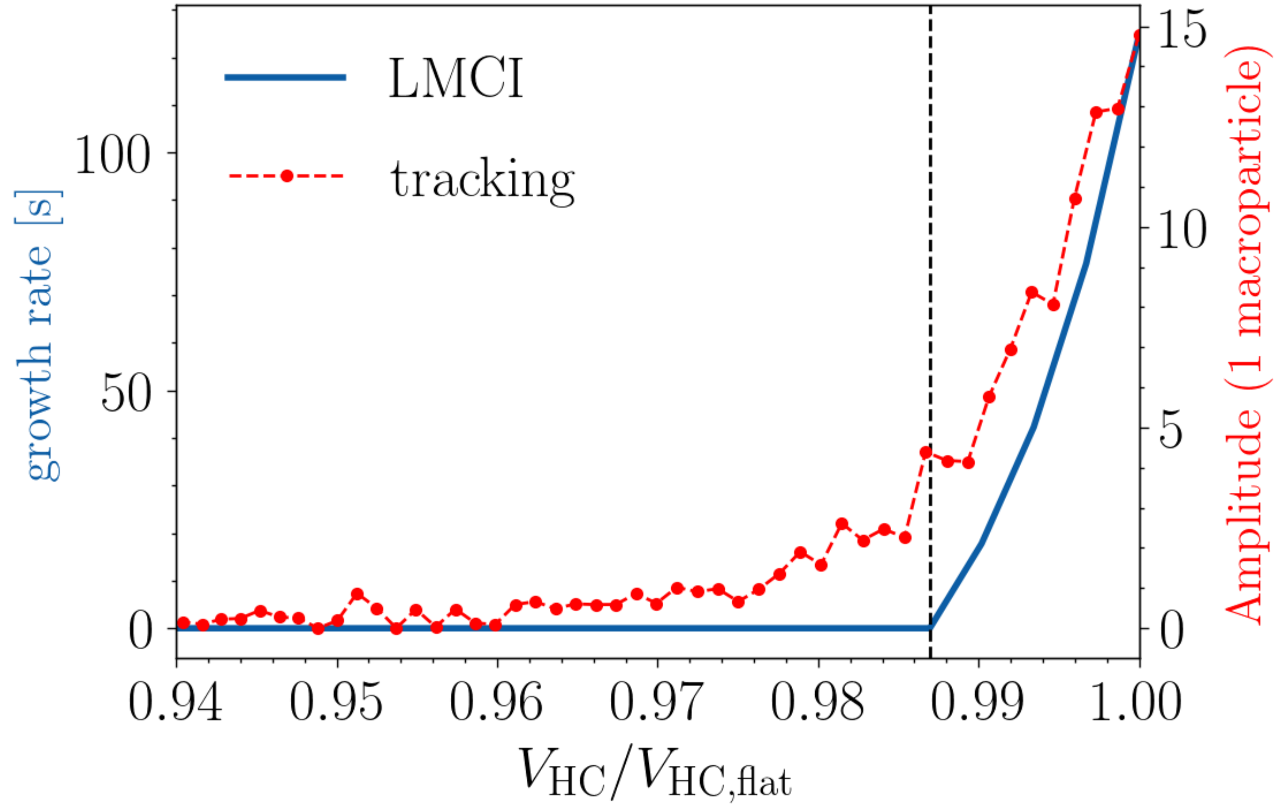
- All the following results were computed with HC-modified bunch length + compatible incoherent synchrotron frequency

LMCI results

LMCI vs. Tracking for MAX-IV parameters

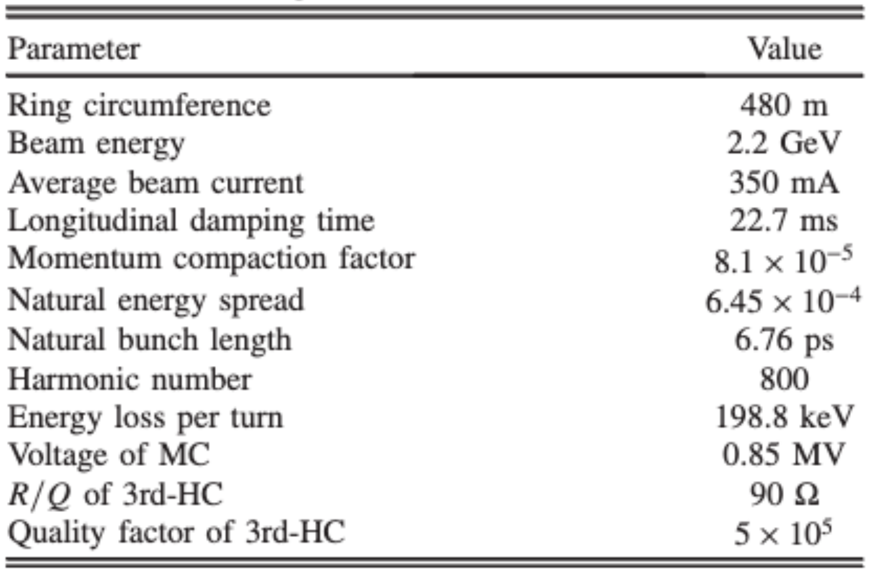
- 300mA
- Main RF voltage: 1.397 MV
- 3 HCs

- Agrees with tracking → mode 1 unstable at flat potential



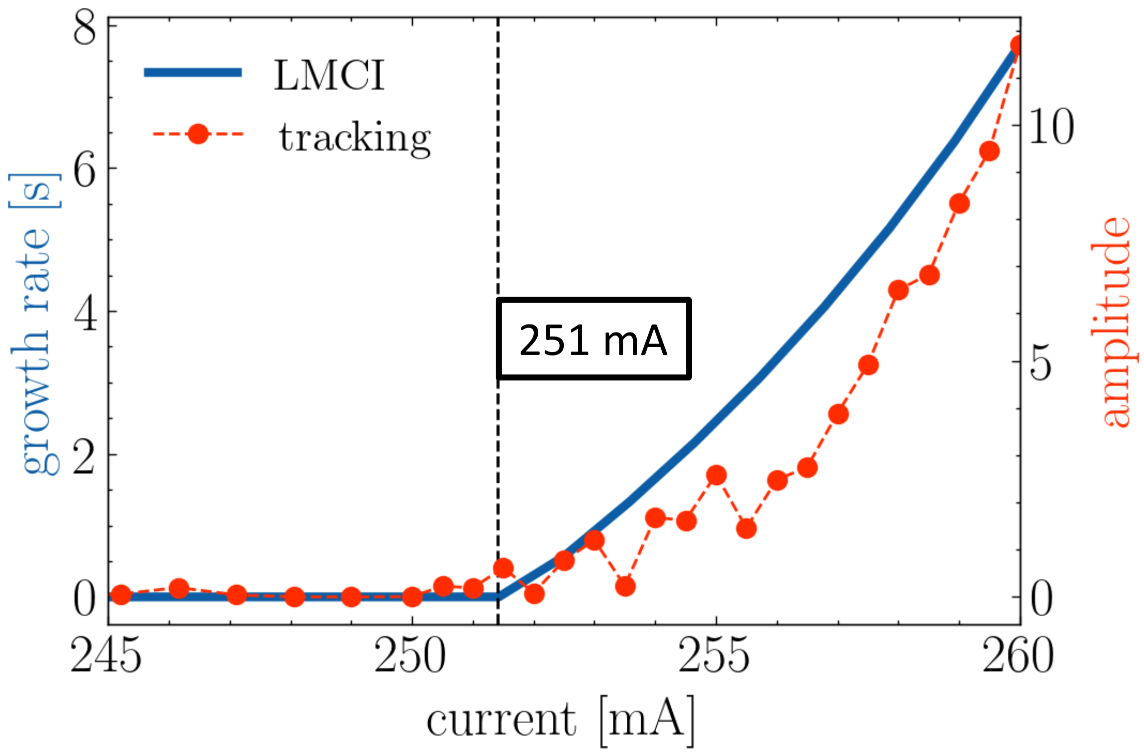
LMCI vs. Tracking for HALF parameters

Parameters for HALF from [1]



Parameter	Value
Ring circumference	480 m
Beam energy	2.2 GeV
Average beam current	350 mA
Longitudinal damping time	22.7 ms
Momentum compaction factor	8.1×10^{-5}
Natural energy spread	6.45×10^{-4}
Natural bunch length	6.76 ps
Harmonic number	800
Energy loss per turn	198.8 keV
Voltage of MC	0.85 MV
R/Q of 3rd-HC	90 Ω
Quality factor of 3rd-HC	5×10^5

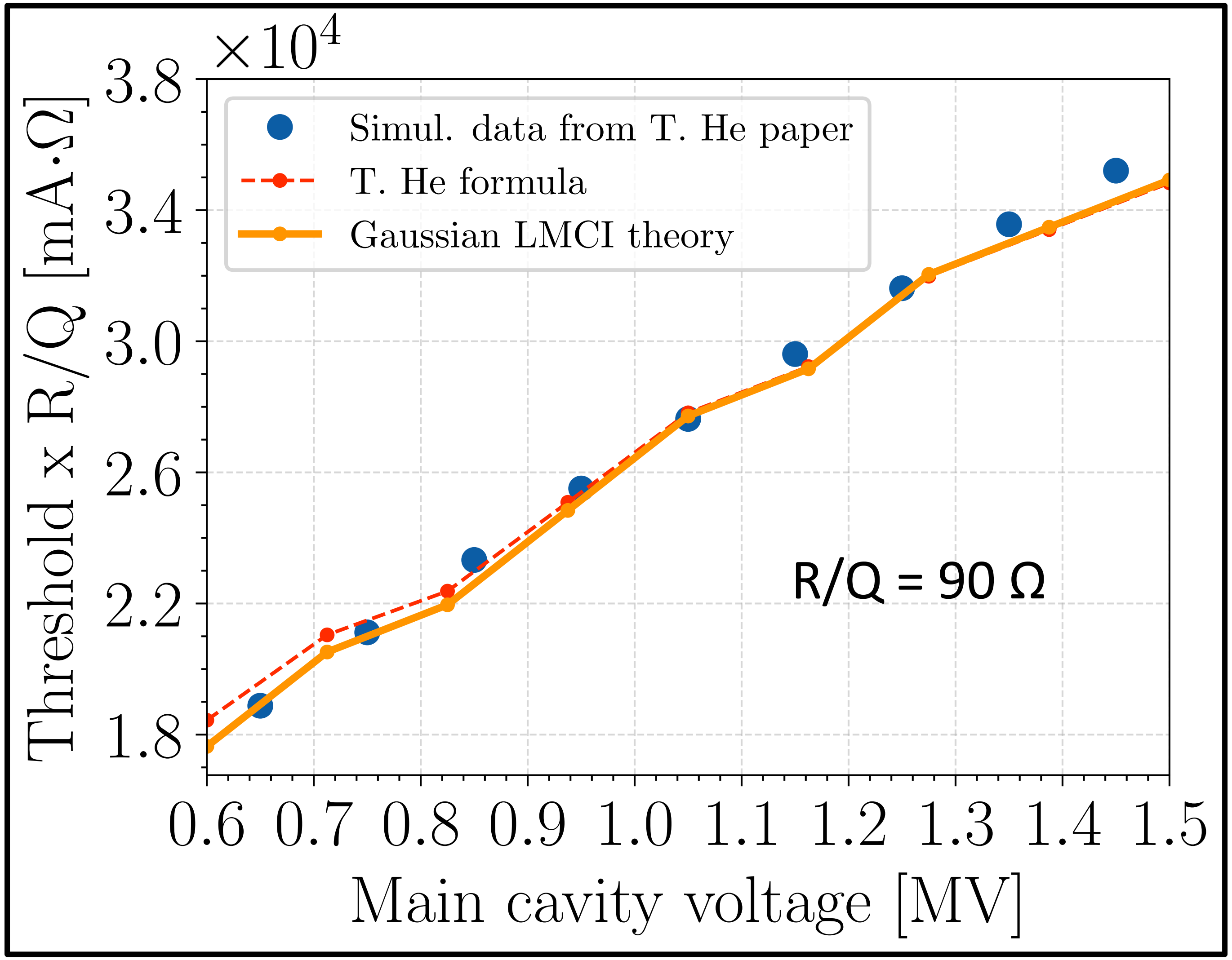
[1] T. He, et al. PRAB **25** 024401 (2022)

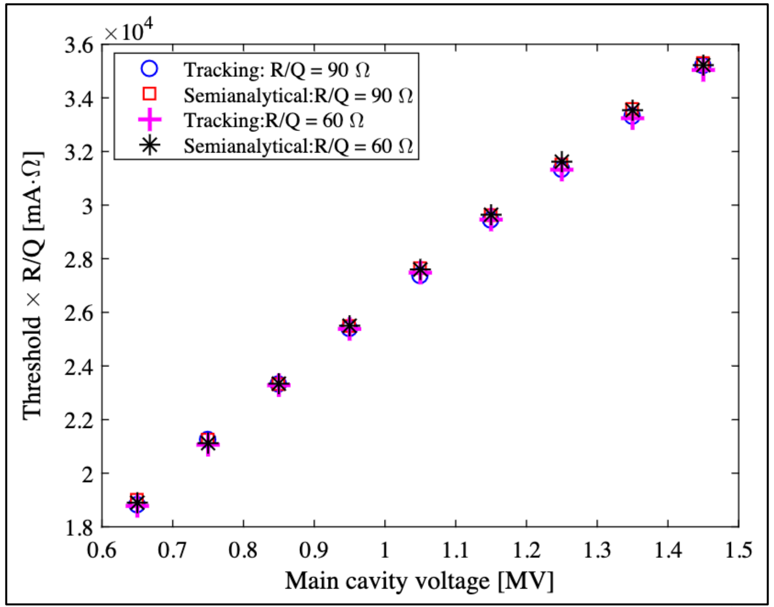


259 mA is the reported threshold in [1]

3% difference between the results

LMCI vs. Tracking/Semi-analytical for HALF



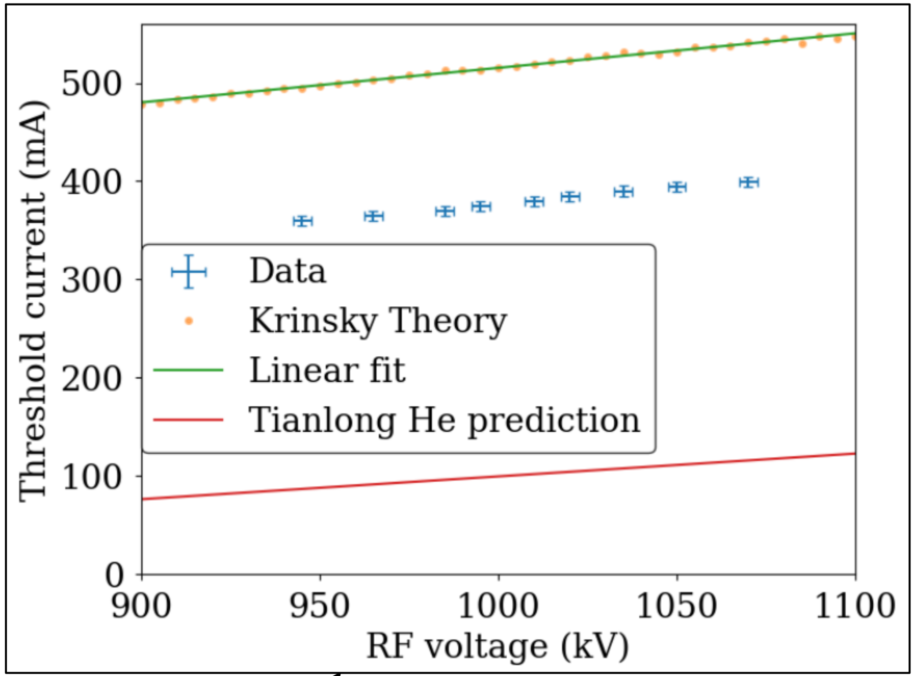


Simulated data from
T. He, et al. PRAB **25** 024401 (2022)
Results from STABLE macroparticle tracking code and semi-analytical method

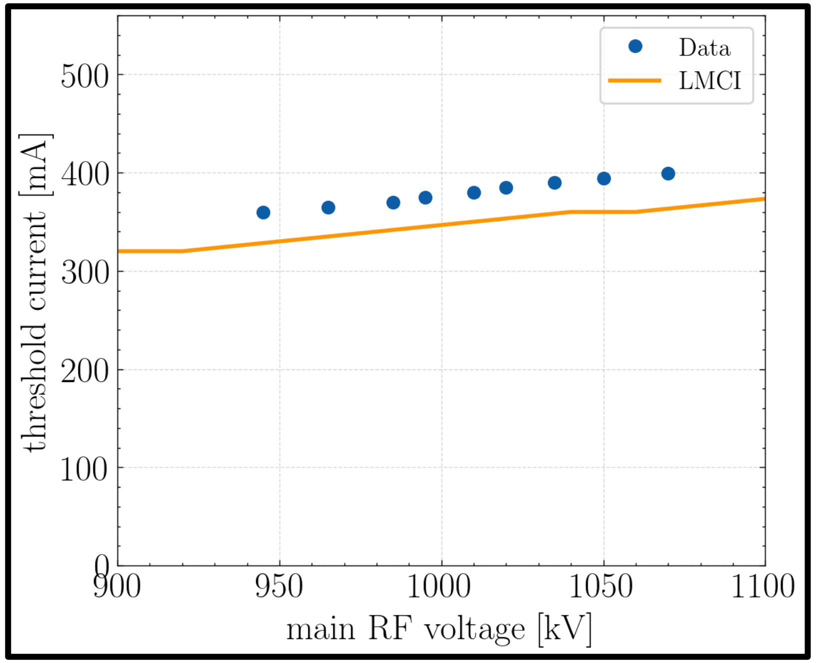
Formula (Eq. 24) from
T. He, PRAB **25** 094402 (2022)

LMCI vs. Experimental data from MAX-IV

- 2 HCs tuned to flat potential voltage

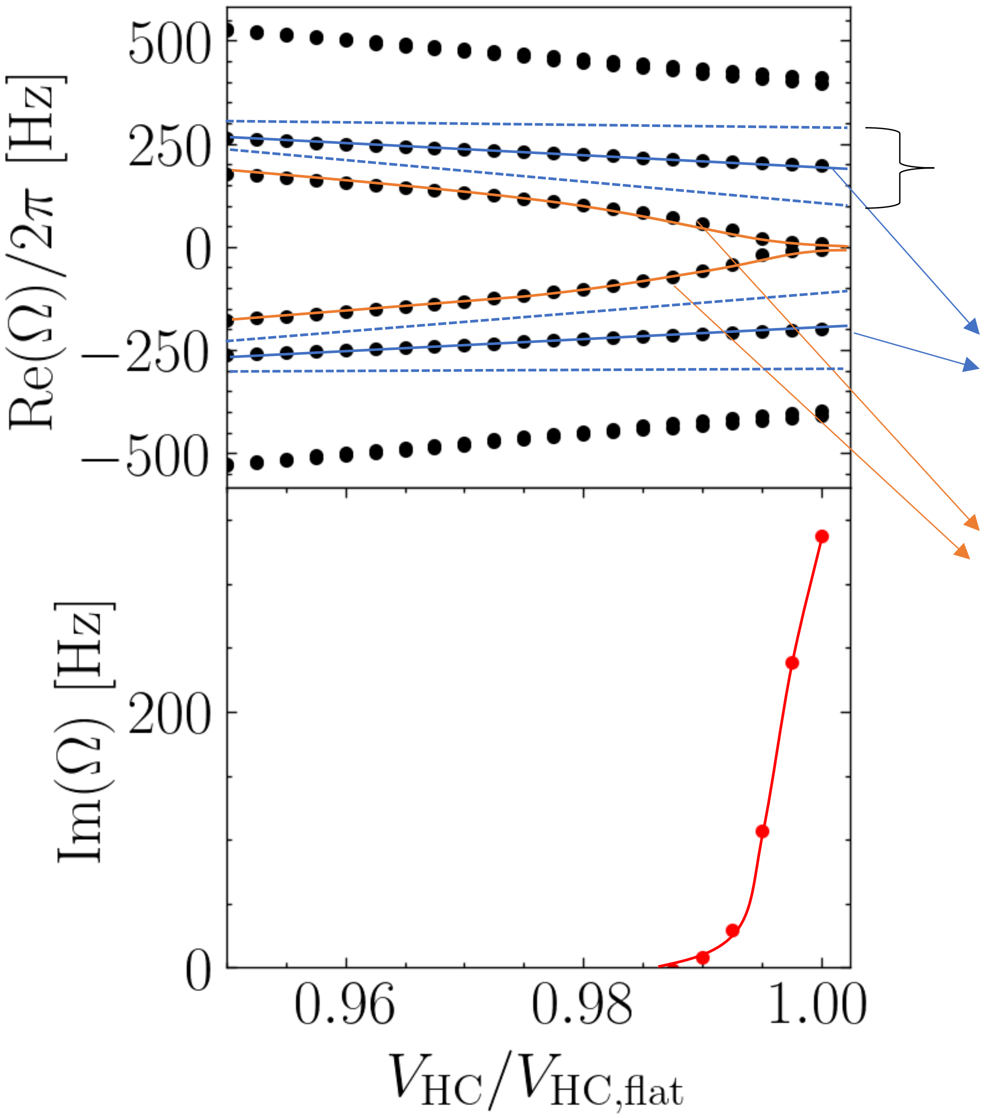


F. Cullinan, et al - Mode 1 Instabilities in Fourth Generation Storage Rings - I.FAST 9th Low Emittance Rings Workshop (2024)



Linear LMCI result agrees with measured values with difference < 10%

Instability mechanism



The hypothesis of non-overlapping coherent and incoherent frequencies is verified

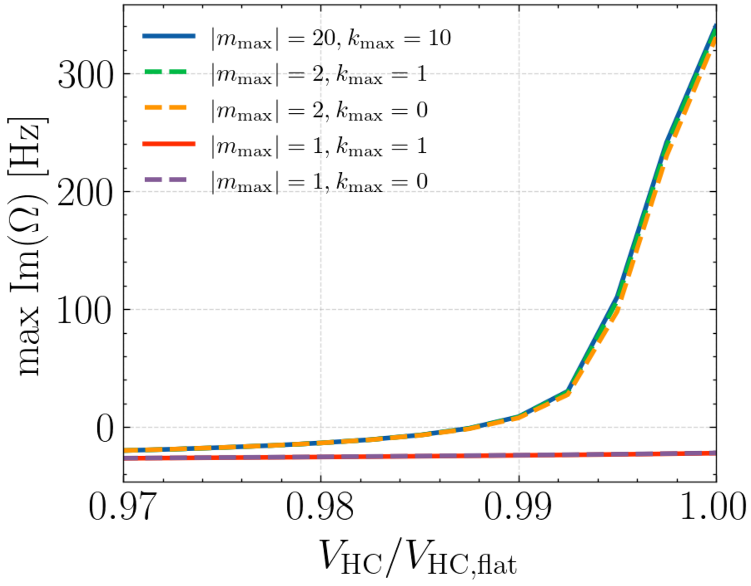
Spread of incoherent synchrotron frequency

Radial modes following the incoherent frequency $\pm\omega_s$

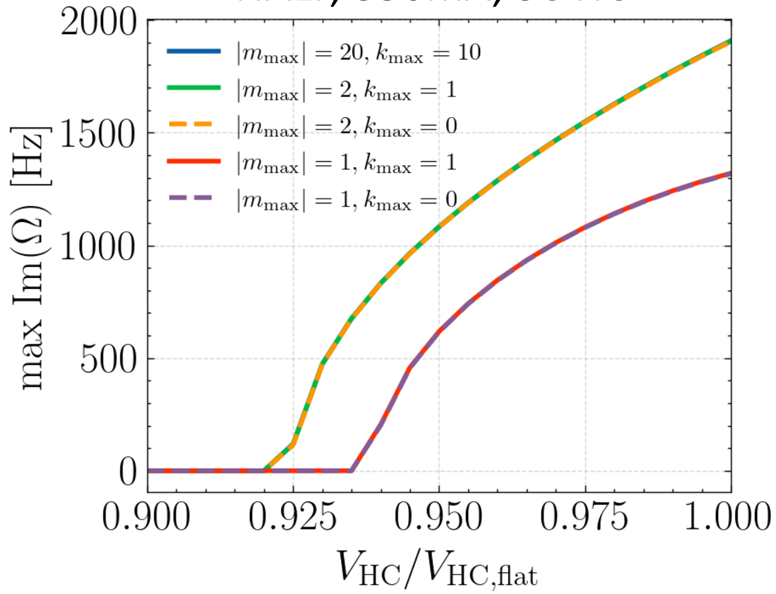
Radial modes with additional shift $\omega_s - \Delta\Omega$ induced by the imaginary part of the HC impedance

Low dipole coherent frequency for mode 1,
very weak longitudinal focusing, instability builds up

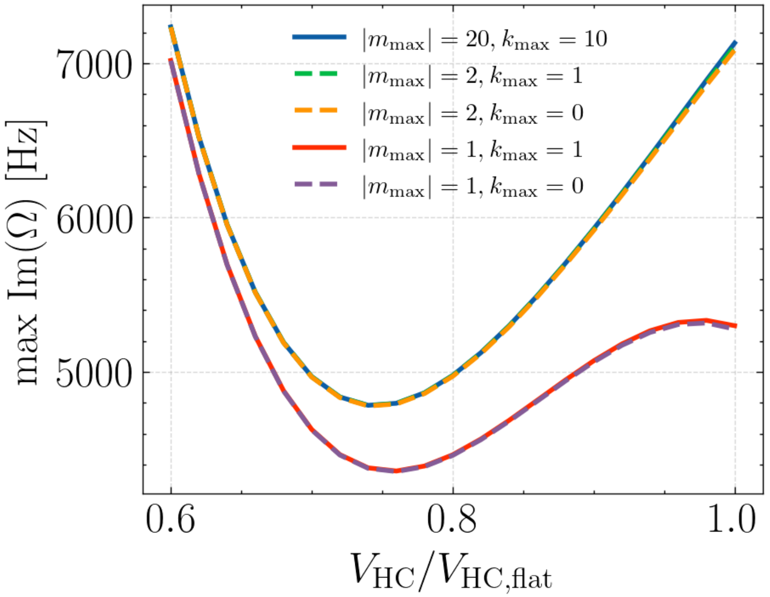
Tests of convergence


MAX-IV, 400mA, NC HC

Only |m|=1 is insufficient
|m|=2 required for instability


HALF, 350mA, SC HC

|m|=1 is sufficient
|m|=2 required for accuracy


ALS-U, 500mA, NC HC

|m|=1 is sufficient for instability
|m|=2 not very relevant

Maybe this explains why theories that consider only |m|=1 terms might fail in accurately predicting the mode 1 instability

Summary

- Tracking with one macroparticle predict mode 1 instability → simple mechanism
- Linear theory of mode-coupling with HC-modified parameters also predicts mode 1 instability. Good agreement with macroparticle tracking and experimental data.
- The mode 1 instability builds up when the coherent dipole frequency is too low (weak focusing)
  - Harmonic cavity → lower incoherent synchrotron frequency and longer bunch
  - Imaginary part of the impedance → negative shift to the coherent frequency
- This mechanism is insightful to understand the dependences of mode 1 instability/PTBL
  - Main RF cavity voltage
  - HC R/Q and detuning
  - Momentum compaction and energy spread
- A simplified theory that neglects non-linearity/Landau damping still provided useful information for the mode 1 instability with HCs. Results rely on the definition of a fictitious single-rf system that represents a double-rf system. Further studies to explore this approach are required.

Thank you for your attention

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